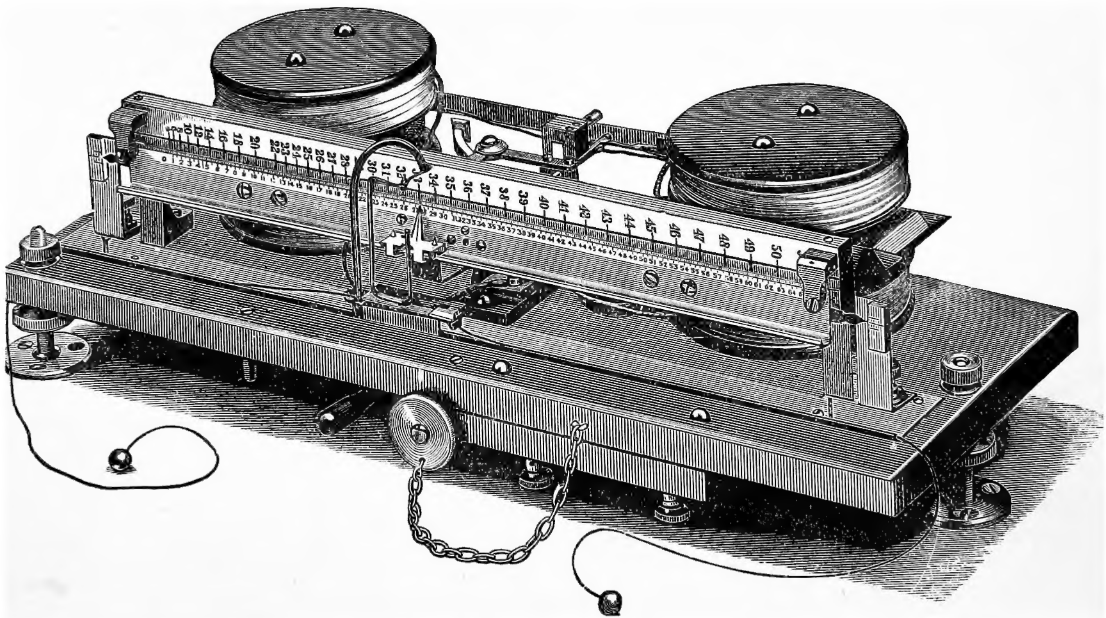
MAGNETISM AND ELECTRICITY



Kelvin Centiampere Current Balance. (See p. 383.)

MAGNETISM

AND

ELECTRICITY

AND THE PRINCIPLES OF ELECTRICAL MEASUREMENT

BY

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PREFACE

The object of this book is to provide a sound and systematic course of study in the main principles of Electricity and Electrical Measurement. It is written to meet the requirements of students who have already, in a first year's course, made some acquaintance with the elementary descriptive parts of the subject, and are commencing the study of Electricity in its more quantitative aspects, either as a branch of pure Physics or as a preparation for a course of Applied Electricity.

In the choice of subject-matter regard has been paid to the demand made on the student's knowledge of auxiliary and correlative subjects. Most students obtain, in conjunction with their first year's course, a knowledge of the elements of Algebra, Mechanics, and Heat; such preparation has therefore been assumed. On the other hand, the student cannot at this stage be expected to have acquired familiarity with the principles of Thermodynamics, the Theory of Wave Propagation, or the Kinetic Theory of Gases. Portions of the subject requiring a knowledge of these or similar branches of higher Physics have therefore been strictly excluded. By this means, also, a more satisfactory treatment of the main principles, and a more adequate explanation of points of special difficulty, have been made possible.

For convenience of reference a brief résumé of the fundamental properties of magnets, electric charges, and currents is given in Chapters I, VIII, and XVIII. A few longer or more difficult propositions, a digression

into which would be apt to divert attention from the main line of reasoning, are given in the Appendix.

Where a choice of experimental methods has arisen, I have selected those which illustrate the principles in question most directly, and at the same time logically. The consideration of elaborate methods adopted to ensure the highest experimental accuracy is best left for a later period of study.

The book will be found suited to the requirements of students preparing for the Board of Education Examinations, Stage II (Day and Evening), the University Intermediate (Pass and Honours), or other examinations of a similar standard. From the first-named source, with kind permission of the Controller of H.M. Stationery Office, a large number of the questions and examples have been drawn.

I am indebted to the courtesy of the publishers for the use of a number of illustrations from *Modern Electric Practice* and some from Dr. Draper's *Heat*; also to the proprietors of the *Electrician*, the *Philosophical Transactions*, and *Wiedemann's Annalen* for permission to reproduce diagrams, and to the following firms who have kindly lent blocks:—Messrs. Kelvin & James White, Ltd., Glasgow; W. & J. George, Ltd., London; Nalder Bros. & Co., London; W. G. Pye & Co., Cambridge; The Union Electric Co., Ltd., London; and the Cambridge Scientific Instrument Co., Ltd. The remaining 184 illustrations are from original drawings.

My thanks are also due to Mr. J. A. Cooper, B.Sc., Mr. A. Dennison, and Mr. A. Ward, B.Sc., for some assistance during the preparation of the work, and for verifying the numerical results.

S. S. R.

LIVERPOOL, *January* 1908.

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MAGNETISM AND ELECTRICITY

PART I.—MAGNETISM

CHAPTER I

FUNDAMENTAL PHENOMENA

1. Magnets, Natural and Artificial.

One of the ores from which iron is smelted is sometimes found to possess a remarkable power of attracting iron. These somewhat rare specimens of the ore are termed *lodestones.* The ore itself—whether possessing this property or not—is called magnetite; it is composed chiefly of the black oxide of iron (Fe_3O_4). The attracting power is more marked at certain points of the lodestone than at others. If the lodestone is dipped into iron filings and then withdrawn, the filings will be found attached to the specimen in dense clusters at two or more points. Powdered magnetite may be used in place of filings.

If a strip of steel, a knitting-needle say, is drawn several times in the same direction over one of the points of the lodestone where the attraction is strongly marked, it will acquire a like property. The strip so treated and the lodestone are examples of *magnets*, and this term is applied to all bodies which possess a special power of attracting iron and small particles of their own material.

Lodestones are *natural* magnets, and the piece of steel treated as above is an *artificial* one. Artificial magnets are now gener-

ally made with the aid of electric currents, as they can by this means be rendered very powerful. The usual forms are the "bar" and the "horse-shoe"; the horse-shoe magnet being preferred where both ends are required to act on the same object.

The portions of the surface of a magnet to which neighbouring particles of iron are attracted are termed the polar surfaces.

It is found that a magnet, however constructed, has *at least* **two** polar surfaces. If there are more than two, the magnet is termed *complex*; if there are two only, the magnet is a *simple* one. The polar surfaces are for brevity termed the **poles.**

Materials which a magnet can attract are called magnetic substances.

The substances distinctly magnetic are comparatively few in number, namely: iron, steel, nickel, cobalt, chromium, manganese, and magnetite. On other substances a magnet has little or no action. (See Arts. 66, 76.) In accordance with our definition, a magnet must always be formed of a magnetic substance.

2. Orientation of Suspended Magnets.

Suspend a simple bar magnet, with poles at its ends, by a bundle of unspun fibres of silk or hemp. To do this conveniently a small stirrup of copper wire may be attached to the magnet at the centre. The magnet will oscillate and finally assume a direction nearly north and south. If the magnet is reversed end for end, it will swing back so that the same pole always points to the north.

This experiment indicates a difference in the nature of the poles: they are named accordingly N.-seeking and S.-seeking, or briefly, N. and S. poles respectively.

The following experiment shows that, in the case of a complex magnet, the setting is definitely related to the N.–S. direction.

Suspend a lodestone in a copper frame by unspun fibres. Mark the portions facing N., E., S., W. respectively with chalk. Now

invert the lodestone in the frame and allow it to come to rest again. You will find that the same portions as before face N. and S. respectively, but those facing E. and W. have changed places.

3. Symmetrical Magnets.

We shall use this term to denote straight bars or strips of steel which are symmetrical both in form and in respect of magnetic properties. The centre line or axis of figure will then form a *magnetic axis*. The symmetry of a magnet may be roughly tested as follows:—

(i) Dip the magnet in fine ground iron and notice the distribution of the attracted mass over the surface (fig. 1).

(ii) Suspend the magnet and observe the direction in which the geometrical axis sets. Invert, and notice if the axis assumes the same direction.

(iii) Place a card over the magnet. Sift iron filings over the card, tapping the latter gently. The filings should arrange themselves in curves symmetrical about the axis.

In fig. 1 (*a*) is a symmetrical simple magnet, (*b*) a symmetrical complex magnet, (*c*) is unsymmetrical and simple.

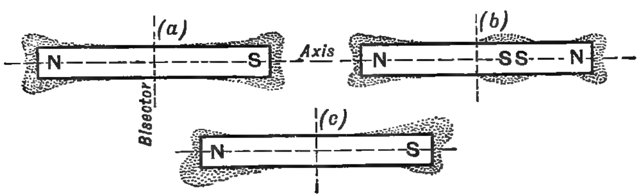


Fig. 1.—Attraction for Iron

Unsymmetrical magnets possess a magnetic axis, but this does not in general coincide with the axis of figure (Chap. III).

If a symmetrical simple magnet is magnetized as regularly as possible, it attracts iron equally at all corresponding points of its surface equidistant from its centre. It may then be said to be *magnetically balanced* (fig. 1 (*a*)).

We shall, in what follows, always suppose the magnets to be of this kind unless the contrary is expressly stated.

4. Magnetic Meridians.

When a magnet is suspended it does not come to rest in an exactly N.–S. direction at most places; but the departure from the geographical meridian is generally less than 30°.

> **Vertical planes parallel to the direction assumed by the axis of a suspended magnet are termed magnetic meridians.**
>
> **Horizontal lines in these planes are called magnetic meridian lines.**
>
> **The angle between the geographical and magnetic meridians is called the declination.**

(In Navigation this is called "variation" of the needle to avoid confusion with astronomical "declination".) In England at the present time the declination is about 19°.

A *compass-needle*, as used in magnetic experiments, is a symmetrical simple magnet, of a shape which combines lightness with rigidity. It is best balanced on a hard steel point by means of an agate cup fitted through its centre.

5. Mutual Action of Poles.

Bring the N. pole of a magnet near the N. pole of a compass-needle. Repulsion occurs. Next bring the S. pole of the magnet near the S. pole of the needle. Repulsion again occurs. Now hold the N. pole of the magnet near the S. pole of the needle. This time the result is attraction. These results may be briefly summed up thus:—

> **Like poles repel, unlike poles attract.**

In accordance with the general law, "action and reaction are equal and oppositely directed" (Newton's Third Law), the forces between magnet poles must be mutual. A strong pole attracts a weak one with the same force that the weak pole exerts on the strong one. The same is true of a magnet acting on a piece of iron.

We may use the above rule to distinguish the N. and S. polar surfaces of a complex magnet like lodestone. If you move a small compass-needle (a "charm" compass will do

excellently) all round a lodestone, and use the above law, you will easily detect the polarity of the different portions of the surface. The polar surfaces tested in this way will be found to occupy the whole surface of the magnet (though the attraction of ground iron is scarcely noticeable in places). Boundary lines may be marked on the surface of the magnet, dividing the N. polar parts from those of S. polarity; these are termed **neutral lines.**

If two simple magnets which attract iron equally are placed with their N. and S. poles in contact, or close proximity, the combination exerts little or no force on a piece of iron. We therefore say that *equal unlike poles neutralize each other's powers.*

The force which one magnet exerts on another diminishes rapidly as the distance between the magnets increases.

6. Theory of Molecular Magnets.

Break a simple bar magnet, say a magnetized strip of watch-spring, into two pieces. You will find that each piece is a complete magnet, with poles nearly as strong as those of the original piece. New poles are developed at the broken edges so that each piece has N. and S. poles. Each piece may be broken again and again with the same result.

The process seems only limited by our limited means of mechanical division. This result has given rise to the theory that "the molecules of a magnet are themselves perfect magnets".

Further, when we magnetize a bar of steel by stroking it with a magnet, the latter does not lose its power in the process. Magnetization is therefore not regarded as a transference of anything from one body to the other. It is considered rather as the *development* of a condition *latent* in the magnetic substance. It is supposed that the molecules of a piece of neutral iron or steel are already magnetized. The neutral behaviour of the whole piece is explained by supposing that the molecules are arranged indiscriminately; on the average as many N. poles point in a given direction as S. poles. (See

fig. 2 (*a*), where the *axes* of the molecules are indicated by straight lines.)

When we stroke the N. pole of a magnet along a bar of neutral steel, the S. poles of the molecules tend to follow the magnet. The axes of the molecules thus acquire a general direction parallel to the bar, and the latter shows polarity (fig. 2 (*b*)). This also agrees with the experimental result that the end of the bar which leaves the magnet last is of unlike polarity to the pole used for magnetizing the bar.

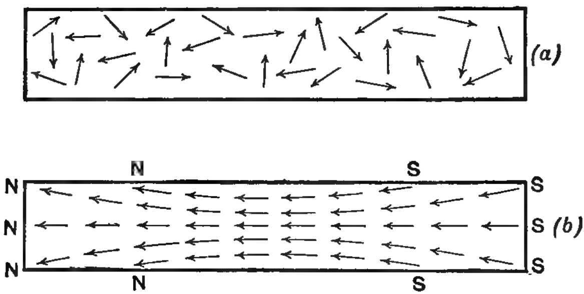


Fig. 2.—Molecular Theory

The magnet so formed, when magnetized as strongly as possible, has its molecules arranged so that their axes form long "open" chains terminating on the polar surfaces of the magnet. Forces tending to produce this regular arrangement may be termed *magnetizing forces*; those tending to upset it, *demagnetizing forces*.

7. Coercive Power.

A steel magnet retains its power, although in a somewhat diminished degree, when subjected to shocks, vibration, moderate heat, and other influences which tend to produce disturbances among the molecules. To account for this we must suppose that internal forces are brought into play which prevent the rotation of the molecules and thus preserve the open-chain arrangement. This resistance to molecular rota-

tion will evidently oppose the magnetizing as well as the demagnetizing process.

We therefore make the following definition:—

> **The coercive power of a substance is its capacity of resisting magnetization and demagnetization.**

A quantitative meaning is assigned to this in Art. 60.

It is difficult to magnetize steel, and when it is once well magnetized it is difficult to render it neutral. On the other hand, iron is readily magnetized, but loses its polarity almost immediately the magnetizing influence is removed. Hence we say that steel possesses a large amount of coercive power, whilst iron has very little.

Coercive power was originally attributed to molecular friction. Prof. Ewing has, however, shown that the phenomena associated with coercive power may be satisfactorily explained by the *magnetic* attractions and repulsions between the poles of neighbouring molecules. (See Art. 59.) It is therefore unnecessary to assume the existence of a purely mechanical resistance to the rotation of the molecules.

The property which we here term coercive power is in different works variously referred to as "coercive force", "coercive capacity", "coercivity", "retentivity", and "molecular rigidity". The oldest name is "coercive force".

Observe that the two following causes *assist* either magnetizing or demagnetizing influences:—

(*a*) Shocks and vibrations of all kinds;
(*b*) Rise of temperature (if not too great).

The coercive power of the substance is therefore called into play against these influences as well as against magnetizing or demagnetizing force.

If steel or iron is raised to a bright red heat it ceases to be magnetic. A magnet therefore cannot attract a piece of red-hot iron. The same is true of other magnetic substances.

8. Magnetic Induction.

We have already seen that the magnetization of a bar by stroking processes can be explained as due to a rearrangement of the molecules. Bearing this in mind, we may conveniently distinguish the two following effects of a magnet on a piece of iron (or steel):—

(i) The production of a definite molecular arrangement in the iron;

(ii) The forces exerted on the iron as a result of this arrangement.

The process of arranging the molecules, or developing polarity, is called *magnetic induction*, and we speak of the magnetization as being *induced* in the iron. The use of this term calls attention to the fact that the magnetic condition is developed in the iron, not "imparted" to it.

But magnetization can be induced without the stroking process. Hold a piece of steel, say two inches of thick pianoforte wire, in contact with one pole of a strong magnet (or very near it). Dip the other end in iron filings, and remove the wire and magnet together. A small bunch of filings remains attached to the wire. Think this over in connection with the molecular theory. The unlike poles of the molecules are attracted towards the magnet pole, so arranging the axes in line. At the end of the wire the molecules all present a pole of the same kind, and the iron is attracted by their joint action.

The induced pole at the end of the wire nearest the magnet is of opposite nature to the inducing pole. The attraction exerted on this near end is greater than the repulsion exerted on the distant end; hence the net result is attraction. In this and similar instances it is evident that *attraction accompanies induction.* In the phrase, "a magnet attracts neutral iron", the term "neutral" must be understood to refer to the *previous* condition of the iron. Both attracted and attracting bodies are magnets for the time being.

Magnetic influence can act *through* all substances. If you

hold a sheet of cardboard, or copper, or any non-magnetic substance, between a magnet and a compass-needle, the latter is deflected; and the deflection is as great as if nothing intervened. If the substance interposed is magnetic, it becomes magnetized by induction, and the effects become more complicated. (See Chap. VII.)

9. Susceptibility.

Instead of using steel wire in the experiment described above, use a piece of iron wire of the same size on the same part of the magnet. You will find a much larger bunch of filings attracted.

Iron is therefore said to have a greater *magnetic susceptibility* than steel; stronger poles are induced in it under equal conditions. (A quantitative meaning is assigned to this term in Art. 65.)

Non-magnetic substances possess no susceptibility.

We may compare the susceptibilities of equal pieces of iron and steel by placing them at equal distances on each side of the pole of a long compass-needle (fig. 3). The pole moves towards the more susceptible piece.

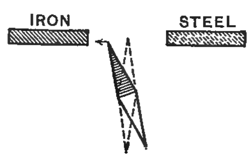


Fig. 3.—Susceptibility

If we make use of the powerful magnetic force of a strong electric current, we find that a given specimen of iron or steel cannot be magnetized beyond a certain limit. When this limit is reached the substance is said to be *saturated.* The degree of magnetization at the saturation-point depends on the chemical composition and physical condition of the steel or iron specimen.

10. Temporary Magnetization.

In the experiment with iron and steel wires described above, you will have noticed that, on removing the magnet, the filings nearly all drop away from the iron wire. This is very noticeable with a rod of soft iron, *e.g.* a key. With a steel rod you will notice that the filings mostly remain attached. The

magnetization is in both cases to some extent *temporary*; in the iron it is almost entirely so.

If you attempt to magnetize a bar of very soft iron by the ordinary stroking process you do not succeed. Whilst the inducing pole is being drawn along the bar the latter is strongly magnetized; but every time the pole is lifted the magnetization induced disappears. In a steel bar, on the other hand, a considerable proportion of the magnetization induced remains. The strength of the poles induced increases, until the additional power gained during a stroke is balanced by the loss on removal of the inducing magnet.

The proportion of the whole magnetization, which we must regard as temporary, varies greatly. Even with a given substance it varies with the shape of the specimen and the conditions under which it is placed. The properties of magnetic materials can only be fully investigated with the aid of electric currents. But the following distinctions should be here carefully observed:—

(i) When no special care is taken to protect the substance from ordinary demagnetizing influences the amount of retained magnetization depends mainly on coercive power. This applies to the rods, magnets, needles, etc., in common use. If they are to remain magnetized for long periods they must be made of a material having great coercive power. Hard steel is generally used. Its retentive capacity may be increased by alloying it with 5 to 10 per cent of the metal tungsten. The term *permanent* magnetization is used, in a relative sense, to denote the magnetic condition which can only be destroyed by the application of a demagnetizing force or long-continued vibration.

(ii) But even if we take care to protect a magnet from vibration or heating, there is still a demagnetizing influence arising from the poles. The nature of this will be perceived from fig. 2 (*b*). Notice that the "free" poles at the end of the bar attract or repel the poles of a molecule near the middle of the bar. These forces tend to reverse the molecules near the middle, and so upset the "chain" arrangement. This

polar demagnetizing force is assisted by vibration and opposed by coercive power, and it is stronger in short bars than in long ones.

You may easily show by using iron filings that a soft-iron rod loses nearly all its magnetization when the inducing magnet is removed, *even if kept quite still.* This loss occurs because the coercive power of soft iron is too weak to withstand the polar or self-demagnetizing force until the poles have been much weakened. The effect is very noticeable in short bars.

Short magnets are subject to a stronger polar force than long ones, and should be made of the hardest steel.

(iii) It is interesting to enquire whether iron would lose any magnetization if it were protected from all demagnetizing influences, including polar force. This can be done by the use of iron rings and the application of electric currents. It has been shown that in all cases *some* magnetization disappears when the inducing influence is removed, but a large percentage is retained. Soft iron retains from 80 to 90 per cent of the magnetization induced. The amount so retained is called the *remanence*; the portion which disappears has been termed the *evanescent* magnetization. Thus for practical purposes we may roughly divide the total magnetization induced into three portions in order of increasing stability:—

(*a*) *Evanescent* magnetization, which disappears on removal of the magnetizing force, and without the aid of any demagnetizing influence whatever.

(*b*) *Sub-permanent* magnetization, which is destroyed by the polar force of the specimen, or by vibration of moderate duration and intensity.

(*c*) *Permanent* magnetization, as defined in (i) above.

The remanence evidently consists of (*b*) and (*c*) together. The temporary magnetization consists of (*a*) and (*b*).

The residual magnetization is usually less than the remanence and greater than the permanent magnetization; it depends on the proportion of sub-permanent magnetization which the experimental conditions leave unneutralized.

Most iron rods, plates, and wires will be found slightly magnetized when tested with a compass-needle. They possess some coercive power owing to the hardening effects of the rolling or wire-drawing process they have undergone. Again, some kinds of mild steel possess very little coercive power. When we make a distinction between *steel* and *iron* for theoretical purposes, we refer to very hard steel and the softest annealed iron.

11. Tests for Polarity.

The iron-filings test will only succeed with strong magnetization, and gives no indication of the nature of the poles. But the existence of polarity, and its nature, may always be detected with a compass-needle.

Bring one end of the specimen to be tested near one pole of a compass-needle. Repulsion indicates at once the existence of polarity and its nature. Attraction, however, may be due to the inductive action of the needle on the specimen. In this case, test the same end of the specimen with the other end of the needle. If attraction occurs at this end also it indicates that the material is neutral. Do not try to make this test by reversing the rod or other specimen near one end of the needle; attraction with both ends might in this case be due to consequent poles (Art. 12).

If the body to be tested is of soft iron and likely to be weakly magnetized, bring it gradually near to the needle, from a distance, and watch carefully for the *first* effect produced. (Quite close to the needle the weak polarity might be overcome by the inductive action of the needle.) Also hold the specimen at right angles to the magnetic meridian to avoid the inductive action of the earth.

12. Methods of Magnetization.

The steel used for making short magnets, say of length not more than six times their breadth, should be first rendered "glass-hard". This is done by first heating the metal to redness and then suddenly plunging it into cold water, or better, into mercury, which is a good conductor of heat. The sudden cooling makes the steel very hard and brittle. For longer magnets the steel may be "let down" or "tempered". The

tempering is done by heating bright glass-hard steel until it assumes the yellowish-brown or blue tint generally seen on watch-springs. The blue tint is attained at a higher temperature than the yellow. Glass-hard steel is brittle, and has great coercive power, but tempered steel is more or less elastic (*e.g.* watch- and clock-springs), and possesses less coercive power. (If the steel is heated to a bright red heat and allowed to cool very slowly it becomes softened or annealed. It is then very tough but inelastic, and is not suitable for permanent magnets.)

The following are two methods of magnetization in common use:—

(*a*) *Single Touch.*—Hold a knitting-needle between the thumb and first finger and draw it ten or twelve times across the pole of a magnet, always in one direction. Dip it into iron filings. They will cluster at one end, but at the other—which was held in the hand—they will be thinly distributed. The needle is symmetrical but not "magnetically balanced" about its centre (Art. 3). Now hold the needle by the opposite end and draw it an equal number of times over the other pole of the magnet. On testing the needle again with filings it will be found that they are equally distributed at the ends, and a "well-balanced" magnet is obtained.

Thicker bars of steel may be magnetized in this way, but a horse-shoe electro-magnet should be used. The bar may be moved forwards across one pole of the horse-shoe and backwards across the other.

(*b*) *Divided or Separate Touch.*—This method yields very symmetrical magnets, and is useful where there is an obstruction at the centre, as in compass-needles.

Place the N. and S. poles of two equal bar magnets just on each side of the centre of the needle (which should be placed on a block of wood, a groove being cut just large enough to receive it). Draw the poles apart until they reach the ends. Lift the magnets, replace them at the centre as before, and repeat the stroke some fifteen times.

If portions only of a steel bar are stroked they become independently magnetized. If adjacent parts are stroked in opposite directions with the same pole of a magnet, then like poles are induced in the two portions at the common boundary,

These are called *consecutive* or *consequent* poles (fig. 1 (*b*)). Very complicated magnets may be produced in this way. The molecular axes in adjacent portions point in opposite directions. Consequent poles are generally objectionable in magnets.

If we stroke a thick bar of steel with a magnet we only succeed in magnetizing the *outer layers*. These act as a kind of shield for the inner layers. But if an electric current is used the bar becomes magnetized throughout. A coil of thick insulated wire should be wound in a close spiral round the bar or horse-shoe: a current may then be sent round the coil strong enough to saturate the steel. This is the most satisfactory way of magnetizing fairly thick bars of steel.

Powerful permanent magnets are sometimes made in a *laminated* form. That is, a number of pieces of steel of bar or horse-shoe shape are first separately magnetized and are then screwed together with their like poles in contact, so forming a very strong magnet. When the magnet becomes weakened with time the parts may be unscrewed and conveniently remagnetized. Such magnets are known as *lamellar* or *compound*.

The retentive capacity of a magnet is increased by dipping it once or twice into boiling water immediately after magnetization. This is called *ageing* the magnet. It destroys a little of the magnetization, but the remainder is more permanent.

The retentive capacity is also increased by providing the magnets with *keepers*. These are soft-iron bars placed across the poles. One keeper is required for a horse-shoe magnet, but bar magnets are used in pairs, with unlike poles adjacent, and two keepers are used. The polarity induced in the keeper neutralizes the polar force of the magnet (Art. 10 (ii)).

13. The Earth as a Magnet.

A compass-needle has perfect freedom of movement in a horizontal plane only, and does not fully indicate the earth's magnetic influence. This can be ascertained by supporting a needle so that it can freely assume *any* direction. This may be done with the arrangement shown in fig. 4. The

horse-shoe-shaped strip is suspended by unspun silk thread. The axle of the needle rolls on two agate "knife edges" attached to the horse-shoe. The needle must be carefully balanced, before it is magnetized, by filing its heavier end. It must then be magnetized by separate touch and resuspended.

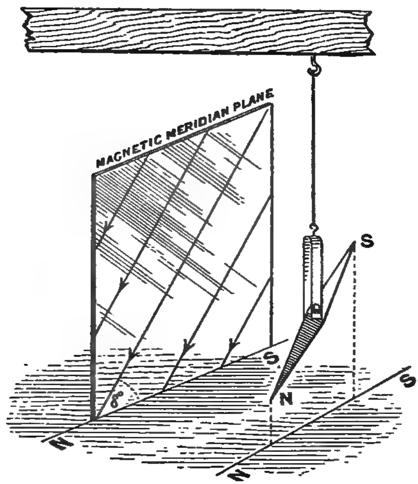


Fig. 4.—Meridian Plane. Lines of Dip

It will be found that the needle sets in the magnetic meridian, and is *inclined to the horizontal.*

The inclination assumed by a freely suspended magnetized needle is called the angle of dip.

This varies, though somewhat irregularly, with the latitude of the place of observation. Speaking generally, we may say that the N. pole dips in the northern hemisphere and the S. pole in the southern hemisphere, and that the angle increases as we travel from the equator towards either pole. The dip is 90° at two places situated comparatively near the N. and S. geographical poles. These places are termed the "N. and S. magnetic poles of the earth" respectively. The

dip is 0° at places situated on a line termed the "magnetic equator". One half of this runs a little north of the geographical equator and the other half a little south of it.

Lines of dip at any place are lines parallel to the direction assumed by the axis of a freely suspended needle.

They lie in the magnetic meridian planes, and are inclined at the angle of dip (fig. 4).

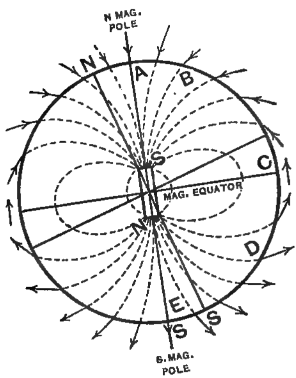


Fig. 5.—The Earth as a Magnet

The magnetic properties of the earth can be *very roughly* represented by a comparatively short magnet placed at the centre of the earth, its axis being inclined to the earth's axis of rotation (fig. 5). The existence of such a magnet or magnetized mass in the earth's interior is of course quite imaginary. The internal temperature of the earth is probably far above that at which known substances retain their magnetization. The earth's magnetic action may be due to currents of electricity flowing round it, at or near the surface. The notion of an internal magnet is, however, a useful one, by which the main facts may be remembered.

CHAPTER II

MAGNETISM—MAGNETIC FORCE

14. Magnetism.

We now proceed to the consideration of magnetic measurements. As a guide in this part of our subject we find it necessary to adopt some definite theory as to the origin of

magnetic actions. In order to fix our ideas, we imagine that the surface of a magnet is, as it were, coated with an invisible "substance" which we term *magnetism.* We further imagine that there are two kinds of magnetism which coat the north and south polar surfaces of a magnet and are named accordingly. It is then possible to think of the attractions and repulsions observed as being exerted on the magnetism or produced by it.

These forces are subject to the two following laws:—

> I. **Like kinds of magnetism repel each other, unlike kinds attract, the forces being always equal and oppositely directed.**
>
> II. **The force exerted between two particles of magnetism is inversely proportional to the square of their distance apart.**

These may be termed the first and second laws of magnetism respectively. The simplest expression satisfying the first law is (in symbols)—

$$f \propto mm',$$

where f is the force, and m, m' are defined as quantities of magnetism. The value of m is taken positive for N. magnetism and negative for S. magnetism, and thus a positive value of f denotes repulsion and a negative value attraction.

The second law may be expressed—

$$f \propto \frac{1}{d^2},$$

and is known as the **Law of Inverse Squares.**

The two relations may be expressed in one formula, thus—

$$f \propto \frac{mm'}{d^2}.$$

This formula (sometimes called Coulomb's Law) forms the starting-point in magnetic calculations; but we must prove by experiment that the laws it expresses are true for actual magnets. Experiments leading to the first law have been

considered in Art. 5. It remains to test the truth of the second—the law of inverse squares. We shall first describe a simple but efficient apparatus devised by Mr. W. Hibbert for measuring the force between two magnetic poles.

15. Hibbert's Magnetic Balance.

A thin steel knitting-needle AB (fig. 6) about 20 cm. long is magnetized and carefully balanced at the centre on a "knife edge" (or suspended at the same point in a cradle by means of silk fibres). A second magnetized needle is now placed with one of its pole centres above or below A, so that A is urged in a downward direction by the repulsion or attraction. In order to measure this force

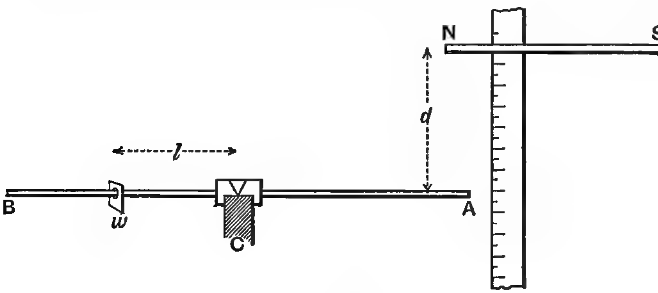


Fig. 6

a small weight w is hung on the left-hand arm and is moved along this arm until AB again becomes horizontal. The horizontal position is indicated by a mark on the fixed upright scale.

If f is the force (say repulsion) exerted on A by the second magnet, a the distance from the pole centre at A to the fulcrum C, w the small weight, and l the distance of this from the fulcrum, we have for equilibrium—

Moment of repulsion about C = moment of weight about C.

$$f \times a = w \times l.$$

If the second magnet is now moved nearer to A the repulsion increases (say to f_1), and A is driven downwards. To restore the balance we must slide w farther along CB, say to a distance l_1.

Then $$f_1 \times a = w \times l_1.$$

Therefore by division, we have—

$$\frac{f_1}{f} = \frac{l_1}{l}.$$

The forces are therefore proportional to the distances of the sliding weight from the fulcrum, and for purposes of comparison may be represented by those distances.

Fig. 7 shows a finished form of the balance.[1] The needle is supported on agate bearings and protected by a glass case. The sliding weight is moved along the beam by a lever B, whilst a lever A is used to lift the weight from the beam before moving it along.

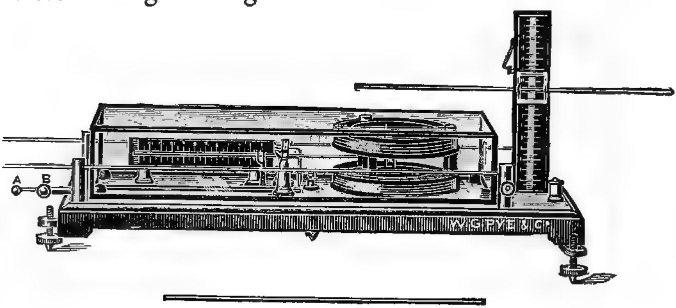


Fig. 7.—Magnetic Balance

To prove the law of inverse squares, place the second magnet at different heights on the vertical scale, keeping its pole exactly over the pole at A. Determine, for each distance between the poles, the position in which the sliding weight must be placed. Then, if the law of inverse squares is true, the product $f \times d^2$ should be constant. The following table shows the result of an experiment:—

Reading for f.	Reading for d.	$f \times d^2$.
7·55	11·66	1026
12·93	8·8	1001
10·0	10·1	1020
9·0	10·7	1030
8·0	11·3	1021
7·0	12·05	1016

[1] Patent, made by Messrs. W. G. Pye & Co., Cambridge.

The products $f \times d^2$ do not vary more than 1·7 per cent from their mean value, a variation which is well within the limits of experimental error in this class of experiment.

The balance may also be used for the absolute measurement of pole-strength, comparison of pole-strengths, absolute measurement of current, and comparison of currents. For current measurements two fixed coils are provided, as shown in fig. 7. For the details of these experiments see Hibbert's *Magnetism and its Elementary Measurement.*

Coulomb established the law of inverse squares by experiments with the torsion balance. This instrument, except in the hands of skilled experimenters, seldom yields satisfactory results. An accurate but indirect method of testing the law, due to Gauss, is given later (Art. 48).

16. Unit Quantity of Magnetism.

This is defined as follows:—

> **The unit quantity of magnetism is such that it repels an equal quantity at a distance of 1 cm. in air with a force of 1 dyne.**

This unit has no special name. When it is necessary to distinguish this from other units, it is called the C.G.S. electromagnetic unit of magnetism. By Coulomb's Law we have—

$$f \propto \frac{mm'}{d^2}.$$

$$\text{Therefore } f = (\text{a constant}) \times \frac{mm'}{d^2}.$$

If f is in dynes, d in centimetres, and m in terms of the unit defined above, then—

$$f = \frac{mm'}{d^2} \quad \text{.........................} \quad (1)$$

Examples.—1. A point S. pole of quantity 10 repels another at a distance of 5 cm. with a force of 3 dynes. Find the strength of the second pole.

Here $m = -10$, $d = 5$, $f = 3$.

$$\therefore 3 = \frac{-10 \times m}{25}, \text{ and } m = -7{\cdot}5 \text{ C.G.S. units.}$$

The negative sign represents a S. pole.

2. At what distance will a N. pole of strength 5 attract a pole strength 338 with a force of 10 dynes?

$$\text{Here } m = 5,\ m' = -338,\ f = -10.$$

$$\therefore\ -10 = \frac{5 \times (-338)}{d^2}, \text{ and } d = 13 \text{ cm.}$$

We can only use formula (1) when the magnetism is collected practically at *points.* In ordinary magnets the magnetism is spread over a polar surface of considerable extent. If two such magnets are placed near together, we cannot in general find the action of one on the other; for in such a case there would be an infinite number of forces to consider. We should have to take into account the forces exerted by *every* element of the polar surface of one magnet on *every* element of the polar surface of the other.

Ordinary magnetic calculations are therefore restricted to cases where the magnetism may be supposed collected at points fixed in the magnet. Before proceeding to these (Art. 21), we shall find it helpful to consider how magnetic effects are transmitted through space.

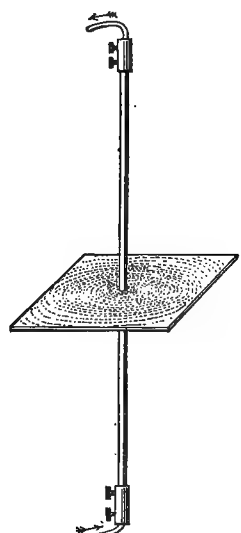


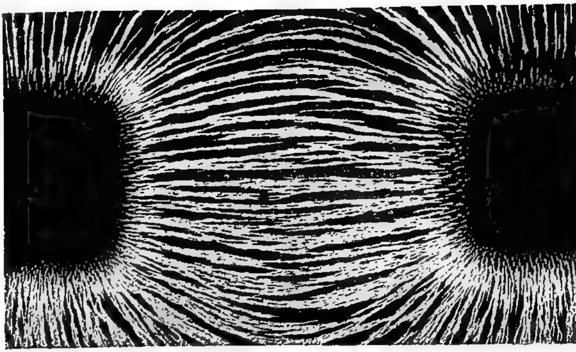
Fig. 8.—Field of Linear Current

17. Magnetic Field.

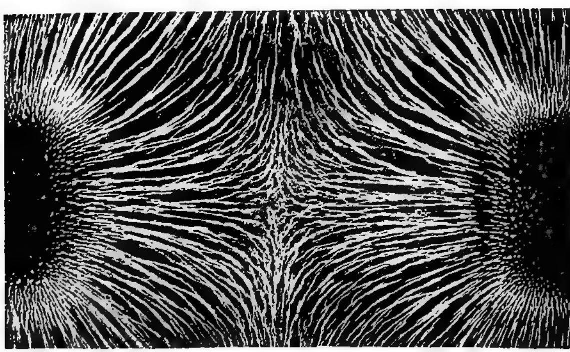
This term is applied to any region where a magnet is subjected to forces of the same nature as those which would be produced by a neighbouring magnet.

Such fields may readily be produced and varied by the agency of electric currents.

The whole field may be mapped out by *lines of force.* These indicate at every point the direction in which a small, freely-suspended compass-needle comes to rest. It is usual in practice to mark



(*a*) Unlike Poles



(*b*) Like Poles

Fig. 9

the lines with arrows, showing which way the N. pole of the compass points. Arrows for the S. pole would of course point in the opposite direction, but to avoid confusion they are not shown in the figures.

You may easily obtain a map of the horizontal field due to a magnet or current by the aid of a very short, delicately balanced compass-needle.

Place a sheet of cardboard in the region to be tested. Use a small "charm" compass, and move it from point to point in the

field. Note carefully the directions assumed by the needle, and mark these down on the card. Smooth curves may then be drawn following these directions. These are the horizontal lines of force.

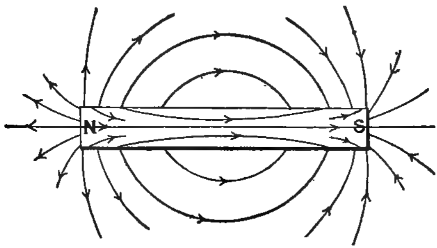


Fig. 10.—Polar Field of Bar Magnet

If the field is strong enough, a map may be readily obtained with iron filings.

Place a card horizontally over the magnet or near the current. Scatter fairly coarse *filings* (not powder) from a sieve somewhat sparingly over the card. Each filing becomes a magnet by induction, and you have thousands of "compass-needles" all over the card. The setting of the filings is assisted by gently tapping the card. Owing to friction with the latter, no definite arrangement of the filings is produced by weak fields—for example, by the earth's field.

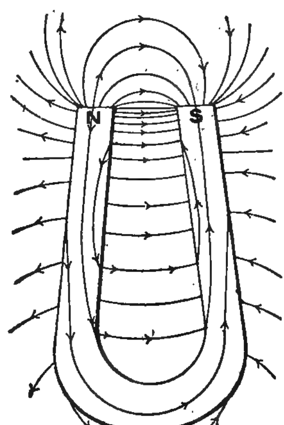


Fig. 11.—Polar Field—Horse-Shoe Magnet

Fig. 8 shows the method of obtaining filings figures for a current in a straight wire or rod, and fig. 9 shows the lines obtained (*a*) between unlike poles of two magnets, (*b*) between like poles. The lines are drawn for one plane only, but they radiate in all directions from the polar surfaces through the surrounding space. Figs. 10 and 11 show the lines of force for a bar magnet and horse-shoe magnet respectively.

18. The Ether.

Notice the difference in the figures for a pair of like poles and for unlike poles (fig. 9). The shape of the lines in (*b*) suggests repulsion, and the figure helps us to realize how one pole can push the other away through the agency of an invisible medium between them. Similarly (*a*) enables us to perceive how the two poles are pulled together by the agency of the medium. By a careful study of what happens in the medium between the poles, we may arrive at a physical *explanation* of attraction and repulsion. We must make a careful distinction between such explanations and the mere statement of natural laws by means of formulæ and equations. Thus equation (1) expresses the law of attraction and repulsion without explaining it. In the "physical" view of natural phenomena we take it as an axiom that "nothing acts except where it is". When one body attracts or repels another across space, it does so *not directly*, but by means of some *medium* between the two. When a compass-needle is deflected by a magnet, it is not the magnet which moves the needle, but some medium in contact with the needle itself; and the force exerted on any *part* of the needle is due to the medium *immediately in contact with* that part. The lines of force which we draw in mapping out the field are a great help in the study of this action in the medium.

Now, what is the medium which transmits magnetic effects? Not *air*, for a compass-needle is deflected as strongly in a vacuum as in air. If you hold a magnet near an electric glow-lamp you will notice that the white-hot filaments are pulled aside, although the space inside the lamp is a very "good" vacuum. We must, therefore, attribute the effects to a medium which, although we are obliged to picture it mentally as a substance of some kind, is different from all ordinary matter—solid, liquid, or gaseous. To this medium the name of *the ether* has been given. A vacuum is therefore not an absolutely void space, as might at first be supposed. The ether is, moreover, supposed capable of penetrating the densest solids.

Other branches of science besides electricity and magnetism lead us to recognize the existence of a medium occupying space, even if this is vacuous. Light and heat radiations travelling through the vast vacuous spaces separating our earth from the sun and stars are manifestations of the transmission of energy. This necessitates the existence of a medium which must fill all known space and be capable of transmitting energy. The properties of such a medium are difficult to investigate; but it is probable that the properties necessary to explain one set of phenomena also serve, wholly or in part, for the others. Hence we assume that it is the same medium—the ether—which is concerned in the various phenomena.

19. Magnetic Force.

You should endeavour to explain the effects you observe with magnets as much as possible by the properties of lines of force. Try to picture each magnet as carrying its own set of lines with it everywhere. Remember that the lines represent some altered condition of the ether. What the nature of the alteration may be is at present largely a matter of speculation, but it is sometimes convenient to imagine that the magnetic field consists of a whirling motion of the ether round the lines of force as *axes*.

The form assumed by groups of lines (fig. 9) shows a tendency to contract lengthwise and expand sideways, so accounting for the attraction and repulsion. This property of longitudinal contraction and lateral expansion is a useful one to bear in mind. It can be applied when we wish to determine *which way* a magnet (or current) will be deflected in a given experiment. But if we wish to determine *how much* the magnet is deflected, it is necessary to consider that the forces are produced in a somewhat different manner, which we proceed to explain.

A compass-needle, or other magnet, is never displaced or rotated by its *own* field. (The attractions and repulsions between the different portions of its own magnetism are always balanced by stresses called into play in the steel.) If the needle is placed near a magnet, the deflection produced may be considered due to the action of the magnet's

field on the *poles* of the needle, the *field of the needle being left out of account.* The N. magnetism at one end of the needle is pulled in one direction along the lines of force, whilst the S. magnetism at the other end is pulled in the reverse direction. The compass, therefore, sets along the lines of force.

A small needle (of hard steel and nearly saturated), if caused to oscillate, will do so much more rapidly near the poles of the magnet than at some distance. This shows a difference in the **intensity** or **strength** of field at different distances from the magnet. If the needle were magnetized less strongly it would oscillate less rapidly at all points in the field. This must be due to the smaller amount of magnetism in the latter case. From experiments of this kind we acquire the idea that the force exerted on the needle depends on two factors—

(i) *The intensity or strength of the field;*
(ii) *The amount of magnetism which the field acts on,* i.e. *the pole-strength of the needle.*

We may express this in the form of an equation—

$$(\textit{Force exerted on pole}) = (\textit{Field-strength}) \times (\textit{Quantity of magnetism});$$
$$\text{or } f = \mathrm{F} \times m \quad\quad (2)$$

If we take a unit quantity of magnetism, $f = \mathrm{F}$. We may therefore make the following definition:—

The measure of the field-strength at any point is the force exerted per unit quantity of magnetism placed at the point.

This definition involves the *assumption* that F is constant and independent of m. This may be proved from Coulomb's Law and its extension through magnetic shells to currents. The unit of magnetic field-strength is sometimes termed a *gauss.*

EXAMPLES.—1. A pole of strength 5 is subject to a force of 60 dynes. Find the field-strength.

$$\text{Here F} = \frac{60}{5} = 12 \text{ dynes per unit of magnetism.}$$

2. A field of strength 20 dynes per unit pole produces a force of

350 dynes on one end of a compass-needle. Find the pole-strength of the needle.

$$m = \frac{350}{20} = 17{\cdot}5 \text{ C.G.S. units of magnetism.}$$

Observe that the field-strength is a quantity possessing direction as well as magnitude, *i.e.* it is a *vector* quantity. Its direction at any point is the direction in which it urges N. magnetism. When we wish to indicate the direction as well as the magnitude of this quantity, we use the term **magnetic force** in place of "field-strength".

Observe carefully the distinction between magnetic force and ordinary mechanical force. The magnetic force is a measure of the condition of the ether in the field, and you may picture the field stronger or weaker at some parts than others without supposing anything placed in the field. But the mechanical force can only act on the compass-needle or other magnet placed in the field, and is equal to the product of the magnetic force into the pole-strength of the needle.

Notice, also, in the examples above, that mechanical force is expressed in *dynes*, whilst magnetic force is always expressed in *dynes per unit pole*, or *gausses*.

In many cases it will be unnecessary to use the words "magnetic" or "mechanical"; it will be understood from the context which kind of force is referred to.

20. Uniform Fields.

When the lines of force are parallel the field is of uniform strength: a compass-needle will vibrate at the same rate at all parts of the field. The earth's field is uniform over districts many miles in extent. No field is absolutely uniform; on the other hand, very small portions of *any* field are practically uniform (with a few exceptions in current electricity).

The forces exerted on a magnet by a uniform field form a couple.

This important result may be proved thus:—

(1) Suspend a bar of steel in a carrier of suitable form by a long silk thread. Let the thread pass through a small hole in a hori-

zontal plate attached to the suspension frame. The thread must pass through the hole without touching the plate. Now magnetize the steel. Carefully replace it in the carrier in exactly the same position as before. It will be found that the thread still hangs "plumb" and clear of the edges of the hole. This shows that no resultant horizontal force acts on the magnet as a whole.

(2) Weigh a piece of steel carefully in a balance free from magnetic substances, magnetize the steel, and then re-weigh it. The weight will be found unaltered. Thus there is no resultant vertical force on the magnet as a whole.

It follows, therefore, that the forces have no single resultant. They form a couple which tends to rotate the magnet but not to move it bodily in any direction. The action of a uniform field is said to be *directive* only.

But in a non-uniform field the forces which act on a magnet are equivalent to—

(*a*) a couple; and
(*b*) a force acting at the centre of gravity.

If you attach a magnetized needle to a cork and float it on a large dish of water, the needle simply rotates into the meridian. But if a magnet is brought near, the needle (*a*) rotates and (*b*) moves towards the magnet. The latter movement indicates a force acting on the needle *as a whole.*

21. Centres of Magnetism, or Pole-Centres.

A magnet placed in a uniform field is subjected to a system of *parallel* forces. The force exerted on any small portion of the magnet is (Eqn. 2) *proportional to the quantity of magnetism on that portion.* If we rotate the magnet into a new position, the forces continue to act *parallel to their previous directions.* The action of the field in this case resembles the action of gravity on matter, quantity of magnetism corresponding to quantity of matter. The latter case is usually dealt with in mechanics by compounding all the forces on the material particles into a single force—the weight of the body. Now, however you place an object with respect to the vertical, the resultant weight acts at a definite point in the object, which

is called the centre of gravity or centre of mass of the latter. In like manner we may regard all the N. magnetism on a magnet as collected at one point, and the S. magnetism similarly collected at another point. The resultant forces on each kind of magnetism act through these points, however the magnet may be rotated in the field (supposed uniform). In fig. 12 (*a*) the resultant is indicated by thicker lines. We may now make the following definition:—

> **The pole-centres of a magnet are the centres of "mass" of the N. and S. magnetisms respectively.**

Pole-strength is thus analogous to mass, pole-centre to centre of gravity, and the magnetic force to intensity of gravity. We may suppose all the magnetism of each kind collected at its own pole-centre, just as we suppose the mass of a body collected at its centre of gravity.

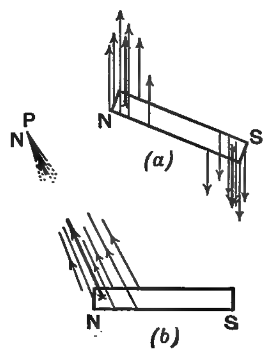


Fig. 12.—Centres of Magnetism

There is a second case in which the magnetism may be supposed collected at the pole-centres. This occurs in the calculation of the field produced by a magnet, at a point some distance from the latter (fig. 12 (*b*)). Suppose a unit of magnetism placed at the point. The repulsions of the particles of magnetism on the N. polar surface act along lines drawn from these particles to the point; and since the point is distant, these lines are *nearly parallel*. Moreover, from Eqn. (1) the forces are *proportional to the quantities of magnetism on each element* of the polar surface. Hence, comparing this with the conditions stated at the beginning of this article, we see that the conditions are similar; and we may suppose the magnetism collected at the pole-centre, for all positions of the point in the distant parts of the field.

Calculations referring to permanent magnets may be classified as follows (excepting calculations requiring difficult mathematics). We may consider—

(1) *Forces exerted on magnets by uniform fields.*

(2) *Fields produced by magnets at a distance from the polar surfaces.*

(3) *Combinations of* (1) *and* (2).

The definition of pole-centres and the methods of calculation depending on it are applicable to any magnet, however complex.

22. Equality of Poles—Definitions.

The two poles of a magnet may be proved to be of equal strength by the results of the preceding article.

Let m and m' be the pole-strengths (numerical values) of the N. and S. ends of a magnet. Then in a uniform field of strength F the mechanical forces on the poles are (Eqn. 2) $m\mathrm{F}$ and $m'\mathrm{F}$. But these form a couple. Hence—

$$m\mathrm{F} = m'\mathrm{F},$$

$$i.e. \quad m = m'.$$

This result is true, however complex the magnet may be.

The magnetic length of a magnet is the distance between the pole-centres.

The magnetic centre is a point half-way between the poles.

The magnetic centre does not coincide with the geometrical centre except in bisymmetrical magnets.

The magnetic axis is the line joining the pole-centres: it may be supposed produced indefinitely in either direction.

This definition applies to all magnets, but it is only in symmetrical ones that it coincides with the geometrical axis.

The magnetic moment is the product of the pole-strength into the magnetic length.

We shall use the symbol M for this quantity. Thus—

$$M = m \times l.$$

The magnetic moment and magnetic axis may be taken as the magnitude and direction of one quantity, also termed the magnetic moment. For further consideration of magnetic moment as a vector see the next chapter.

23. Resultant and Component Fields.

A needle is sometimes acted on by several magnets at once. In this case it will assume the direction of the *resultant* force due to the combined action of all the magnets.

Suppose the N. pole of the needle to be situated at O, near the magnets A and B (fig. 13). The lines of force of these

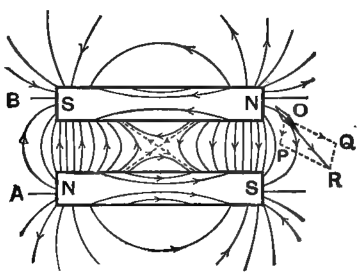


Fig. 13.—Resultant Field

magnets run through O in the directions OP and OQ respectively. Let OP and OQ be drawn to such lengths that they are proportional to the forces exerted on the pole by the two fields. Then OR, the diagonal of the parallelogram OPRQ, represents the direction and magnitude of the resultant mechanical force to the same scale.

If the pole-strength of the needle be increased, OP and OQ will be *both* increased, and in the same ratio. The direction of the resultant is therefore unaltered. If the needle has unit strength, OP and OQ represent the *magnetic* forces of the separate fields, and OR represents the resultant magnetic force (or field-strength). The parallelogram of forces may be applied

to compound or resolve magnetic forces (field-strengths), just as it is applied to ordinary mechanical forces.

The following is an example of the resolution of field-strengths. The earth's total field is represented by lines of force inclined at the angle of dip (fig. 4). Resolve this into a vertical component and a horizontal one, and consider the action of these components on an ordinary compass. The vertical field tends to make the needle dip, pulling the N. pole down and the S. pole up. This tilting of the needle is, however, prevented by the weight of the needle and its pivoting. The needle has perfect freedom of movement in a horizontal plane only, and its movements are mainly controlled by the horizontal component of the field.

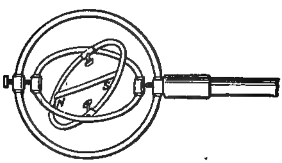


Fig. 14.—Exploring Needle

In most magnetic experiments the needles are balanced or suspended like compass-needles, and therefore in the majority of cases we consider only **horizontal components.**

EXAMPLE.—At a place (A) the dip is 45° and the earth's total force ·536 dyne per unit pole. At another place (B) the dip is 60° and the total force ·555 dyne per unit pole. At which place will a compass oscillate the more rapidly.

Resolve the resultant force R into horizontal and vertical components. If δ is the dip,

$$H = R\cos\delta;\ V = R\sin\delta.$$

Since the needle is controlled by H, we have—

$$\begin{aligned}\text{Controlling force at A} &= \cdot536 \times \cos 45^\circ \\ &= \cdot379 \text{ dyne per unit pole.} \\ \text{Controlling force at B} &= \cdot555 \times \cos 60^\circ \\ &= \cdot277 \text{ dyne per unit pole.}\end{aligned}$$

Thus the needle will oscillate more rapidly at the place where the total force is weaker.

A magnetic needle may be mounted so as to show the direction of the resultant field in space. The arrangement resembles the gimbals of a mariner's compass, and is shown in fig. 14.

If such a needle is brought near the field-magnets of a dynamo, or near a conductor carrying a strong current, and moved into various positions, it will be found to follow the changes in direction of the lines of force in a manner which is very interesting to observe.

24. Methods of Tracing Lines of Force.

1. *Experimental.*—The form of the horizontal lines of force, if the field is fairly strong, may be readily obtained by the use of a compass-needle or filings. In a weak field, or at a distance from a magnet, filings assume no particular arrangement, and a compass-needle is deflected from the direction of the field under test, because the effect of the earth's weak force becomes appreciable.

The earth's influence may be eliminated in the following manner. (Suppose the distant field of a magnet required.)

Place the magnet in the middle of a sheet of cardboard lying on the table. Mark down its outline on the card so that it may be replaced, if accidentally disturbed. Now proceed to obtain the direction of the lines; but at each step in the experiment, before marking down the direction of the compass-needle, turn the whole arrangement round by moving the card until the needle lies in the meridian with its N. pole to the north. The card is now so placed that the magnet's lines at the point tested coincide in direction with the earth's field, and the latter produces no disturbing effect.

If the distant field is mapped with the aid of a compass whilst the magnet remains fixed, a complex field is obtained which is the *resultant* of the earth's field and that of the magnet. Figs. 15 (*b*), 16 (*b*) show the resultant fields obtained for two positions of the magnet.

2. *Theoretical.*—An idea of the general shape of the lines may in most cases be obtained theoretically by supposing a single N. pole free to move in the space round the magnet. The path traced out may be ascertained by considering the repulsion of N. poles and the attraction of S. poles on the moving test-pole.

The form of the lines of force *inside* the magnet *must* be found

theoretically. The internal lines are indicated in figs. 10 and 11. Imagine that a particle of N. magnetism can move freely through the magnet under the influence of the polar magnetism. Starting from the N. pole the particle is first strongly repelled, but at a little distance the attraction of the S. pole becomes evident, and the particle describes a curved path as shown in the figures.

The student must be careful here not to be misled by loose statements, such as "lines of force pass more readily through iron

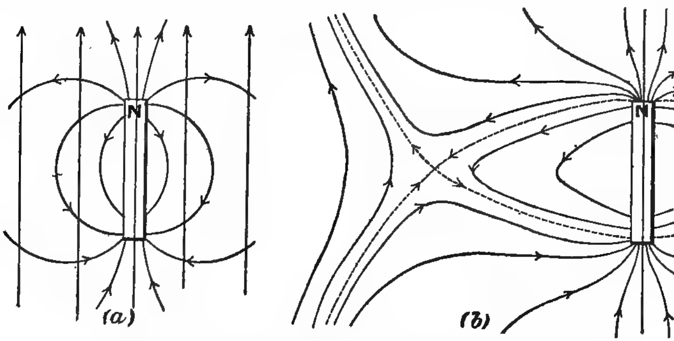


Fig. 15.—(*a*) Component Fields, N. Pole to North—(*b*) Resultant

or steel than through air". This refers to lines of *induction* (Chap. VII). In tracing lines of *force* by the above method the lines must be supposed to pass indifferently through all substances.

The general form of a resultant field may be ascertained by noticing (1) at what parts of the field the components oppose each other, producing a weak resultant and lines far apart (Art. 72); (2) where the components directly help each other—resultant strong and lines close together; (3) where one force acts across the other, giving a resultant inclined to both components.

Notice these points carefully in connection with figs. 15, 16. The component fields are shown separately at (*a*). At certain points the component fields exactly balance each other, and there is no resultant. These are called *neutral points*. Lines of force always form a cross at the neutral points. This is shown by the dotted lines in the figures, which form a useful

guide in drawing the remaining lines. Observe that the lines of force do not actually cross at the neutral point, since the strength of the force vanishes just at this point. In fact, lines of resultant force never intersect or meet, because there can only be one direction of the resultant at each point. Another example of a neutral point occurs in fig. 9 (*b*).

The field between like poles has always the general form of fig. 9 (*b*), and between unlike poles that of fig. 9 (*a*).

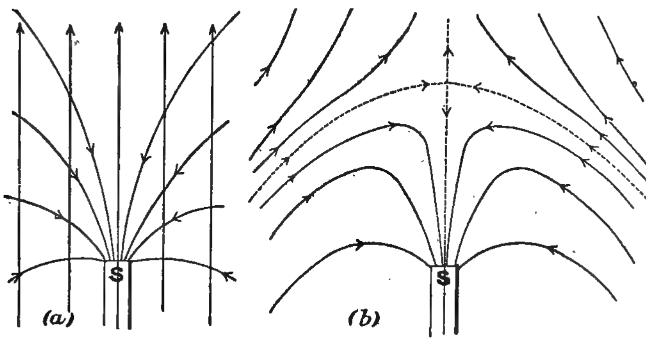


Fig. 16.—(*a*) Component Fields, N. Pole to South—(*b*) Resultant

25. Theories of Magnetism.

A complete theory of the cause of magnetic phenomena has not yet been formed. The theory of molecular magnets merely relates to the internal structure of the magnet, and makes no assumption as to the cause of polarity in the molecules themselves. Each molecule is simply regarded as the whole magnet in miniature. Modern research has shown that the polarity of the molecule may be due to the revolution of electrically-charged particles within the molecule.

This may be regarded as the modern development of the older theory of Ampère, which considered each molecule to be the seat of a minute electric current. It has always been found convenient, however, to ascribe the properties of magnets to a "substance" termed magnetism, and this hypothesis will probably always be found the most useful one for

purposes of calculation, even when the nature of molecular polarity becomes better understood. At one time magnetism was supposed to actually exist in a magnet, and was spoken of as a "fluid". This view is now quite discarded, and we must regard magnetism as a purely imaginary substance—a mathematical fiction—invented chiefly for the purpose of calculation. Lord Kelvin has termed it "imaginary magnetic matter".

Adopting the molecular theory, we must regard each molecule as possessing equal and opposite quantities of magnetism on its "poles". In a chain of molecules the effect of a pole of one kind is everywhere neutralized by the effect of a pole of the opposite kind except at the ends of the chain. The terminal poles of all the chains of molecules taken together form the *free magnetism* which we have been considering in this chapter. The internal magnetism will be considered in Chap. V.

Note carefully that there is no relation between the shape of a magnet and the distribution of magnetism on it. In electricity we find a definite connection between the shape of a conductor and the distribution of charge on it, but there is no parallel to this in magnetism. This is illustrated by fig. 1, representing magnets of the same size, shape, and material, but with very different magnetic distributions.

26. Examples.

1. A magnetized knitting-needle has pole-strength 50. What is the strength of the field due to one pole of the needle at a distance of 80 cm. from this pole, and what force would be exerted by this field on the pole of a needle of strength 200 units?

The field-strength is the force exerted on unit pole.

$$\therefore F = \frac{m \times 1}{d^2} = \frac{50}{80^2} = \frac{1}{128} = \cdot 0078 \text{ gauss.}$$

The mechanical force on pole-strength 200 is—

$$f = F \times m = \cdot 0078 \times 200 = 1 \cdot 56 \text{ dyne.}$$

2. A triangle ABC has sides AB = 3, BC = 4, CA = 5 cm. respectively. Point-poles of strengths 3 and 4 units are placed at A and

C respectively. Find the resultant force on unit-pole at B, all the poles being N.-seeking.

Since $5^2 = 3^2 + 4^2$, the angle at B is a right angle.

$$\text{Repulsion due to A} = \frac{3 \times 1}{3^2} = \tfrac{1}{3} \text{ dyne.}$$

$$\text{Repulsion due to B} = \frac{4 \times 1}{4^2} = \tfrac{1}{4} \text{ dyne.}$$

Produce AB and CB and draw the parallelogram of forces at B. Since the parallelogram is a rectangle, the resultant force

$$= \sqrt{(\tfrac{1}{9} + \tfrac{1}{16})} = \tfrac{5}{12} \text{ dyne.}$$

3. A thin uniform magnet 1 metre long is suspended from the north end so that it can turn freely about a horizontal axis which lies magnetic east and west. The magnet is found to be deflected from the perpendicular through an angle θ ($\sin \theta = \cdot 1$, $\cos \theta = \cdot 995$). If the weight of the magnet is 10 grams, the horizontal component of the earth's field ·2 C.G.S. unit, and the vertical component ·4 C.G.S., find the moment of the magnet. (Bd. of Educ. 1906.)

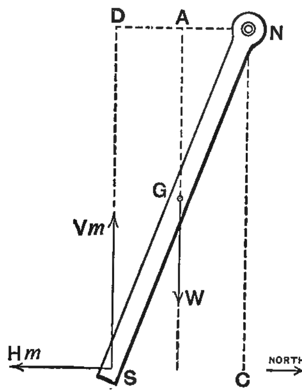


Fig. 17

Fig. 17 will help you to realize how the forces act. NS is the magnet hinged at N. In England the vertical field points downwards. Hence the *south* pole of the magnet is urged upwards by the vertical field, and towards the south by the horizontal field. Since the earth's field is uniform, the mechanical force may be supposed to act at the pole (Art. 21). The forces at the hinged end produce no effect.

To realize the effect of the forces, suppose two threads tied to the point S, and let one be pulled horizontally and the other vertically. The horizontal pull, even if weak, will deflect the bar in consequence of its leverage or moment. The power to rotate the bar is reckoned by the moment of the force, which is—

$$(\text{force}) \times (\text{perpendicular distance from the axle}).$$

The vertical component also tends to rotate the magnet and in the

same direction (clockwise). The weight of the magnet acts at the centre of gravity G, and tends to bring it back to the vertical position. Its moment balances the moments of the magnetic forces.

Let H and V be the horizontal and vertical field-strengths, w = the weight of the bar. Since 1 gram weighs 980 dynes, this weight = 980 × 10 dynes.

Horizontal pull = Hm; vertical pull = Vm (Art. 19).

$$\begin{aligned}
\text{Moment of horiz. pull} &= \mathrm{H}m \times \mathrm{NC}. \\
\text{Moment of vert. pull} &= \mathrm{V}m \times \mathrm{ND} = \mathrm{V}m \times \mathrm{SC}. \\
\text{Moment of weight} &= w \times \mathrm{NA} = w \times \tfrac{1}{2}\mathrm{SC}.
\end{aligned}$$

∴ equating moments,

$$\mathrm{H}m\,.\,\mathrm{NC} + \mathrm{V}m\,.\,\mathrm{SC} = \frac{w}{2}\,.\,\mathrm{SC}.$$

Divide by SN.

$$\mathrm{H}m\,.\,\frac{\mathrm{NC}}{\mathrm{SN}} + \mathrm{V}m\,.\,\frac{\mathrm{SC}}{\mathrm{SN}} = \frac{w}{2}\,.\,\frac{\mathrm{SC}}{\mathrm{SN}};$$

$$\text{or,}\quad m(\mathrm{H}\cos\theta + \mathrm{V}\sin\theta) = \frac{w}{2}\,.\,\sin\theta.$$

Substituting the values given—

$$\begin{aligned}
m(\cdot 2 \times \cdot 995 + \cdot 4 \times \cdot 1) &= 4900 \times \cdot 1 \\
m \times \cdot 239 &= 490 \\
m &= 2050.
\end{aligned}$$

The magnetic moment

$$= m \times l = 2050 \times 100 = 205{,}000 \text{ units},$$

assuming the magnetic length is equal to the actual length.

QUESTIONS

(Questions taken from the Board of Education Examination Papers are indicated by the date in brackets.)

1. How could the law of the inverse squares of the distance as applied to magnetic poles be experimentally proved? What is meant by a pole of unit strength? (1902.)

2. How would you proceed to prove that every magnet exhibits the two opposite magnetic properties in an equal degree? (1900.)

3. Two magnets are placed horizontally on a large sheet of white paper. You are supplied with a small compass, a pencil, and a watch. Assuming that the earth's field may be neglected, how would you trace the lines of force due to the magnets and deter-

mine points at which the intensity of the magnetic field was equal? (1898.)

4. A magnet is placed so that its centre is due magnetic east of the centre of a compass-needle, which is so mounted that it cannot dip. The magnet is made to rotate in a vertical plane about a horizontal axis, which lies east and west, and passes through its centre. Describe the behaviour of the compass-needle during one complete revolution of the magnet. (1895.)

5. A short bar magnet is placed at the centre of a circle 3 feet in diameter, its axis being in the magnetic meridian. Trace the changes in the direction in which a compass-needle points as it is carried round the circumference of the circle. (1902.)

[Notice that the circle is a large one, and remember that the needle points in the direction of the resultant force.]

CHAPTER III

MAGNETS IN UNIFORM FIELDS

27. The conditions under which the forces exerted on permanent magnets may be subjected to calculation have already been explained (Art. 21). We shall now deal more fully with the first of these, namely, the case of a magnet in a uniform field. It will be convenient to take the horizontal component of the earth's field as the controlling force, but the results and formulæ obtained will apply equally to any other uniform field. The fields inside a solenoid, and over a metal sheet carrying a current in the same direction at all points, are other examples of uniform fields of considerable extent. It will also be necessary to suppose that the magnets are made of hard steel, and the field too weak to affect the strength of the poles by induction.

28. **Moment of the Restoring Couple.**

When a magnet is displaced from its position of rest (by a rotation), it is acted on by a couple tending to restore it to this position.

Let A′B′ be a magnetic meridian line and NS a deflected magnet (fig. 18). The angle of deflection is δ. If m is the strength of the pole and H the intensity of the earth's horizontal field, the mechanical force f acting on each pole is given by $f = m\text{H}$.

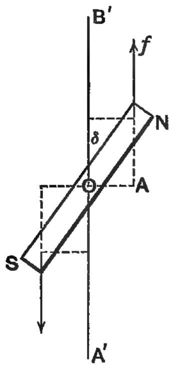


Fig. 18.—Restoring Couple

Since these forces act parallel to the meridian in all positions of the magnet, they form a couple. The moment of this couple (or the "torque")—

$$\begin{aligned} \text{G} &= f \times (\text{arm of the couple}). \\ \text{Now, (arm)} &= 2 \times \text{AO} \\ &= 2 \times \text{ON} \sin \delta \\ &= l \sin \delta;\ \text{where } l = \text{length of the magnet.} \\ \text{Hence G} &= f \times l \sin \delta \\ &= \text{H}ml \sin \delta \\ &= \text{H}\,.\,\text{M}\,.\sin \quad \dots\dots\dots\dots(1) \end{aligned}$$

Thus, the moment of the couple tending to restore the magnet to the meridian is proportional to the continued product of the magnetic moment, the intensity of the field, and the sine of the angle of deflection.

We proceed to describe an instrument to which the formula may be applied.

29. The Torsion Balance.

This is an instrument in which the moment of a couple (or torque) is measured in terms of the angle through which it will twist a wire. The usual form for magnetic experiments is shown in fig. 19. A glass case, about 30 cm. wide, carries a glass tube some 20 cm. high. The magnet is supported by a copper or brass stirrup, which is suspended from the top of the tube by a fine wire. The upper end of the wire is attached to a disc which can rotate in a horizontal plane. This disc is divided at its edge into a scale of degrees. A pointer is attached to the tube, and serves as a reference mark in reading the degrees. These fittings at the top of the tube constitute the "torsion head"; by this means the angle through which the upper end of the wire is turned in any experiment may be

determined. The angle through which the magnet is deflected is read off from a circular scale attached to the base.

The instrument is first placed so that a centre line passing through the zero of the lower scale lies in the magnetic meridian. When the magnet rests with its axis pointing along the meridian, there will be no twist on the wire. We shall call this the "normal position".

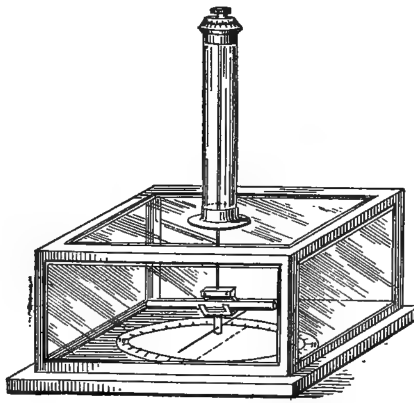


Fig. 19.—Torsion Balance

Starting with the normal position, let the torsion head—and therefore the top of the wire—be rotated through A°. If the bottom end of the wire followed the rotation through the same angle, there would be no twist on the wire; but the bottom end of the wire—and therefore the magnet—follows the rotation through a less angle, δ. Hence there is a twist remaining in the wire equal to $(A - \delta)$, which tends to increase the deflection of the magnet. The wire therefore exerts a torque on the magnet, and this balances the earth's restoring couple. Hence, for purposes of comparison, $(A - \delta)$ may be taken as representing the moment of the restoring couple.

To prove that $G \propto \sin \delta$.

Rotate the torsion head from the normal position through any angle, A_1. Read the corresponding deflection of the magnet, δ_1. Repeat the experiment, increasing the rotation to A_2. Let the new deflection of the magnet be δ_2. The results of the experiment will be in accordance with the relation—

$$\frac{A_1 - \delta_1}{A_2 - \delta_2} = \frac{\sin \delta_1}{\sin \delta_2}.$$

$$\therefore \frac{G_1}{G_2} = \frac{\sin \delta_1}{\sin \delta_2}.$$

To compare the moments of two magnets.

Turn the torsion head until the magnet is deflected through (say) 30°. Note the reading, A_1. Remove the magnet from the stirrup and substitute the other magnet to be compared with it. Rotate the torsion head until the same deflection is obtained as before. Note the reading, A_2.

$$\text{Then } \frac{G_1}{G_2} = \frac{A_1 - 30°}{A_2 - 30°}.$$

$$\text{Also } \frac{G_1}{G_2} = \frac{M_1 H \sin 30°}{M_2 H \sin 30°}.$$

$$\therefore \frac{M_1}{M_2} = \frac{A_1 - 30°}{A_2 - 30°}.$$

This is not the most accurate method of comparing magnetic moments (on account of experimental difficulties).

To compare the horizontal intensities of the field at two places.

In this experiment use only one magnet. Find the rotation of the torsion head necessary to deflect it (say) 90°. Transfer the whole instrument to the second locality, taking care with the magnet, and repeat the experiment. Let the rotations of the torsion head at the two places be A_1 and A_2 respectively. Then—

$$\frac{A_1 - 90°}{A_2 - 90°} = \frac{MH_1 \sin 90°}{MH_2 \sin 90°};$$

$$\text{or } \frac{H_1}{H_2} = \frac{A_1 - 90°}{A_2 - 90°}.$$

30. Small Deflections—Oscillating Magnets.

The sine of a very small angle is practically equal to the circular measure of the angle. Hence, if a magnet is displaced only a few degrees from the meridian, the formula—

$$G = MH \sin \delta$$

$$\text{becomes } G = MH\,\delta;$$

$$\text{or } \frac{G}{\delta} = MH \quad \text{.......................... (2)}$$

Since M and H are constant for all values of δ, the above relation shows that *for small displacements the restoring moment is proportional to the angle of deflection.* The closeness of the

approximation for different angles may be seen from the following table:—

Angle (Degrees).	Angle (Circular Measure).	Sine.
1	·01745	·01745
2	·03491	·03490
3	·05236	·05234
4	·06981	·06976
5	·08727	·08716

It is proved in works on Mechanics that when a body oscillates under the influence of a couple proportional to the displacement, the motion is of the kind called *harmonic motion.* The oscillation of a pendulum through a small angle is the simplest example of harmonic motion. The above table shows that if the deflection does not exceed 5°, the proportion is exact to less than 1 per cent. Hence, if a compass-needle is allowed to oscillate through a small angle, its motion is, like that of a pendulum, practically harmonic.

The extent of swing on each side of the mean position is called the *amplitude.* The duration of one complete to-and-fro swing is called the *period.* One characteristic of harmonic motion is that the *period is independent of the amplitude.* Thus a compass-needle swinging through an angle of 5° performs each oscillation in the same time as when it oscillates through only 3°. In the former case the angle covered is greater, but at the same time the average speed is greater in the same proportion; hence the time or duration of the swing is the same.

Note.—It is necessary to remind the student that, if the magnet is suspended by a thread, the motion we are speaking of is always such as exists if the magnet is balanced on a pivot—not the swaying of the magnet (which must be avoided).

31. Period of Oscillation—Moment of Inertia.

It can be shown that the period of a harmonic oscillation is given by the formula—

$$t = 2\pi\sqrt{\frac{K}{c}};$$

where c is the ratio of the moment of the couple to the displacement, and K is the *moment of inertia* of the body.

The student who is not already familiar with the meaning of moment of inertia should consider the following experiment carefully:—

EXPERIMENT.—Choose a bar of wood about 40 cm. long and 5 cm. wide. Cut a groove in the upper surface so that weights placed on the bar will not slip. Attach a brass wire 1 metre long to a stirrup at the centre of the bar, and fix the upper end of the wire to a suitable support. Allow the bar to oscillate under the action of the controlling forces of the wire, equal weights being placed near each end. Replace these weights by heavier ones. The bar now oscillates more slowly, *i.e.* the period is greater. Move the weights nearer the middle of the bar; this makes the bar oscillate more quickly. The rate of oscillation therefore depends—

(i) *on the mass of the body,*
(ii) *on the distance of the mass from the axis of oscillation.*

Both conditions are taken into account in the quantity called the moment of inertia of the body. This quantity is increased by increasing the mass, or by distributing it farther from the axis of oscillation. Two bodies may have the same mass or weight, but different moments of inertia according to their size and shape.

The moment of inertia can be calculated for certain bodies of regular shape. Bar magnets are nearly always rectangular or of circular section. If the bars are suspended horizontally at the centre of their length, their moments of inertia are—

(*a*) Rectangular bars, $K = \text{mass} \times \left(\frac{l^2 + b^2}{12}\right)$;

(*b*) Round bars, $K = \text{mass} \times \left(\frac{l^2}{12} + \frac{r^2}{4}\right)$.

l = length, b = breadth, r = radius of cross-section.

The formula for the period of a harmonic oscillation may be applied to a magnet oscillating through a small angle. If the magnet is suspended by a fine silk fibre or balanced on a hard steel point, *the motion is controlled only by the magnetic field.*

$$\therefore c = \frac{\text{couple}}{\text{displacement}} = \frac{G}{\delta} = MH.$$

Therefore the formula for the period becomes—

$$t = 2\pi\sqrt{\frac{K}{MH}} \quad \text{(3)}$$

This is one of the standard formulæ in Magnetism, and we proceed to describe an instrument to the action of which it may be applied.

32. Oscillation Magnetometer.

This instrument is shown in fig. 20. It consists of a rectangular box with removable glass sides. The cover carries

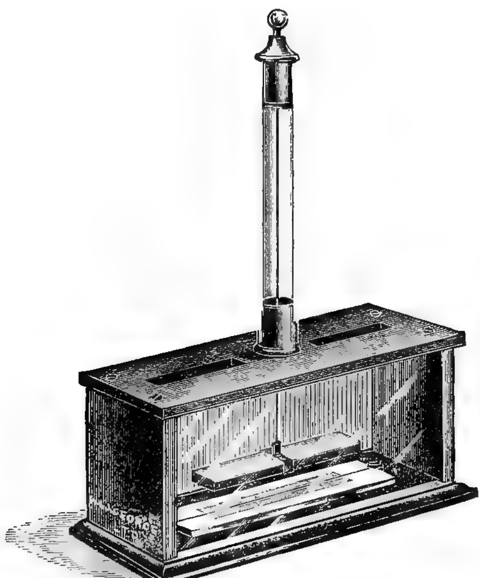


Fig. 20. Oscillation Magnetometer

a glass tube with a cork at its upper end. The magnet is laid in a copper stirrup, and is suspended in the middle of the box by a silk thread attached to a hook on the bottom of the cork, and passing centrally down the tube. The base

of the box is covered with a strip of mirror glass, which has a scratch along its centre line parallel to the longer sides of the box. This serves as a reference mark, and to avoid parallax the magnet is viewed through openings in the top of the box, so that it appears to exactly cover its image. The ends of the magnets are sometimes provided with brass pointers to facilitate this adjustment.

The instrument is first placed with its centre line lying along the meridian, and the magnet is made to oscillate through a few degrees. The instant at which it crosses the centre line is noted with a stop-watch. An exact number of oscillations is then counted, say 30. At the 30th transit when the magnet crosses the centre line *in the same direction as at first*, the time is again noted. The interval (in seconds) divided by the number of oscillations gives the period. The gradual decrease of amplitude due to friction with the air does not affect the result.

To compare the magnetic moments of two magnets.

Weigh the magnets and obtain their masses in grams. Measure accurately the length and breadth of each. Calculate the moments of inertia K_1, K_2 from the formulæ given above.

Place one of the magnets in the stirrup and obtain its period t_1. Similarly, find the period t_2 of the other magnet.

$$\text{Then } t_1 = 2\pi\sqrt{\frac{K_1}{M_1H}}; \quad t_2 = 2\pi\sqrt{\frac{K_2}{M_2H}}.$$

Squaring and transposing we have—

$$HM_1 = \frac{4\pi^2K_1}{t_1^2}; \quad HM_2 = \frac{4\pi^2K_2}{t_2^2}.$$

∴ by division—

$$\frac{M_1}{M_2} = \frac{K_1t_2^2}{K_2t_1^2}.$$

In cases such as this, where we are only concerned with comparisons, the *ratios* may be expressed in terms of the number of oscillations in any convenient fixed time, say one minute. The period is *inversely* proportional to this number. If n_1 and n_2 are

the number of vibrations made in one minute in the above example, then—

$$\text{Since } t_1 : t_2 = n_2 : n_1,$$

$$\text{we have } \frac{M_1}{M_2} = \frac{K_1}{K_2} \cdot \frac{n_1^2}{n_2^2} \qquad \text{(4)}$$

If the magnets are of equal size and weight—

$$\frac{M_1}{M_2} = \frac{n_1^2}{n_2^2} \qquad \text{(5)}$$

To compare the intensities of two fields.

In this case only one magnet is used. Its period is found, as above, in the first field. The instrument is then transferred to the second field, and the period of the same magnet is found again.

Then, adopting the usual symbols—

$$H_1M = \frac{4\pi^2K}{t_1^2}; \quad H_2M = \frac{4\pi^2K}{t_2^2}.$$

$\therefore$ by division—

$$\frac{H_1}{H_2} = \frac{t_2^2}{t_1^2}, \text{ or } \frac{H_1}{H_2} = \frac{n_1^2}{n_2^2} \qquad \text{(6)}$$

This is a ready and accurate method of comparing the intensities of horizontal fields, provided the fields are not so strong that they will affect the moment of the magnet by induction.

The present chapter refers only to uniform fields, but we may here note that the relation—

$$\frac{H_1}{H_2} = \frac{n_1^2}{n_2^2}$$

is often made use of to compare the intensities at different points of a *non*-uniform field like that of a permanent magnet, the oscillating magnet used being a *very short* one. This use of the formula in reference to a non-uniform field is justified by the circumstance that the field is practically uniform throughout the limited region in which the short magnet swings.

A short magnet generally oscillates very quickly. Hence, to make the swings conveniently slow, the moment of inertia must be increased by loading the needle. The magnet may

be formed of a bundle of four or five pieces of knitting-needle 2·5 cm. long, fitted in a lead tube 2·5 cm. long and 3 mm. diameter. This is hung by a silk fibre inside a small glass bottle, which may be placed at the part of the field to be tested. For the reason already mentioned, the method cannot be used for the strong parts of the field.

We shall refer to the instrument just described as an "exploring magnetometer".

33. Systems of Magnets—Resultant Moment.

A. **Two Magnets.**—If two simple magnets are rigidly fixed together and suspended in a uniform field, they are equivalent to a single magnet whose axis lies in a definite direction with respect to the combination. We shall prove this for a particular example, and the student will see that the method is generally applicable.

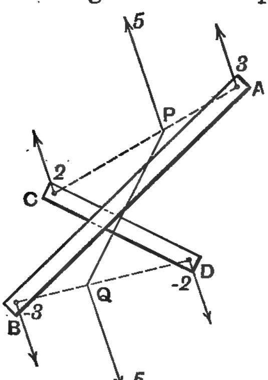


Fig. 21

Let AB, CD (fig. 21) be two magnets rigidly connected as shown, A and C being north poles. The strength of the pole A is (say) 3 units, and of C, 2 units.

Now, in a uniform field *like* parallel forces are exerted at A and C, whose relative strengths are 3:2. Hence the resultant acts at a point P in the line joining A and C such that—

$$AP : CP = 2 : 3.$$

The relative strength of the resultant force is 5, since the components are parallel and similarly directed.

Similarly, the resultant of the forces on B and D acts through Q in BD such that—

$$BQ : DQ = 2 : 3.$$

Thus the resultant forces are such as would act on a magnet

PQ of pole-strength 5. The moment (5 × PQ) of this magnet is called the *resultant moment*, and PQ is the *resultant axis*.

If the system is suspended, or fixed to a cork and floated, it will turn round until PQ lies along the meridian. The resultant moment is not equal to the sum of the separate moments except when the magnetic axes are parallel. If δ is the angle between the *resultant* axis and the meridian, the moment of the couple acting on the whole system is—

$$G = MH \sin \delta,$$

where M is the resultant moment.

B. Three or more Magnets.—A rigid combination of three or more magnets may be dealt with by extending the above construction. First find the resultant of two of the given magnets as above. Then find the resultant of a third magnet and the resultant of the first two; and so on, until all the magnets have been taken into account.

EXAMPLE.—Three equal bar magnets, of equal pole-strength, are arranged along the diagonals of a hexagon, N. and S. poles occurring alternately at the corners of the hexagon. Find the resultant moment.

Let each pole have strength n. Take into account first all the N. poles, A, C, E (fig. 22). The resultant of A and C is a pole, strength $2n$, at the middle point M of AC. From the geometry of the figure it is evident that this is also the middle point of OF.

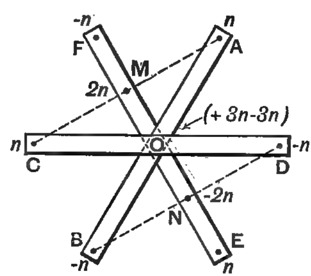


Fig. 22

Next, to compound the pole $2n$ at M with the pole n at E, we must divide EM in the ratio 2 : 1. The resulting pole $3n$ is therefore situated at O.

Similar reasoning shows that the resultant S. pole is also at O, and of strength $-3n$. Hence the resultant magnet has no length or axis, and its total moment $3n \times 0 = 0$. The arrangement, if suspended or floated, would remain at rest in any position. It forms what is called an *astatic* system.

34. **Alternative Method.**

In a given field the couple acting on a magnet depends on M and δ. Hence a magnet may be supposed replaceable by one of unit pole-strength, and of such a length that its moment is equal to the given one. Thus a magnet of pole-strength 3, length 10 cm., displaced 45° from meridian, is equivalent to one of length 30 cm., pole-strength 1 also deflected 45° from the meridian, the value of—

$$MH \sin \delta$$

being the same in both instances.

If we replace a system of magnets in this way, the *lengths of the separate magnets are then proportional to their magnetic moments.* We shall apply this result to establish a second rule for the resultant moment.

I. *The axes of the magnets intersect.*

Let AB, CD (fig. 23) be magnets of moments 3 and 5. First slide each magnet along its own axis until its S. pole lies at the point of intersection O of the axes. Produce AB, CD. Let OM : ON = 3 : 5, and let OM = length of AB × pole-strength. Then ON = length of CD × pole-strength.

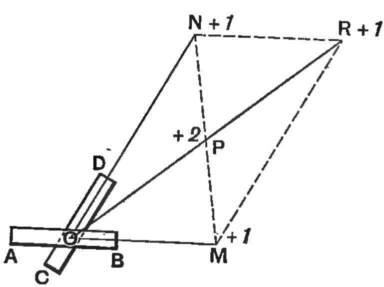


Fig. 23

Then the given magnets are equivalent to magnets OM, ON having unit poles, M and N being north poles. The resultant of M and N is a pole of strength 2 at P, the middle point of MN. The two south poles at O are equivalent to a pole of strength -2 at O.

The resultant magnet is OP, pole-strength 2. But this is equivalent to a magnet OR of strength unity where OR = 2 OP.

Thus, if ON, OM represent the component moments, OR represents the resultant moment to the same scale. OR is also the resultant axis.

Join NR, MR. Then NM and OR are each bisected at P. Hence the figure ONRM is a parallelogram. (The diagonals of a parallelogram bisect each other.)

It will be seen that the figure is similar to the "parallelogram of forces" in Mechanics. Hence we have an *alternative* method for finding the resultant axis and moment of a number of magnets, which may be stated thus:—

When the axes of magnets intersect, the magnetic moments may be compounded like forces by the parallelogram law.

The numerical value of the moment corresponds to the magnitude of the force, and the direction of the axis (pointing from the S. pole towards the N. pole) to the direction of the force.

EXAMPLE.—Two magnets, moments 5 and 12, are fixed together at right angles. How will the combination set with regard to the meridian if floated horizontally?

Draw a rectangle, sides 5 and 12 units respectively. The diagonal is $\sqrt{5^2 + 12^2} = 13$ units long. Hence the resultant moment is 13, and the system sets with the longer magnet (moment 12) at an angle with the meridian whose tangent is $\frac{5}{12}$.

II. *The axes are parallel.*

The moment MH sin δ of the restoring couple acting on a magnet is unaltered by a change in its position provided the inclination δ to the meridian is unaltered, for the values of M, H, and sin δ respectively are unchanged by the displacement.

The behaviour of the magnet is therefore the same. (Moment of inertia, which is a mechanical, not a magnetic property, is not involved in the comparisons of the present article.)

If we replace each magnet of a parallel system by one of unit strength, and then arrange all the latter in one line to

form a chain with the N. pole of each magnet coincident with the S. pole of the next, it is evident at once that—

> **The resultant moment of a system of parallel magnets is equal to the algebraic sum of the separate moments.**

This again is similar to the corresponding rule for forces. Magnets whose axes point along parallel lines in opposite directions are regarded as having moments of opposite sign. Thus, if the upper and lower needles of an astatic galvanometer have moments 30 and 45 respectively, the system acts with respect to the *earth's* field like a weak magnet of moment 15, its N. pole pointing the same way as that of the lower magnet.

III. *The axes are inclined but do not intersect.*

This case may be dealt with by combining I and II.

EXAMPLE.—An astatic pair, with needles similar in all respects, is slightly injured, so that the needles, though still moving in horizontal planes, are slightly inclined to each other. How will the system come to rest?

Suppose the upper needle displaced downwards until it intersects the lower needle. The behaviour of the system is (by II) unaltered. Next, by the "parallelogram of magnetic moments", find the resultant. This is represented by a very short line bisecting the very obtuse angle of the parallelogram. This resultant must lie in the meridian, and therefore the general direction of the needles in the position of rest is E. and W.

Note.—The student will find that the axis determined by the method of Art. 33 has a definite *position*, whilst the method of Art. 34 only gives the *direction* of the axis. The former, however, is arrived at by the mathematical device of a pole, whose position may be assumed, but cannot be determined experimentally. From the experimental point of view the magnetic moment has direction and magnitude, but no definite position. It is an example of a pure vector.

If three or more magnets are to be compounded by the parallelogram method, the order of Art. 33 B must be followed, just as with forces.

35. Astatic Systems.

If the resultant moment of a system of magnets is zero, the system will remain at rest if suspended in any manner in a uniform field. Such a system is said to be "astatic".

An instance has already been given (Art. 33). An astatic pair consists of two parallel magnets of equal moments, with their axes pointing in opposite directions. If the magnetic lengths are equal, as is usually the case, the poles must also be equal; but if one magnet is, say, 3 times as long as the other, its pole-strength must be $\frac{1}{3}$ of that of the other magnet.

A perfectly astatic system would not long remain so, since actual magnets slowly lose their strength at unequal rates. A combination of magnets is most useful in a galvanometer when it is nearly, but not quite, astatic.

To ascertain if any given arrangement of magnets is astatic, find the resultant moment by the method of Art. 33 or Art. 34.

It may sometimes happen that a system, mounted on an axis which is fixed relative to the field, will remain at rest in any of its possible positions about this axis. Such an arrangement is temporarily in an astatic condition, but does not form an astatic system in the true sense of the term. In fact, *any* magnet, or system of magnets, is in an astatic condition if it can only rotate about an axis whose direction coincides with that of the field.

36. Unsymmetrical Magnets.

We have hitherto supposed that the magnets used were symmetrically magnetized (Art. 3). This is seldom the case with actual magnets, although with care in magnetization very close approximations can be obtained. In accurate work we must never *assume* that the magnets used are symmetrical. If a magnet has consequent poles, or is otherwise irregularly magnetized, its resultant poles may be far out of the centre line of the bar, but the position of the resultant axis may always be determined by the methods we are about to describe. The same methods will apply to the combinations of magnets which we have dealt with theoretically in Arts. 33, 34.

The student may here notice that the method of finding the axis is also that of finding the meridian, or direction of the field. We cannot accurately determine the direction of the axis by simple suspension unless we know the exact direction of the meridian, and to find the latter we must know the direction of the axis of the magnet. The dilemma is overcome by a method of inverting the magnet which determines both directions.

37. Determination of Axis and Meridian.

It is only convenient to suspend magnets so that they rotate in a horizontal plane. Otherwise, if the centre of gravity is not exactly in the axis of suspension, the position assumed by the magnet does not depend entirely on magnetic forces. This difficulty is experienced with the dip-circle.

A suitable copper or aluminium frame may be constructed to carry the magnet, and if this is suspended from a fixed support by a bundle of unspun fibres, the centre of gravity of the suspended mass always falls vertically below the point of suspension, and the direction of setting is then decided by magnetic forces only.

1. Axis of a magnetized steel disc.

Suspend the disc in a suitable carrier, having previously marked one of its diameters (as a reference line) by an arrow. Attach a

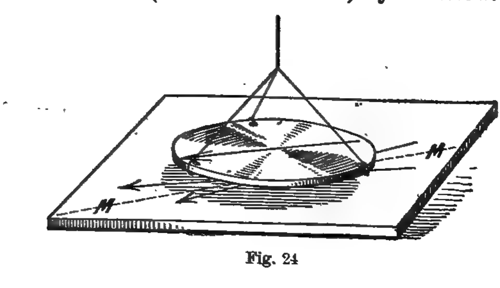


Fig. 24

sheet of paper to the table just below the disc. When the latter comes to rest, draw a line on the paper exactly under the reference line and mark it with a similar arrow. Now *invert* the disc in its carrier and allow it to come to rest again. It will generally do so in a position different from that first assumed. Draw a second

line on the paper under the new position of the reference line and mark it also by an arrow. The line bisecting the angle between the two lines on the paper is a *meridian* line. (The angle chosen for bisection is that bounded by lines on which the arrows *both* point away from the intersection.) A line drawn on the disc (at rest) exactly over the meridian line is the *magnetic axis* of the disc (fig. 24). The reference line only—not the axis—is shown in the figure.

2. Axis of a magnetized steel sphere.

When the meridian line has been determined as above, the direction of the resultant axis of a sphere is easily found.

Attach a hook to the sphere and suspend the latter centrally over the meridian line. When it comes to rest, its axis lies in a vertical plane through this line. Draw a great circle on the sphere in this plane. Now attach a hook at a different point, say, 90° from the first, and suspend the sphere again over the meridian line. Mark another great circle on the sphere in the meridian plane. The axis lies in the plane of this circle. The resultant axis therefore passes through the points where the two circles intersect.

If two other hooks are attached to the sphere at points diametrically opposite to the first two, the axis may be found, without a previous knowledge of the meridian, by applying the method of inversion used with the steel disc. We leave the details of the experiment as an exercise for the student.

The method adopted to find the resultant axis of a sphere may be applied to a magnet of any shape. In most experiments with bar magnets a knowledge of the horizontal component axis only is required, and the method described for the steel disc may be applied.

38. Determination of Dip.

The direction of the meridian does not completely specify the direction of the earth's resultant field; the angle of dip is required as well as the angle of declination. The measurement of dip is effected with a **dip-circle** (fig. 25). This consists of a vertical brass circle, mounted so that it can be rotated about a vertical axis, its position in azimuth being indicated by an attached pointer, which moves over a fixed

horizontal circle. The vertical circle is graduated in degrees, the extremities of the *horizontal* diameter being marked 0°. Two uprights carry two agate prisms or knife edges. A hard steel pivot passes through the centre of the needle and *rolls* on the knife edges, so reducing friction to a minimum.

When the plane of the circle is *in the meridian*, the horizontal and vertical components are both effective, and the needle sets at the angle of dip.

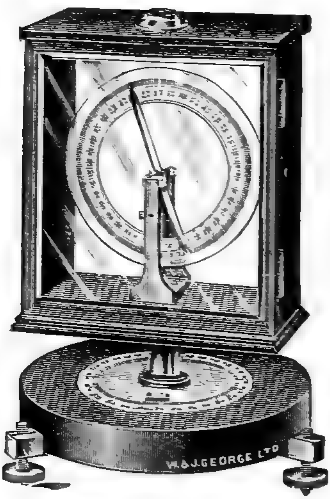


Fig. 25.—Dip-Circle

When the plane of the circle is *at right angles to the meridian*, the horizontal component is ineffective, and the needle stands vertical.

In intermediate positions the needle takes a direction between the vertical and the angle of dip.

When the plane of the meridian has been determined, the observation of dip is made in the following manner:—

(*a*) The positions of both ends of the needle are read.

(*b*) The needle is reversed face for face, and the readings (*a*) are taken again.

(*c*) The vertical circle is turned through 180°, and the readings (*a*) (*b*) are repeated.

(*d*) The needle is remagnetized, so that its polarity is reversed, and observations (*a*) (*b*) (*c*) are repeated.

The mean of the sixteen readings is the angle of dip.

The object of the above procedure is to eliminate the various errors which may affect the readings.

(1) *Error of Centering.*—If the axle of the needle does not coincide with the centre of the vertical circle, the reading will be too great at one end and too small at the other (fig. 26 (*a*)).

If the excentricity is not great, the error is eliminated by taking the mean.

(2) *Error of Magnetic Axis.*—If the magnetic axis does not coincide with the centre line of the needle, the observed dip will be too great or too small. This error is eliminated (as in determinations of meridian) by reversing the faces of the

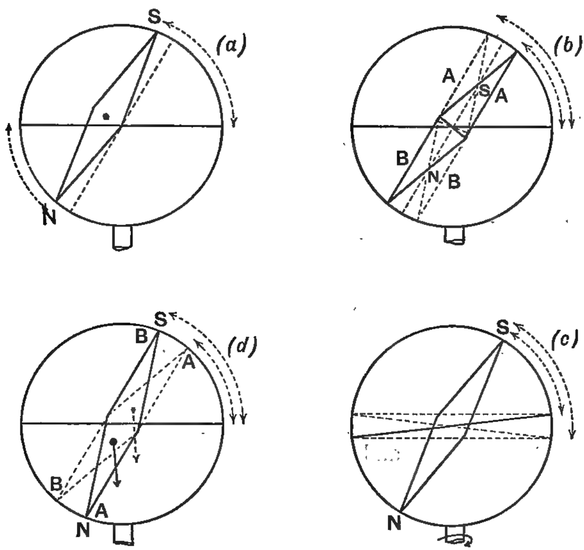


Fig. 26.—Dip Needle Errors

needle (fig. 26 (*b*)). The *magnetic* axis sets along the line of dip in each case.

(3) The line joining the zero points of the vertical scale may not be truly horizontal (fig. 26 (*c*)). This error is eliminated by operation (*c*).

(4) *Error of Centre of Gravity.*—In practice this is the most serious error. A displacement of the centre of gravity along a line at right angles to the magnetic axis is corrected in the observation (*b*). But if the centre of gravity is displaced along the magnetic axis, the error can only be eliminated by reversing

the polarity of the needle (fig. 26 (*d*)). If the lower half of the needle is the heavier the observed dip is too great; but when the polarity is reversed the heavier half is brought to the top, and the observed dip is then too small.

The meridian position of the vertical circle is determined as follows:—The circle is adjusted until the needle is *vertical*, eight readings being taken by following the order (*a*) (*b*) (*c*) above. The mean of these is the reading for the magnetic *east–west* position. The meridian position is 90° from this.

39. Examples.

1. Show how the magnetic moments of two unequally strong magnets may be compared by mounting them in the manner of astatic needles (i) with like, (ii) with unlike poles together, and observing their oscillations when so mounted. (1899.)

This is known as *Sum and Difference* method of comparing magnetic moments. It avoids the trouble of finding the moments of inertia, K being for the whole system the same in both experiments.

Let a and b be the separate moments, a being the greater. Then if M represents the resultant moment—

$$M_1 = a + b, \text{.................(Art. 34 II.)}$$

$$\text{and } M_2 = a - b.$$

$$\text{Also } \frac{M_1}{M_2} = \frac{n_1^2}{n_2^2}.$$

$$\therefore \frac{a+b}{a-b} = \frac{n_1^2}{n_2^2},$$

$$\text{or } \frac{a}{b} = \frac{n_1^2 + n_2^2}{n_1^2 - n_2^2}.$$

The comparison may be made in a similar manner by using the torsion balance. In this case—

$$\frac{a+b}{a-b} = \frac{A_1 - \delta}{A_2 - \delta}.$$

2. A bar magnet is suspended by a wire so as to hang horizontally. By how much must the top of the wire be twisted for the magnet to be deflected 90° from the meridian, when for a deflection of 30° it has to be twisted 120°? (1900.)

The torsion in the given case is 120° − 30° = 90°. Hence 90°

represents the couple MH sin 30°. If A is the required angle, then—

$$A - 90^\circ \text{ represents MH sin } 90^\circ.$$

$$\therefore \frac{A - 90^\circ}{90^\circ} = \frac{\text{MH sin } 90^\circ}{\text{MH sin } 30^\circ} = \frac{2}{1};$$

$$\therefore A = 270^\circ.$$

3. A small compass-needle makes 15 complete oscillations in 1 min. 30 sec. under the influence of the earth's field alone. A bar magnet is then placed along a meridian line, south of the needle, with its S. pole towards the north. The needle (whose direction is not reversed) now makes 26 oscillations in 650 seconds. Compare the field at the needle due to the magnet with the earth's field.

This example refers to the use of an exploring magnetometer (Art. 32.)

In the second instance the magnet's field is opposed to the earth's, but is weaker than the latter, since the needle is not reversed. Hence, if F is the intensity of the magnet's field at the needle, and H the intensity of the earth's field, the resultant field is H − F. Thus—

$$\frac{H}{H - F} = \frac{n_1^2}{n_2^2}.$$

Take 1 minute as the standard period for comparison. Then—

$$n_1 = \frac{15}{90} \times 60 = 10 \text{ oscillations per minute.}$$

$$n_2 = \frac{26}{650} \times 60 = 2{\cdot}4 \text{ oscillations per minute.}$$

$$\therefore \frac{H}{H - F} = \frac{100}{5{\cdot}76};$$

$$\therefore \frac{F}{H} = \frac{94{\cdot}24}{100}.$$

i.e. the magnet's field is 94·24 per cent of the earth's field.

QUESTIONS

1. A bar magnet is suspended in the earth's field, and the number of oscillations made per minute is ascertained. Will the rate of oscillation be altered if a second bar magnet, exactly similar in all respects, is laid on the top of the first, so that their like poles are coincident at each end?

2. What data are necessary to determine the magnetic moment of a magnet by observations of its rate of swing in a field of known strength?

3. A bar magnet is suspended horizontally in the magnetic meridian by a wire without torsion. To deflect the bar 10° from the meridian the top of the wire has to be turned through 180°. The bar is removed, remagnetized, and restored, and the top of the wire has now to be turned through 250° to deflect the bar as much as before. Compare the magnetic moments before and after remagnetization. (1894.)

4. Two magnets, the moment of one of which is double that of the other, are rigidly connected, with the centre and axis of the one vertically above the centre and axis of the other, and the whole suspended by a fine metallic wire. When the magnets are oppositely directed the top end of the wire has to be twisted through 75° in order to deflect the combination through 30° from the meridian. By how much must the top end of the wire be twisted when the magnets are similarly directed in order that the combination may take up a position perpendicular to the magnetic meridian? (1897.)

5. Two pieces of steel, of which one is a little longer than the other, are magnetized so that, when attached by their centres to a piece of wire and suspended so that both are horizontal and in the same vertical plane, the combination is astatic. Will it be astatic if suspended by its centre of gravity so that the magnets are in a horizontal plane? Give reasons. (1891.)

6. What data are necessary in order that the moments of two magnets may be compared by observation of their times of oscillation in a constant magnetic field? Describe the experiment. (1903.)

CHAPTER IV

REMOTE FIELD OF A MAGNET—DEFLECTIONS

40. We now proceed to the second case in which we can make calculations on magnets and apply the idea of a concentrated pole, namely, when we are dealing with the field *due to* a magnet at some distance (Art. 21). The poles are then fixed points.

41. Field-strength along the Axis.

Let N, S, fig. 27, be the poles, and P a point on the axis. Let d be the distance from P to the magnetic centre, and let l = *half*

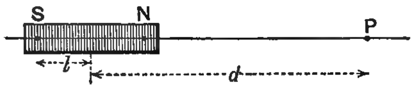


Fig. 27

the magnetic length. Remember that d is supposed great in comparison with the dimensions of the chief portion of the polar surface.

$$\text{Then PN} = d - l;\ \text{PS} = d + l.$$

$$\therefore \text{ repulsion of N on unit pole at P} = \frac{m \times 1}{(d-l)^2};$$

$$\text{and attraction of S on unit pole at P} = \frac{m \times 1}{(d+l)^2}.$$

$\therefore$ resultant force (magnetic)—

$$\begin{aligned} F &= \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2} \\ &= \frac{m(d^2 + 2dl + l^2 - d^2 + 2dl - l^2)}{(d^2 - l^2)^2} \\ &= \frac{2(m \,.\, 2l)d}{(d^2 - l^2)^2} \\ &= \frac{2Md}{(d^2 - l^2)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1) \end{aligned}$$

With ordinary bar magnets this formula is sufficiently true when d is greater than five or six times the length of the magnet.

Field-strength across the Bisector.

Fig. 28 refers to this case.

The component forces due to N and S are equal, but not quite in the same line. Hence we must find their resultant from a parallelogram of forces. Let PR represent (in magnitude and direction) the repulsion due to N,

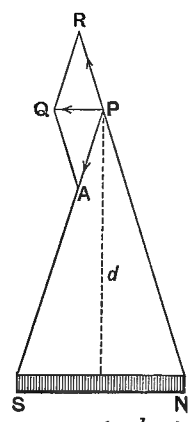


Fig. 28

and PA the attraction due to S. Then the diagonal PQ of parallelogram PRQA is the resultant required.

$$\frac{\text{F (resultant)}}{\text{Repulsion}} = \frac{\text{PQ}}{\text{PR}}$$
$$= \frac{\text{NS}}{\text{NP}},$$

because the triangles PNS and RPQ are similar.

$$\text{Now, NS} = 2l;\ \text{NP} = \sqrt{d^2 + l^2};$$
$$\text{and repulsion of N} = \frac{m \times 1}{\text{NP}^2} = \frac{m}{(d^2 + l^2)}.$$

Therefore, substituting, we have—

$$\frac{\text{F}}{\text{Repulsion}} = \frac{2l}{\sqrt{d^2 + l^2}}.$$
$$\text{F} = (\text{repulsion}) \times \frac{2l}{\sqrt{d^2 + l^2}}$$
$$= \frac{2ml}{(d^2 + l^2)\sqrt{d^2 + l^2}}$$
$$= \frac{\text{M}}{(d^2 + l^2)^{\frac{3}{2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Notice that this magnetic force acts parallel to the magnet, *i.e.* at right angles to the bisector.

42. Field at Greater Distances.

The formulæ may be simplified if the point is very distant from the magnet in comparison with the length of the latter. l^2 becomes so small in comparison with d^2 that it may be left out of the formulæ.

EXAMPLE.—A magnet is 10 cm. long, and the point is half a metre away.

Here l (half the magnetic length) is somewhat less than 5 cm. $\therefore l^2 < 25$, and $d^2 = 2500$.

$\therefore l^2 < \frac{1}{100}$th of d^2, and may, therefore, for many purposes, be neglected At greater distances the ratio becomes smaller still.

Omitting l^2 from the formulæ (1) and (2), we have—

$$\text{F} = \frac{2\text{M}}{d^3} \ldots \text{(on the axis)}, \ldots\ldots\ldots\ldots\ldots(3)$$

$$\text{and F} = \frac{\text{M}}{d^3} \ldots \text{(on the bisector)} \ldots\ldots\ldots\ldots\ldots(4)$$

In connection with these simplified formulæ you should notice the following points:—

(i) *The force at a point on the bisector is only half as great as the force at an equally distant point on the axis.*

(ii) *A small displacement of the magnet "parallel to itself" will not affect the distant field,—the small change in* d *so produced may be neglected for the same reason that* l *is neglected.*

(iii) *The force depends on* M. *Hence the force is unaltered by replacing the magnet with another of different length but equal moment, with its axis lying in the same direction.*

E.g. a magnet of pole-strength 5 and length 12 has the same effect at a distant point as one of pole-strength 3 and length 20, or pole-strength 1 and length 60.

These results depend on the circumstance that the point in question is so distant that lines drawn from it to all parts of the magnet are practically parallel. Contrast this with the assumption made in Art. 41, where the parallelism only extends over the strongest portion of the polar surface.

43. **Field at any very Remote Point.**

Equations (3) and (4) may be applied to find the field-strength at points *off* the bisector or axis. We shall first show that for the purposes of this calculation the *moment of a magnet may be resolved into components.*

This we have already seen is true when the magnet is acted on by a uniform field (Art. 34), and we shall now prove it true for the second standard case, mentioned in Art. 21.

Let SN be the magnet, P the point (fig. 29). SP makes an angle θ with the axis SN. Let m and l be the pole-strength and length of the magnet. Draw NK perpendicular to SP. A unit north pole being placed at P, we get repulsion along NP and attraction along SP. We might construct a parallelogram of forces at P, but the triangle of forces is here more convenient. Let AP = attraction, PR = repulsion; then AR = required resultant (which, however, acts at P). Let angle RAL = ϕ.

$$\mathrm{PN} = d;\ \mathrm{SK} = l\cos\theta;\ \mathrm{NK} = l\sin\theta;$$
$$\mathrm{SP} = d + l\cos\theta.$$

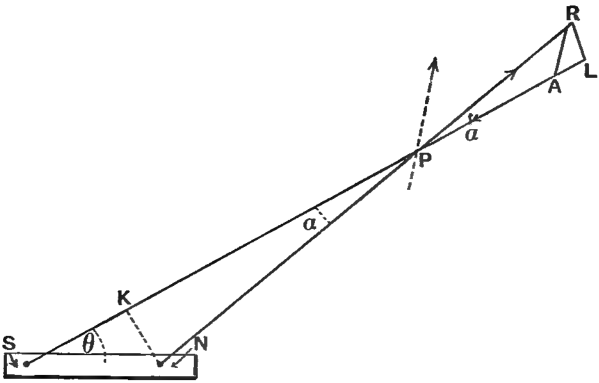


Fig. 29

On account of the smallness of the angle α, we have, approximately, RP = PL and $\sin \alpha = \alpha$ (Art. 30). The resultant AR may be resolved into components AL and LR.

$$\text{Now, AL} = \text{PL} - \text{PA} = \text{PR} - \text{PA}$$
$$= \frac{m}{d^2} - \frac{m}{(d + l\cos\theta)^2}$$
$$= md^{-2} - m(d + l\cos\theta)^{-2}.$$

Expanding the second term by the binomial theorem, and neglecting terms containing l^2 and higher powers—

$$\text{AL} = \frac{2ml\cos\theta}{d^3} = \frac{2\text{M}\cos\theta}{d^3} \quad \text{.................(5)}$$

$$\text{Next, RL} = \text{RP}\sin\alpha = \text{RP}\cdot\frac{\text{NK}}{\text{NP}}$$
$$= \text{RP}\cdot\frac{l\sin\theta}{d}$$
$$= \frac{m}{d^2}\cdot\frac{l\sin\theta}{d}$$
$$= \frac{\text{M}\sin\theta}{d^3} \quad \text{...................(6)}$$

$$\text{Also, } \tan\phi = \frac{\text{RL}}{\text{AL}} = \frac{1}{2}\tan\theta \quad \text{...........................(7)}$$

Equations (5) and (6) show that the resultant force is such as would be produced by two magnets of moments M cos θ

and M sin θ, placed along the line SP and perpendicular to it respectively. (Since the lines SP and NP are practically parallel, we may take a line drawn to the middle of NS instead.) But these magnets would be the components of the given magnet. The parallelogram law is therefore applicable.

EXAMPLE.—A short magnet has moment 500. Find the direction and magnitude of the resultant force at a point situated on a line making an angle of 60° with the axis, and 30 cm. distant.

Let O be the middle of the magnet.

Component magnet along OP has moment $M \cos 60° = \frac{M}{2}$.

Component magnet at right angles to OP has moment

$$M \sin 60° = \frac{M\sqrt{3}}{2}.$$

Component forces at O—

$$\text{Force along OP} = 2 \cdot \frac{M}{2} \cdot \frac{1}{d^3} = \frac{M}{d^3}.$$

$$\text{Force perp. to OP} = \frac{M\sqrt{3}}{2d^3}.$$

$$\therefore \text{Resultant} = \frac{M}{d^3}\sqrt{\left(1 + \frac{3}{4}\right)} \ldots\text{(By Euc. I, 47.)}$$

$$= \frac{500 \times \sqrt{7}}{27000 \times 2} = \frac{\sqrt{7}}{108} \text{ gauss.}$$

For the direction of the resultant we have (Eqn. 7)—

$$\tan \phi = \tfrac{1}{2} \tan \theta = \tfrac{1}{2} \cdot \sqrt{3}.$$

44. Remarks Concerning the Remote Field.

The angle ϕ in equation (7) is the angle between the lines of force in the field and a radial line from the middle of the magnet. Equation (7) shows also that this angle does not depend on the distance. Hence a radial line from the magnet cuts all the lines of force in the remote field at the same angle (fig. 30).

It follows also from this that the remote fields of all magnets are similar, however different the fields may be close to the magnets. If two small magnets are superposed with their axes parallel, their remote fields will coincide in direction at

all points. The strengths of these fields will be different, being proportional to the moments of the magnets.

The formulæ relating to remote fields may be regarded as absolutely true for *molecular* magnets.

Distinguish carefully between the laws expressed by the equations—

$$(a)\ F = \frac{m}{d^2};\ (b)\ F = \frac{2M}{d^3},\ \text{or}\ \frac{M}{d^3}.$$

The first equation refers to the force produced by a *single pole*, *m* being the *pole-strength* and F the magnetic force. It is

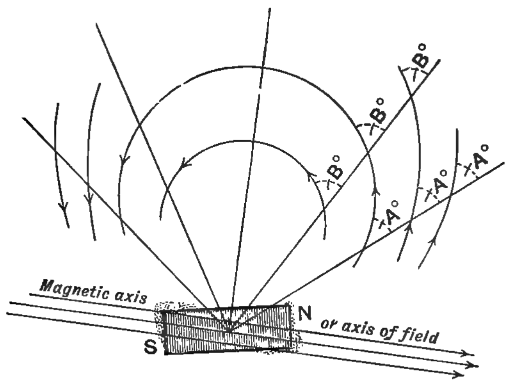


Fig. 30

known as the **Law of Inverse Squares**, and forms the basis of most calculations on permanent magnets.

The second equation refers to the force produced by both poles, *i.e.* by the *whole magnet*, M being the *magnetic moment*. It is known as the **Law of Inverse Cubes.**

If a magnet is broken into halves, the pole-strength of each half is practically as great as in the original piece; but at a given distance the field due to each half is weaker than the field of the original magnet. The poles are brought nearer together, and therefore more nearly neutralize each other's component forces. Hence, the shorter the magnet for a given

pole-strength the weaker is its field. Thus the field depends on the moment as shown by equations (*b*).

DEFLECTION EXPERIMENTS

45. The Tangent Law.

In many instruments a compass-needle is deflected away from the magnetic meridian by a uniform field acting at right angles to this plane. In such cases each pole of the needle is acted on by two forces—

(a) *a controlling force parallel to the meridian;*
(b) *a deflecting force at right angles to the meridian.*

The needle, therefore, sets in the direction of the resultant of these two forces. Thus, if NC is the controlling force tending to pull the needle back to the meridian (fig. 31), and ND the deflecting force, we may construct the parallelogram of forces NDRC, and so determine the direction of the resultant NR. The axis SN of the needle will point along NR.

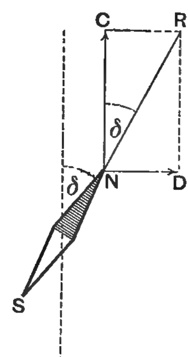


Fig. 31

If δ is the angle of deflection, then $\delta = \angle CNR$. Also—

$$\frac{\text{Deflecting force}}{\text{Controlling force}} = \frac{ND}{NC} = \frac{CR}{NC} = \tan\delta.$$

If m is the pole-strength of the needle, H the intensity of the controlling field, F the intensity of the deflecting field, we have—

$$\text{Deflecting force} = Fm,$$
$$\text{Controlling force} = Hm.$$
$$\therefore \frac{Fm}{Hm} = \tan\delta,$$
$$\text{or } \frac{F}{H} = \tan\delta \quad \text{.................. (8)}$$

This important result may be stated thus:—

If a magnet is subject to the action of a field of intensity H, and is deflected from the direction of this field by a second field of intensity F acting at right angles to the first, the tangent of the deflection is equal to the ratio of F to H.

46. Application to Magnetic Experiments.

When the deflecting field is due to a magnet, the formula can only be applied if the magnet is situated at a considerable distance from the needle, or if the needle is exceedingly short; for the field must be *uniform* over the region in which the needle swings.

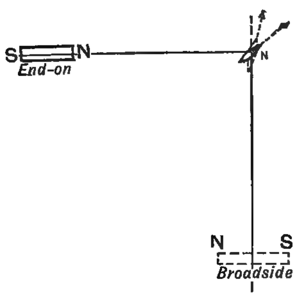


Fig. 32.—Gauss Positions

There are two principal positions in which a magnet may be placed so that its field acts at right angles to the earth's field at the point occupied by the centre of the needle. These are called the *Gauss positions* (fig. 32).

In the "end-on" or "A" position the magnet is placed with its centre on a (magnetic) E.–W. line drawn through the centre of the needle, and with its axis lying along this line.

In the "broadside" or "B" position the magnet is placed with its centre on a (magnetic) N.–S. line drawn through the centre of the needle, and with its axis at right angles to this line.

In both cases the axis of the magnet is at right angles to the meridian.

(The earth's field is here taken as a typical controlling field, but the controlling force is often produced by a magnet instead.)

If the needle is at a sufficient distance from the magnet, we may calculate the deflection thus:—

End-on position. We have, $F = H \tan \delta$.
But since F acts along the axis of the magnet—

$$F = \frac{2M}{d^3}.$$

$$\therefore \frac{2M}{d^3} = H \tan \delta;$$

$$\frac{M}{H} = \frac{d^3 \tan \delta}{2} \qquad \text{.........................(9)}$$

This expresses the ratio $\frac{M}{H}$ in terms of quantities which may be measured.

Broadside. As before, $F = H \tan \delta$.
But F acts across the bisector of the magnet.

$$\therefore F = \frac{M}{d^3}.$$

$$\text{Thus } \frac{M}{d^3} = H \tan \delta;$$

$$\frac{M}{H} = d^3 \tan \delta \qquad \text{.........................(10)}$$

For accurate work we must use, instead of the simpler expressions for F, the more complete equations given at the beginning of this chapter, (1) and (2). We thus obtain—

$$(a)\ \frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan \delta \qquad \text{.........................(11)}$$

$$(b)\ \frac{M}{H} = (d^2 + l^2)^{\frac{3}{2}} \tan \delta \qquad \text{.........................(12)}$$

EXAMPLES.—1. A short bar magnet is placed "end-on" to a compass-needle along the E.-W. line at a distance of 30 cm. from the needle. If the deflection is 30°, find the ratio of the moment of the needle to the earth's field-strength.

Here l, not being mentioned, is supposed negligible. We have—

$$\frac{M}{H} = \frac{(30)^3 \times \tan 30^\circ}{2} = \frac{30^3}{2} \times \frac{1}{\sqrt{3}} = 7794.$$

2. Two magnets of negligible length are placed with their centres magnetic north and south of a compass, the distances being 36 in. and 24 in. respectively. The N. pole of the former magnet points magnetic west, and the N. pole of the latter magnetic east. In this

position the needle is undisturbed. Find the ratio of the magnetic moments.

A sketch of the arrangement will show you that the magnets are in the broadside-on position, and that their fields tend to deflect the needle in opposite directions. Since the needle is undisturbed, we have—

$$\frac{M_1}{d_1^3} = \frac{M_2}{d_2^3}, \text{ or } \frac{M_1}{M_2} = \frac{d_1^3}{d_2^3}.$$

$$\therefore \text{ Required ratio } \frac{M_1}{M_2} = \frac{3^3}{2^3} = \frac{27}{8}.$$

It is immaterial whether the distances are expressed in inches or feet, since we are dealing with a *ratio.*

47. Deflection Magnetometer.

The usual form of this instrument is shown in fig. 33. The essential part is a short needle of hard steel strongly mag-

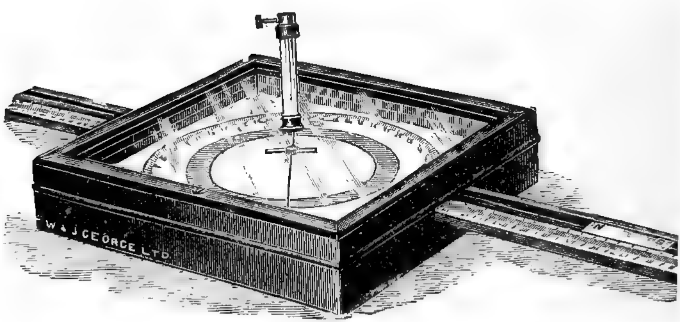


Fig. 33.—Deflection Magnetometer

netized. It is provided with an agate centre, by which it is delicately balanced on a hard steel point. The needle carries a light aluminium pointer, by which the deflections can be read from a large circular scale. The shallow box containing the needle is provided with two long arms on which the magnets may be placed. The distance between the middle of the magnet and the middle of the needle can be read off from the scales attached to the arms.

Method of taking deflections.

The magnets used are not in general quite symmetrical, and in order to ensure accuracy the mean of a number of observations must be taken with the magnet in different positions.

(*a*) Error of centering. The needle may not be pivoted quite at the centre of the magnetometer circle. To eliminate this, read from both ends of the pointer and take the mean of these readings.

(*b*) The centre of the magnet may not be the magnetic centre. Reverse the magnet end for end and take the mean of the deflections.

(*c*) The centre line of the magnet may not coincide with the axis. If the want of symmetry is not great, this may be corrected by turning the magnet over, keeping the N. end pointing in the same direction. This is only necessary in "tangent" observations and with magnets of some width.[1] With knitting-needle magnets it may be omitted.

(*d*) The common zero of the scales may not coincide with the centre of the needle. One means of correcting for this is to make the above readings (*a*), (*b*), (*c*) on each arm at equal scale readings.

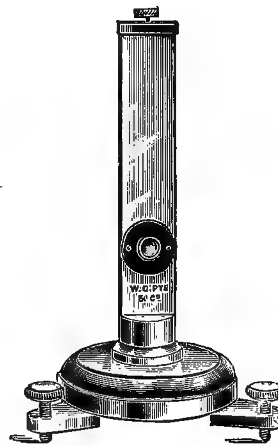


Fig. 34.—Reflecting Magnetometer

The mean of the 16 observations can then be calculated.

For accurate work a **reflecting magnetometer** is used (fig. 34). This consists of two light strips of steel strongly magnetized and cemented to the back of a small concave mirror. The latter has usually a diameter of $\frac{3}{8}$ in. and a radius of curvature 40 in. It is suspended by a single fibre of unspun silk inside an upright brass case, having a plane glass front. A horizontal scale is used to receive the

[1] In magnetometric experiments, where great accuracy is required, the magnet is adjusted to lie on the axis or bisector of the *deflected* needle. These are known as "sine" observations; they eliminate the effect of the transverse component of the magnetic moment.

image or "spot" of light reflected from the mirror. A lens is usually employed near the lamp, and serves a double purpose. (*a*) It focuses an image of the source of light on the mirror, so increasing the illumination; (*b*) it serves as a convenient object, the circular spot of light on the scale being an image of the lens. A fine vertical scratch on the lens shows as a vertical black line on the spot, and so makes accurate reading possible, whilst the large area of the spot enables the figures to be read.

The beam from the lamp is fixed in direction, and it follows that the reflected beam is deflected through an angle twice as great as the deflection of the mirror (or magnets). On account of the great distance of the scale the spot moves a considerable distance for a very small movement of the needle. The reflected beam forms a long weightless pointer, and the instrument is thus very sensitive.

For *comparative* measurements or ratios it is generally sufficiently accurate to take the scale readings as representing the deflection.

For *absolute* measurements the angular deflection of the reflected beam must first be found. If s is the number of scale divisions in the deflection, and L the distance of mirror from scale *expressed in scale divisions*, and 2δ the deflection of beam—

$$\tan 2\delta = \frac{s}{L}.$$

2δ may then be ascertained in degrees from a table of natural tangents. δ is thus ascertained, and its tangent may be found by a second reference to the tables.

The instrument must be set up facing E. or W., and the scale must be quite perpendicular to the line from its zero to the mirror. N.–S. and E.–W. lines may be drawn on the table to facilitate the placing of magnets in the end-on and broadside positions.

The magnets used should be strips of steel, of knitting-needle thickness, some 10 cm. long. These must be glass-hardened by heating to bright redness and sudden cooling in cold water. They should then be uniformly magnetized by the use of a solen-

oid. The last portions of the sub-permanent magnetism may be removed by plunging the strip several times into boiling water and allowing it to cool between the immersions.

The observations are taken thus:—

(*a*) The magnet is placed at a distance from the needle on the N.-S. or E.-W. line, and the deflection δ_1 is noted. It is then reversed end for end and the deflection δ_2 noted. The mean $(\delta_1 + \delta_2)/2$ is then calculated.

(*b*) The magnet is removed to the opposite side of the needle, and the deflection δ_3 is noted; it is then reversed end for end, and deflection δ_4 found. The distance on this side must be adjusted until the mean $(\delta_3 + \delta_4)/2$ equals the former mean. The mean deflection is thus obtained. The distance of the magnet from the needle is half the distance between the two positions of the magnet, these distances being all measured from the middle of the magnet.

48. Magnetometer Experiments.

(i) Gauss's test of the Law of Inverse Squares.

Formulæ (1) and (2) are obtained on the assumption that this law is true, and from these formulæ (11) and (12) are derived. Thus for given magnet in a given locality we have—

$$\frac{\mathrm{M}}{\mathrm{H}} = \frac{d^3 \tan \delta}{2} = d_1{}^3 \tan \delta_1.$$

If, further, the distances are equal—

$$\tan \delta = 2 \tan \delta_1.$$

Thus, if the law of inverse squares is true, the tangent of the deflection in the *end-on* position is *twice* the tangent in the *broad-side* position.

Now it may be shown by methods similar to those used in obtaining the above formulæ, that if the law were $\mathrm{F} \propto \frac{1}{d^3}$, we should have $\tan \delta = 3 \tan \delta_1$; or if the law were $\mathrm{F} \propto \frac{1}{d^x}$, we should have $\tan \delta = x \tan \delta_1$; and so on. Hence, if we can show that the first tangent is *twice* the second, we prove the law of inverse *squares*.

The test should be made with a reflecting magnetometer and well-magnetized bars of hard steel. The distance should

be as great as possible consistent with a good deflection, so that the length of the magnet may be neglected.

(ii) Effective length of a magnet.

The effective or "magnetic" length may be obtained approximately by combining observations in the end-on and broadside positions, using the "moderate distance" formulæ (11) and (12). Thin regular magnets should be used, and a reflecting magnetometer.

Let d be the distance, which is same in both observations, and $2l$ the length required. Then since $\frac{M}{H}$ has the same value in both observations—

$$\frac{(d^2 - l^2)^2}{2d} \tan \delta = (d^2 + l^2)^{\frac{3}{2}} \tan \delta_1.$$

$$\text{Put } \frac{l}{d} = x; \text{ and } \frac{2 \tan \delta_1}{\tan \delta} = z.$$

Then x is a comparatively small fraction, and z is nearly equal to unity (compare (i) above).

Thus $$(1 - x^2)^2 = (1 + x^2)^{\frac{3}{2}} z.$$

Expand both sides by the binomial theorem, and neglect powers of x higher than the second.

Thus $$(1 - 2x^2) = (1 + \tfrac{3}{2}x^2)z.$$

Therefore $$x^2 = \frac{2(1 - z)}{3z + 4};$$

or $$l = d\sqrt{\frac{2(1 - z)}{3z + 4}}.$$

(iii) Ratio of M to H.

To determine the ratio of the numerical values of M and H, first determine the effective length of the magnet. Then, making use of the results obtained in the (say) end-on position, calculate the ratio from the equation—

$$\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan \delta.$$

If the magnet is short and strongly magnetized it may be placed at some distance; l is then negligible, and

$$\frac{M}{H} = \frac{d^3 \tan \delta}{2}.$$

49. Absolute Measurement of M and H.

Our measurements hitherto have all been of the comparative kind, *i.e.* they have merely determined *how many times* one magnetic moment or field-strength was greater than another. We have not yet seen how to determine the moment of any one magnet or the strength of any one field in terms of the units of these quantities. A measurement of this kind is called an *absolute* measurement.

Two experiments are required to measure either M or H. (Compare note at end of Art. 36.)

1. The oscillation experiment.

Determine carefully as described in Art. 32 the time of oscillation in seconds of a uniformly magnetized strip, say 10 cm. long. Find the moment of inertia of the strip from its dimensions and weight.

Then $$\mathrm{MH} = \frac{4\pi^2\mathrm{K}}{t^2} = x \text{ (say).}$$

2. The deflection experiment.

Place the reflecting magnetometer in the position previously occupied by the oscillation instrument. Place the magnetized strip in the end-on position, and determine the mean deflection in the manner already described. The effective length of the strip should be also determined.

Then $$\frac{\mathrm{M}}{\mathrm{H}} = \frac{(d^2 - l^2)^2}{2d} \tan\delta = y \text{ (say).}$$

$$\left[= \frac{d^3 \tan\delta}{2} \text{ approximately.}\right]$$

Now to find the moment of the magnet, multiply x and y.

$$\frac{\mathrm{M}}{\mathrm{H}} \times \mathrm{MH} = \mathrm{M}^2 = xy.$$

$$\therefore \mathrm{M} = \sqrt{xy}.$$

To find the strength of the earth's field, divide x by y.

$$\mathrm{MH} \div \frac{\mathrm{M}}{\mathrm{H}} = \mathrm{H}^2 = \frac{x}{y}.$$

$$\therefore \mathrm{H} = \sqrt{\frac{x}{y}}.$$

The average value of the earth's horizontal field in England is ·17 gauss. It decreases as we go nearer the poles.

The pole-strength of a magnet may be found by determining M as above, and l according to (48, ii).

Then $$m = \frac{M}{l}.$$

50. Couple Due to the Field of a Distant Magnet.

In arriving at the tangent law we considered only the force at one end of the needle. But an equal and opposite force is exerted on the other end of the needle, and these form a couple. The couple exerted on one short magnet by the field of another short magnet at a distance may be found as follows:—The formula of Art. 28 was worked out primarily for the earth's field, but applies to any uniform field, δ being the angle between the axis of the magnet and the direction of the field.

Thus $$G = MF \sin \delta,$$

and F can be determined for the remote field. Thus G is determined.

Example.—Two short bar magnets are placed so that the axis of one (M_1) coincides with the bisector of the other (M_2). If the magnets are prevented from rotating, find the couples tending to rotate each magnet.

(*a*) Field due to M_1 at the second magnet $= \frac{2M_1}{d^3}$ (end-on).

Angle between this field and the axis of the second magnet $= 90°$.

$$\therefore G_2 = M_2 \cdot \frac{2M_1}{d^3} \cdot \sin 90° = \frac{2M_1M_2}{d^3}.$$

(*b*) Field due to M_2 at first magnet $= \frac{M_2}{d^3}$ (broadside).

Angle between this field and the axis of the first magnet $= 90°$.

$$\therefore G_1 = M_1 \cdot \frac{M_2}{d^3} \cdot \sin 90° = \frac{M_1M_2}{d^3}.$$

There is a curious point to be noticed here. The couple exerted

on the second magnet is double that exerted on the first. Hence if the magnets were glued to a board and floated it might appear that there would be a resultant couple. This, however, would keep the board in perpetual rotation; also the component *mutual* attractions and repulsions could not produce a resultant couple. The explanation lies in the fact that the field round each magnet is *not quite* uniform. The forces on the poles of M_2 are equal but not quite parallel; those acting on the poles of M_1 are parallel but not quite equal. If you resolve these forces at right angles to the magnets and parallel to them (fig. 35), you will see that there are *weak* resultant forces which may be referred to the centre of gravity of each magnet (compare Art. 20, latter portion). These weak forces form a couple with a long arm ($=d$). A complete investigation shows that this exactly balances the resultant of the couples already calculated. Hence there is no resultant for the whole system.

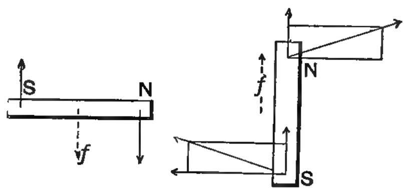


Fig. 35.—Mutual Action of Magnets

51. Unsymmetrical Magnets.

Magnets used in the end-on tangent position must be symmetrical or very nearly so. Very irregular magnets having their axes a long way out of their centre line must be tested for their component moments along and perpendicular to this line. The former is obtained from the end-on experiment, the position being adjusted so that the deflecting force varies as the *sine* of deflection. The latter is obtained by rotating the magnet and adjusting to a sine position in which the centre line is parallel to the deflected needle.

52. Examples.

1. A magnet 10 cm. long is placed in the magnetic meridian, the "north" end of the magnet being to the south. The force

due to this magnet just counterbalances the earth's horizontal magnetic force (·18 C.G.S. unit) at a place 35 cm. from the centre of the magnet (along its axis produced). Find the strength of each pole of the magnet. (1905.)

See fig. 16.

The magnetic force due to the magnet at 35 cm.

$$= \frac{2 \cdot M \cdot 35}{(35^2 - 5^2)^2}.$$

Since this balances the earth's field,

$$\cdot 18 = \frac{70\,M}{(35^2 - 5^2)^2}. \quad \therefore M = 3702.$$

We have to take 10 as the magnetic length.

Hence since $M = 2ml$,

$$m = \frac{3702}{10} = 370{\cdot}2 \text{ C.G.S. units.}$$

2. Prove that the work done in twisting a magnet of moment M through 90° from the meridian in a field of strength H is given by the product MH. (1904.)

Let m be the pole-strength.

Then mechanical force exerted by the field on each pole $= f = Hm$.

Now as the magnet rotates the N. pole moves across the lines of force of the earth's field, as well as along these lines. Its motion at any instant might be resolved into two component *motions*, one E. to W. and the other N. to S. The *force* exerted on the pole is always N.–S., and will therefore neither help nor hinder the former movement.

Hence we have only to consider how far the pole moves in *a direction parallel to the lines of force.* Since the rotation is 90°, this is evidently equal to half the length of the magnet.

Hence work done on the N. pole—

$$= \text{force} \times \text{distance} = Hm \times l.$$

An equal amount of work is done on the S. pole.

$$\therefore \text{Total work} = 2Hm \times l,$$

or, work done on the whole magnet $= HM$.

3. Two short bar magnets, the moments of which are 108 and 192 respectively, are placed along two lines drawn on the table at

right angles to each other. Find the intensity of the magnetic field due to the two magnets at the point of intersection of the lines, the centres of the magnets being respectively 30 and 40 cm. from this point. (1900.)

Both magnets are "end-on" to the point. The lengths of the magnets are not given. Hence, using the remote distance formula, the fields are respectively—

$$\frac{2 \times 108}{30^3} \text{ and } \frac{2 \times 192}{40^3};$$

$$i.e. \quad \frac{8}{1000} \text{ and } \frac{6}{1000},$$

$$\text{or} \quad 4 \times \frac{1}{500} \text{ and } 3 \times \frac{1}{500}.$$

These components act at right angles to each other. The resultant is represented by the diagonal of a rectangle whose sides are in ratio 4:3. To the same scale the diagonal is represented by 5.

$$\text{Hence required resultant} = 5 \times \frac{1}{500}$$
$$= \cdot 01 \text{ gauss.}$$

QUESTIONS

1. Show that the force exerted by a bar magnet, of magnetic moment M, on a unit pole placed at any point in the plane which bisects the magnet at right angles is Mr^{-3}, where r is the distance of the point from either pole, it being assumed that the magnetic strength of the magnet is concentrated at its poles. How may this result be applied in the comparison of the horizontal component of the earth's magnetic field at different points? (1894.)

2. A magnet is placed so that its centre is due magnetic east of the centre of a compass-needle, which is so mounted that it cannot dip. The magnet is made to rotate in a vertical plane about a horizontal axis which lies east and west, and passes through its centre. Describe the behaviour of the compass-needle during one complete revolution of the magnet. (1895.)

3. Prove that the magnetic force exerted by a short magnet at a point A on the line passing through its centre and perpendicular to its axis, is the same as the force exerted at a point on the axis, the distance of which from the centre of the magnet is $\sqrt[3]{2}$ times the distance of A from the centre. (1896.)

4. A short bar magnet is placed on a table with its axis perpendicular to the magnetic meridian, and passing through the centre of a compass-needle. In London the compass-needle is deflected through a certain angle when the centre of the magnet is 25 in. from the centre of the needle. If the experiment be repeated in Bombay the magnet must be moved 5 in. nearer to the needle to produce the same deflection. Use these data to compare the horizontal forces in London and Bombay. (1895.)

5. The centres of two short magnets AB and CD are at a distance r apart. AB lies along the line joining their centres and CD is at right angles to it. Show that the couple due to AB tending to make CD twist round is $2MM'/r^3$, where M and M′ are the magnetic moments of the magnets. (1898.)

6. A magnet placed due east (magnetic) of a compass-needle deflects the needle through 60° from the meridian. If at another station, where the horizontal force of the earth's magnetism is three times as great as at the first, the same magnet be similarly placed with respect to the compass-needle, what will be the deflection of the latter? (1897.)

7. Describe some method of comparing the magnetic moments of two magnetic needles.

8. A short bar magnet, moment 300, is placed at one corner of a square, with its axis lying along a side of the square. Find the direction and strength of the resultant field at each of the remaining corners, if the side of the square is 50 cm. long.

CHAPTER V

MAGNETIZATION

53. Magnetic Force inside a Magnet.

We are already accustomed to regard a magnetized substance as consisting of molecules arranged along definite lines more or less parallel to the length of the magnet. Let us suppose that the N. pole of each molecule is close to the S. pole of the next, as in fig. 2 (b). All through the interior these molecular poles neutralize each other's effects. We may therefore say that the field of the magnet is due to the ex-

posed molecular poles at the ends of each chain of molecules. The exposed poles constitute the "free" magnetism on the ends and sides of the magnet.

The lines of force inside a magnet as well as outside may be considered to arise from the free magnetism of the polar surfaces,[1] the existence of polarized molecules within the bar being completely ignored. The whole set of lines arising from the free magnetism is termed the *polar* field (figs. 10, 11).

The lines of force due to a neighbouring magnet or electric current and the earth's magnetic field must also be considered. These may be termed the *impressed* fields.

The resultant magnetic force at any point inside or outside a magnet is therefore compounded of—

(*a*) the **polar force** at the point,
(*b*) the **impressed force.**

The internal polar force in an ordinary bar magnet exerts a demagnetizing influence. Compare Art. 10 (ii).

So long as we are dealing with the mechanical forces exerted on hard steel magnets, we may neglect the internal molecular condition of the magnets. But in many cases (the most important of which is the study of induced currents) the internal condition must be considered.

54. Intensity and Direction of Magnetization.

Imagine that a small test-piece is cut out from a magnet and removed without change of condition. All the molecules in the piece are arranged in parallel lines, and we say that the specimen is uniformly magnetized. Its magnetic moment depends on its volume. The moment of a piece having unit volume depends on the moment of each molecule and the closeness with which the molecules are packed, and is taken as a measure of the magnetic condition of the substance.

> **The intensity of magnetization at any point of a magnet is the magnetic moment of unit volume taken about that point.**

[1] In the elementary study of magnetization the possible volume distribution of magnetism may be neglected.

The following alternative definition is useful. Let the test-piece be a small cylinder, length l, cut parallel to the lines of molecules. Let the ends of the cylinder have area a, and carry a quantity of free magnetism m. Then if I is the intensity of magnetization, we have by the preceding definition—

$$\mathrm{I} = \frac{\text{Magnetic moment}}{\text{Volume}} = \frac{m \times l}{a \times l} = \frac{m}{a} \quad \text{.........(1)}$$

Thus—

The intensity of magnetization in a given direction is the quantity of free magnetism on unit area of a section perpendicular to the direction.

To deal with the magnetization at a point inside a magnet we may suppose that (instead of removing a test-piece) a saw-

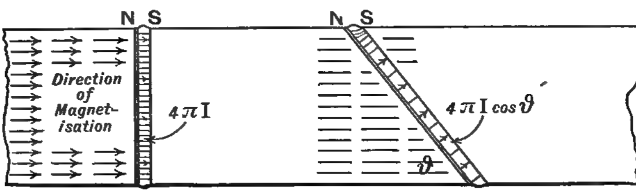


Fig. 36.—Force in a Crevasse

cut is made. The density[1] of free magnetism on the faces of such a gap or crevasse is the intensity of magnetization at right angles to the gap (fig. 36).

Magnetization, like magnetic moment, is a directed quantity, and can be resolved into components by the parallelogram law. If I is the actual magnetization, and I′ the component in a direction inclined θ to this, then—

$$\mathrm{I}' = \mathrm{I} \cos \theta \quad \text{.........................(2)}$$

Example.—A rod uniformly magnetized to intensity 100 in the direction of its length is cut at an angle of 60° with the axis. Find the density of free magnetism on the cut surfaces.

[1] The density of free magnetism is the quantity per unit area. If σ = surface density, then $\mathrm{I} = \sigma$.

The component magnetization perpendicular to the faces of the gap

$$= \mathrm{I} \cos\theta = 100 \cos 30^\circ = 50\sqrt{3}.$$

55. Magnetic Force in the Crevasse.

The free magnetism is spread uniformly over each face of the cut, N. magnetism on one side and S. magnetism on the other. It may be shown from the law of inverse squares[1] that the magnetic force due to a uniform layer of magnetism at points very close to the layer is $2\pi\sigma$ (or $2\pi\mathrm{I}$, since $\mathrm{I} = \sigma$). The magnetic force within the crevasse, due to the joint action of *both* layers of free magnetism, is $4\pi\mathrm{I}$. For if a unit N. pole is placed between the faces of the gap the repulsion of the N. magnetism on one side urges this pole in the same direction as the attraction of the S. magnetism on the other side. The two layers of free magnetism neutralize each other's effects at all points not situated between the layers (fig. 37).

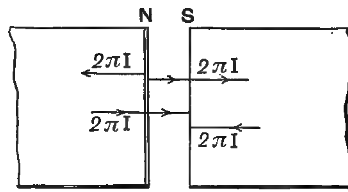


Fig. 37.—Component Fields

At the polar surfaces of a magnet there is a layer of free magnetism which produces a magnetic force $2\pi\mathrm{I}$ at points just inside or just outside the surface of the magnet.

56. Magnetized Ring.

The simplest example of magnetization is a hard steel ring of small thickness in comparison with its diameter, and magnetized so that the molecules are arranged in circles concentric with the ring. This magnetic condition may be produced approximately by stroking the pole of a bar magnet round the ring many times in the same direction. A better method is to wind a solenoid, as shown in fig. 38, and send a strong current through it for a few moments. If such circular magnetization could be obtained with theoretical perfection, the

[1] See Appendix

ring would have no poles, would not affect a compass-needle, or filings, or take up a definite direction when suspended. It is not a magnet according to the definition given in Art. 1; but the magnetized condition may be shown by cutting or

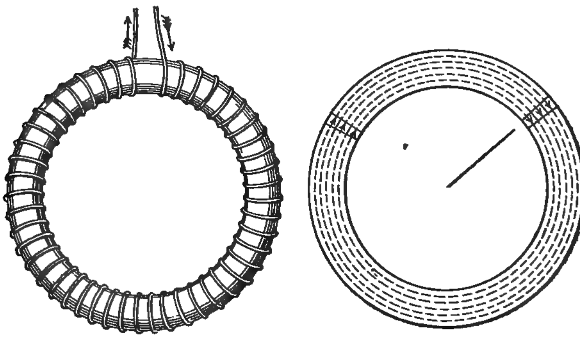


Fig. 38.—Anchor-Ring

breaking the ring, and in other ways, as by change of dimensions. A ring of the kind just described is termed an *anchor-ring* or *toroid.*

57. Distinction between Magnetization and Field.

If a very narrow saw-cut is made across a circularly-magnetized ring, a strong field is produced in the gap, but the field is practically confined to this space. The lines which cross the gap *end* on the cut surfaces. But although they end there as lines of *force,* they may be supposed to be continued in the steel as lines of *magnetization* (or chains of magnetic molecules).

Magnetization must be carefully distinguished from magnetic field. The former is always a condition of certain substances (iron, nickel, cobalt), and is therefore always associated with ordinary matter. If the magnetized material is distorted the lines of magnetization are distorted at the same time. If a flexible steel ring is magnetized (circularly), and then bent into an oval form, the lines of magnetization assume the same shape; but the lines of magnetic field (say the earth's) go straight across the ring in both cases.

Again, if polarized light is reflected from the surface of a polished magnet the light undergoes a change which depends on the magnetization. The effect reaches a limit when the magnet is saturated, although the field may continue to increase in strength. Similar effects are observed in the thermo-electric and some other properties of magnetic metals.

If we cut several radial gaps in the ring the same field appears in each gap. For many purposes it is convenient to *suppose* that this field is continued through the steel. Such an internal field, which is used to *represent* the magnetization, will have intensity $4\pi I$. We shall refer to this as the "magnetization field".

In crossing lines of magnetic force at right angles the changes in the intensity are always gradual (except in ideal limiting cases). But in the case of lines of magnetization there may be sudden changes; there is strong magnetization just within the periphery of the ring and none in the air just outside.

58. Experimental Study of Magnetization.

When a piece of iron is placed in a field of magnetic force it becomes magnetized, and in general any change in the force produces a corresponding change in the magnetization. In quantitative work the field is produced by electric currents, for it is only in this way that we can maintain fields of known strength.

The following method of testing the magnetic properties of materials is known as the **magnetometric** or **one-pole** method.

Wind insulated copper wire (say 18 S.W.G.) over a glass tube about 1 metre long and 2 cm. diameter. The winding must be close and uniform. Mount the coil on a board, and fix it in a vertical position at the E. or W. end of a bench, with the upper end above the level of the bench (fig. 39). Place the deflection magnetometer on an E.–W. line drawn from the solenoid, and at a distance of (say) 40 cm. Connect the coil in series with a variable resistance, a commutator, an ammeter, and 3 or 4 accumulators. The ammeter and all connecting wires must be kept at a distance from the magnetometer. (For description of these instruments

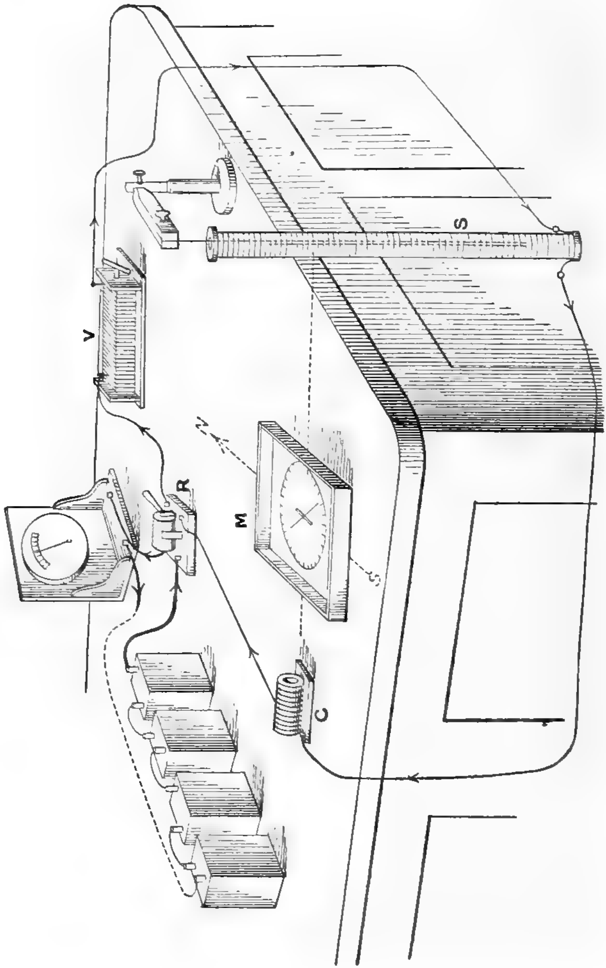


Fig. 39.—Magnetometric Method of Testing Iron

see Pt. III.) Put on the strongest current it is intended to use (say 3 ampères), and notice if the solenoid has any effect on the magnetometer. If so, include a small coil of any form in the circuit, and adjust its position until the magnetometer needle is

undeflected. The position of this compensation coil must not subsequently be altered. For accurate work the magnetometer must be of the reflecting form. The specimen may be an iron wire 80 cm. long and 2 mm. diameter.

When these preliminary adjustments have been made, cut the current off, and suspend the wire vertically with its ends both well within the coil. It will probably possess some residual magnetism, and it should be raised or lowered until it produces a maximum deflection. Its pole is then level with the magnetometer needle. The residual magnetism must now be neutralized. To do this send a current through the coil, work the commutator rapidly, and at the same time gradually decrease the current to zero.

The iron being now neutral, and the reading zero, the observations required may be taken. Put on a weak current. Note the ammeter and magnetometer readings. Now decrease the variable resistance (being careful to always move it in one direction), so increasing the current. Note again the ammeter and magnetometer readings. Repeat this step by step until the magnetization has been increased as far as required.

If stouter wire is used on the coil the current may be increased until the iron is saturated. If necessary the earth's field (vertical component) may be neutralized by an additional solenoid wound over the first, and connected to a separate battery.

The results should be entered as shown in the table below:—

Ammeter Reading.	Magnetometer Reading.	Magnetic Force.	Magnetization.
C	$\tan\delta$	H	I

To obtain the values of H and I in the third and fourth columns we proceed thus:—

1. It is shown in Chap. XXIX that the field (=H) inside a solenoid is given by—

$$H = \frac{4\pi n C}{10} \qquad \text{(i)}$$

where C is in ampères and n = number of turns *per cm.*

This is the impressed field. To obtain the effective or actual force we should subtract the polar (demagnetizing) force. But if

the specimen is of length = 300 or 400 diameters, the polar force may be neglected. This is the reason for using a long rod or wire.

2. If a is the area of cross-section of the rod, and m the pole-strength, we have—

$$I = \frac{m}{a}.$$

If the rod is long the lower pole may be neglected. Thus the field at the magnetometer is practically

$$F = \frac{m}{d^2},$$

where d is the distance from the upper pole to the needle.

Also if H' is the *earth's* field controlling the magnetometer needle, we have—

$$F = H' \tan \delta.$$

Thus—

$$I = \frac{d^2 H' \tan \delta}{a} \quad \text{..........................(ii)}$$

(Be careful to distinguish H, the current field in (i), from H′, the earth's field, in (ii).)

The value of H′ must be determined as described in Art. 49.

To obtain column (3) multiply each of the values of C in the first column by the constant $\frac{4\pi n}{10}$.

To obtain column (4) multiply each of the values of $\tan \delta$ in the second column by the constant $\frac{d^2 H'}{a}$.

If comparative values only are required the first two columns are sufficient.

59. Curve of Magnetization.

The results obtained are in practice always shown by a graph. The general shape of the curve for I and H is shown in fig. 40. There are three well-marked stages.

(i) From O to A the magnetizing force is weak, and produces comparatively weak magnetization.

(ii) From A to B the small increment in force produces large increase in magnetization.

(iii) From B (the "knee" of the curve) to C there is only a small increase. From C, the saturation point, the line is practically horizontal.

The explanation of these changes by the molecular theory has been worked out by Prof. Ewing. It is beyond our scope to enter into the details of this explanation, but its general character may be given as follows.

The molecules in a neutral piece of iron arrange themselves

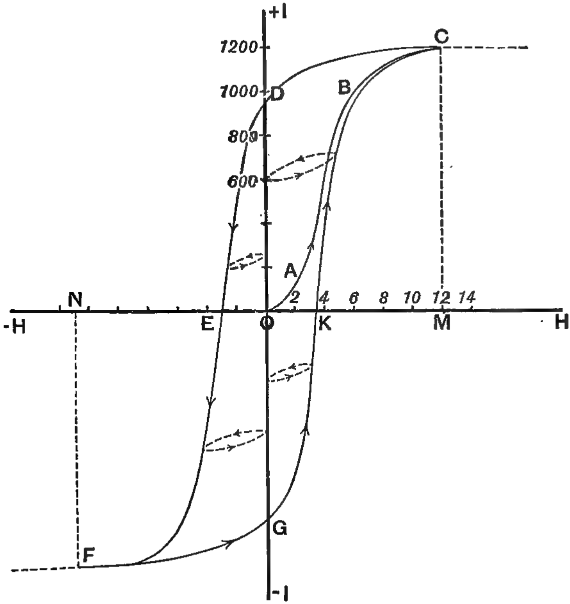
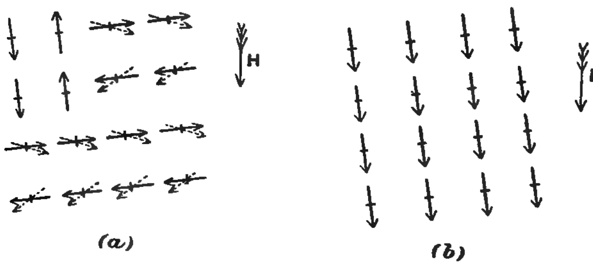


Fig. 40.—Initial Curve and Hysteresis Loop

(in the absence of magnetic force) into groups, under the influence of the mutual attractions and repulsions of their poles. Owing to the multiplicity of directions which the axes of the groups assume, the iron as whole shows no polarity; the moments of the molecules, resolved in any direction, balance. If, now, a weak force is applied, the N. poles of the molecules will all tend to point in the direction of the force, but their mutual forces will tend to hold the molecules in their original positions; hence only a small rotation is produced. The

molecular moments have now a weak resultant in the direction of the force. (The components at right angles to this on the whole balance.) This corresponds to the portion OA of the curve.

A stronger force will cause the molecules to break away from their original arrangement and assume directions nearly



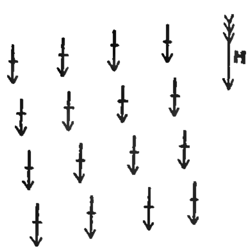


Fig. 41.—Molecular Theory

coinciding with the direction of the force. A *new set of mutual forces* is now brought into play which prevents the molecules from assuming exactly the direction of the applied force. At this stage there is evidently a large increase in the magnetization. This corresponds to the portion AB of the curve.

Further increase in the impressed force will simply tend to bring the molecules more nearly into line with the force. So that the specimen tends to approach a saturation limit, although this limit would be never quite reached theoretically.

These stages are illustrated in fig. 41,[1] which shows a group of sixteen molecules (represented by their magnetic axes), their centres being arranged in squares.

[1] Adapted from Ewing's *Magnetic Induction in Iron and Other Metals* by kind permission.

Prof. Ewing has devised a model consisting of a large number of pivoted compass-needles which illustrate the changes. The model may be placed inside a large coil and subjected to a steadily increasing or decreasing magnetic force. The movements of the needles take place in three main stages as described above.

In Chap. V we always supposed the molecules arranged in well-defined chains in the magnetized substance, whether the magnetization was strong or weak. You should now try to realize that the so-called "molecules" of these chains simply *represent* the resultant moment of groups of the actual molecules of the substance. Curves for different materials show the same general features, but the maximum magnetization varies considerably.

The curve representing an increase of magnetization from zero to saturation is conveniently termed the *initial curve.*

60. Cycle of Magnetization.

In the experiment described in Art. 58 we only considered the current as an increasing one. We may now continue the experiment by decreasing the current. If the (iron) specimen is kept perfectly still it will retain some magnetization, and the return curve is therefore higher than the initial curve. When the current becomes zero the residual magnetization is practically the *remanence* (compare Art. 10). If the current is reversed the remanence is soon neutralized, and the specimen becomes neutral for a certain reversed force. We may now increase the reversed current to a maximum; decrease it to zero; reverse it again and increase to the original maximum. If the maximum values of the current are equal the magnetization curve will return to the point C. The substance has been subjected to a *cycle* of magnetization (fig. 40).

The ratio of the remanence OD to the maximum magnetization MC is sometimes taken as a measure of the "retentiveness" of the material. Thus the retentiveness of soft iron may be, say, 80 per cent for a particular cycle. You must be

careful, however, not to confuse this use of the term "retentiveness" with the power of the material to retain its magnetization against adverse circumstances (Art. 10).

Similarly, the ratio of EO to OD may be taken as an indication of the coercive power of the material; for OD is the remanence, and OE is the force required to neutralize it in the absence of mechanical aids such as vibration.

61. Hysteresis.

In most cases where the magnetic force applied to a specimen is caused to vary, there are turning points where the force after increasing begins to decrease or *vice versa*. In such cases the magnetization for a "return" value of the force is always different from that for the corresponding "outward" value, tending to keep nearer to the magnetization at the turning point. This *lagging* of the return magnetization relative to the outward is termed *hysteresis*.

Hysteresis causes the magnetization curve for any complete cycle to form a loop. The loop is of definite shape after the cycle has been repeated a few times. If the positive and negative values of H are equal, the curve assumes the form shown in fig. 40. The small dotted loops shown in the same figure are obtained by stopping the magnetization at a point of the main curve, then reducing the force to zero, and finally increasing it to its former value. The changes indicated by the arrows on the curve should be carefully compared with the statement at the beginning of this article.

One of the most important practical consequences of hysteresis is that it entails a permanent loss of energy when the substance is taken through a complete cycle. The energy supplied during magnetization cannot all be recovered during demagnetization; the unrecovered portion is dissipated in useless heating of the material. In dynamos, transformers, and other arrangements of applied electricity, masses of iron are subjected to cycles of magnetization, and hysteresis losses must be allowed for. It may be shown that the energy dissipated is proportional to the area of the hysteresis loop.

The phenomenon of hysteresis is well shown by Prof. Ewing's model, mentioned above.

62. Time-Lag or Viscous Hysteresis.

In the magnetometer experiment the readings of deflection should not be taken immediately after each increment of current. The magnetic force requires a certain *time* to produce its full effect, and the magnetometer reading will slowly creep up a little after each deflection. This is called time-lag or viscous hysteresis.

63. Influence of Vibration.

If the specimen is subjected to vibration or tapping after each increment of the current the hysteresis is largely reduced, and in the case of soft iron practically destroyed. The falling and rising curves of the cycle coincide, and the loop vanishes.

A marked change is therefore produced in the zero to saturation curve, or what we have called the *initial* curve. The first portion, OA (fig. 42), is nearly horizontal if the iron is kept quite still; but with vibration the curve rises steeply from the first, and becomes more horizontal as the saturation limit is approached.

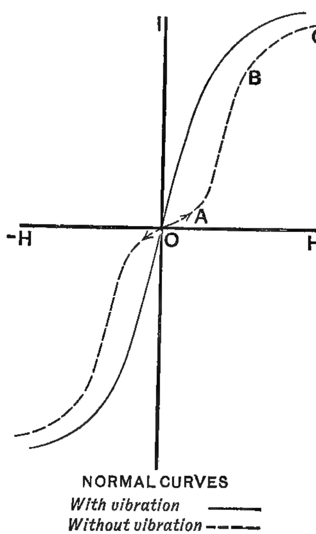


Fig. 42

A similar effect is produced by sending an alternating current of a few ampères through the iron under test during the experiment.

64. Influence of Heat.

The effect of a rise of temperature on magnetization is

peculiar. In weak fields (say, ·5 gauss) heating aids the magnetization, but in stronger fields (50 gausses) opposes it. The saturation values of I become less as the temperature rises. The initial curves of magnetization therefore cross each other at points corresponding to certain values of H (fig. 43).

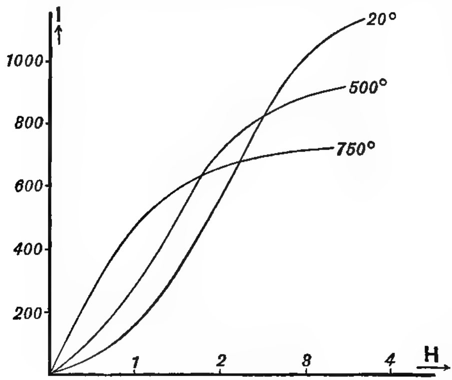


Fig. 43.—Influence of Heat

At a temperature of about 780° iron ceases to be magnetic. At this temperature changes take place also in certain other physical properties of iron. 780° is therefore termed the *critical point* of the metal.

Hysteresis loops close up as the temperature rises, until the area = 0 at a temperature approaching the critical value.

65. Susceptibility.

It is evident from hysteresis curves that the value of I does not depend on the value of H alone. Thus, if we draw a line from some point between O and M, and parallel to OI, it will intersect the curve at several points. The value of I depends not only on the value of H, but also on the whole history of the specimen since it was last neutral in the absence of a magnetic field.

But if we commence with an unmagnetized specimen, then we may consider that during the initial stage of magnetization represented by the curve OABC the value of I depends only on the value of H, and we may make the following definition:—

The ratio of the magnetization to the mag-

netic field which entirely induces it, is termed the susceptibility of the material.

If k is the susceptibility, we have—

$$k = \frac{\mathrm{I}}{\mathrm{H}}, \text{ or } \mathrm{I} = k\mathrm{H} \quad \text{(3)}$$

The susceptibility is small and nearly constant for very weak fields (stage OA), then increases rapidly (stage AB), and finally decreases slowly.

Note.—The curve of initial magnetization is really affected by hysteresis. If it were practicable to counteract hysteresis in all materials by continued vibration or by an alternating current, the susceptibility would be better defined with reference to the curves so obtained (fig. 42).

66. Other Magnetic Substances.

The chief magnetic substances, besides iron and steel, are the metals nickel and cobalt. These elements closely resemble iron in many of their chemical and physical properties. They may be annealed and hardened. They exhibit hysteresis, are capable of forming weak permanent magnets, possess definite saturation limits and critical temperatures. The following table will indicate the main features of the magnetization of ordinary magnetic substances. The values vary considerably in different specimens of the same substance, and the numbers given must be taken as typical or average values:—

Substance.	k (Max. for Initial Curve.)	I (Saturation Value MC.)	I (Remanence OD.)	H^1 (Force to Neutralize Remanence.)	Critical Temperature.
Annealed iron...	245	1200	900	1·7	785° C.
Hard iron	53	1100	400	4·5	...
Cast iron (soft)	32	1200	320	4·0	...
Mild steel	23	1120	840	23·0	720° C.
Hard steel........	9·4	1040	760	45·0	700° C.
Nickel.............	23·5	400	284	7·5	300° C.
Cobalt (cast).....	13·8	800	270	12·0	...

A variety of steel known as dynamo cast-steel is capable of taking a maximum magnetization of about 1500 C.G.S. units.

Certain alloys of magnetic metals possess curious properties. Hadfield's manganese steel (containing 12 per cent Mn, 1 per cent C) is almost non-magnetic. Its susceptibility k = ·024 to ·04, and there is no residual magnetism. k is moreover constant for fields of different strengths. Nickel steel (containing iron alloyed with 25 per cent Ni) is also almost non-magnetic (k = ·032). If cooled to a very low temperature it becomes strongly magnetic, and remains so when allowed to regain the ordinary temperature.

QUESTIONS

1. A thin rod of iron is subjected to a gradually increasing magnetic field. Explain how its magnetic moment could be measured at each stage of the experiment, and the intensity of magnetization deduced. (1904.)

2. What is meant by intensity of magnetization? If a steel ring is circularly magnetized to intensity 300, and a narrow gap then cut in the ring, what force would be exerted on a pole, strength 1000 C.G.S. units, placed in the gap?

3. A ring of non-magnetic material is wound with a solenoid, through which an electric current is then passed. By what properties may the lines of force generated in the ring be distinguished from the lines of magnetization in a circularly-magnetized steel ring?

4. Define susceptibility, and explain under what conditions the definition has a definite physical meaning. What is the approximate maximum susceptibility of a specimen of soft iron?

5. A cylinder of soft iron is wound round with iron wire insulated with asbestos, and is placed with its axis in the east-and-west direction due east of a small compass-needle. A constant current is passed through the wire of such a magnitude as to eventually heat the iron to bright redness. Describe how the deflection of the compass will alter as the iron gets hot. (1903.)

CHAPTER VI

MAGNETIC INDUCTION—MAGNETIC FLUX

67. The term "induction" is used with two distinct meanings. First, it may denote the *process* of inducing magnetism; secondly, it is used with a *quantitative* meaning, which we are about to consider. A similar remark applies to the term magnetization. Whether the "process" or the "quantity" is referred to is always evident from the sense in which the term is used.

Magnetic Induction.

If a conductor carrying an electric current is placed in a magnetic field it is subject to a mechanical force which depends on the length of the conductor, the current strength, and on a measure of the field. If a conductor is caused to move across the lines of the magnetic field an E.M.F. is induced in it, the magnitude of which depends on the same field-measure.

The magnetic quantity which determines the magnitude of the force exerted on the conductor placed at the given point of the field, or the magnitude of the E.M.F. induced in the moving conductor, is the magnetic *induction* at the point.

The *definition* of magnetic induction is given in the chapter dealing with induced currents. At present we are concerned only with the induction in iron and other substances capable of retaining permanent magnetization. It can be shown that the magnetic induction in these substances may be expressed as the resultant of two other quantities, independently defined.

In air and other non-magnetic materials the induction may be considered identical with the magnetic force.[1] But in iron and other magnetic substances the induction and force may differ considerably.

It may be shown by the experimental methods given in

[1] It is to be understood that electro-magnetic measure is here used. See chapter on Units.

Art. 74 that the inductive effect of a magnetization I is equivalent to that of a field of strength $4\pi I$. This, we have seen, is the field-strength in a crevasse cut at right angles to the lines of magnetization. Hence in dealing with magnetic induction the magnetization is represented by this equivalent field.

The magnetic induction at any point within a magnetized body may therefore be expressed as the resultant of—

(*a*) **the magnetic force** (**H**);
(*b*) **the magnetization**—represented by the equivalent **field** 4π**I**.

The induction is usually denoted by the symbol B.

Since the two components (*a*) and (*b*) are not always in the same direction, their resultant must in general be found by the parallelogram of forces. But in very soft iron H and $4\pi I$ are in the same straight line, and their resultant B may be found by simple addition. Thus—

$$B = H + 4\pi I \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In air and other non-magnetic materials $I = 0$ and $B = H$.

A line so drawn that its direction at each point is the direction of the induction at the point is called a *line of induction.*

In non-magnetic substances, lines of force and lines of induction are identical, but in magnetized substances they do not always coincide.

68. **Examples of Induction.**

Since the magnetic force is the resultant of the impressed force and polar force, we have in general three components to consider for the induction—

$$\left.\begin{array}{l}\left.\begin{array}{l}\textbf{Impressed Force } (h)\\ \textbf{Polar Force } (h')\end{array}\right\} = \left\{\begin{array}{l}\textbf{Magnetic}\\ \textbf{Force H}\end{array}\right. \\ \textbf{Magnetization } (4\pi I)\ldots\ldots\ldots\ldots\ldots\end{array}\right\} = \left\{\begin{array}{l}\textbf{Magnetic}\\ \textbf{Induction B.}\end{array}\right.$$

The student should now consider very carefully the following examples, which show how the induction is expressed in some important cases.

(i) **Circularly-magnetized steel ring.**—Here there is no impressed field, and if the ring is complete there are no free poles.

$$h = 0; \ h' = 0; \ \mathrm{H} = 0. \quad \therefore \ \mathrm{B} = 4\pi\mathrm{I}.$$

Here the lines of induction are simply the lines of magnetization.

(ii) **Iron ring temporarily magnetized by current in solenoid.**—The ring whilst magnetized acts like the steel ring in (i); in addition we have the impressed field due to the current.

$$h' = 0; \ \therefore \ \mathrm{H} = h \text{ and } \mathrm{B} = h + 4\pi\mathrm{I}.$$

(iii) **Iron rod placed in straight solenoid.**—Here the conditions are the same as in (ii), with the addition of polar demagnetizing force, which opposes the impressed force and the magnetization.

$$\mathrm{H} = h - h', \text{ and } \mathrm{B} = h - h' + 4\pi\mathrm{I}.$$

(iv) **Steel bar magnet away from other influences.**—Here

$$h = 0, \ \mathrm{H} = -h', \text{ and } \mathrm{B} = 4\pi\mathrm{I} - h'.$$

(There is, of course, always the earth's field as an impressed field, but being weak it has been neglected in the above examples.)

If a strip of magnetized hard steel is cut down to shorter and shorter lengths, the induction in it is decreased. Although the magnetization may remain practically constant, the polar (backward) force increases, for it varies by the law of the inverse square of the distance. Therefore $4\pi\mathrm{I} - h'$ diminishes.

69. Boundary Conditions.

The lines of magnetization end at the surface of the magnet. The lines of induction exist on both sides of the surface, and in general undergo a sudden change of direction in crossing the surface.

Let AB be a small portion of the polar surface of a magnet. Let the lines of magnetization end on AB at an angle α with

the normal. Then the surface density of free magnetism on AB is (Art. 54)—

$$\sigma = \mathrm{I} \cos a.$$

The field arising from this free magnetism is perpendicular to the surface, *in opposite directions* on the two sides, and equal to—

$$2\pi\sigma \text{ or } 2\pi\mathrm{I} \cos a \quad \text{(Art. 55).}$$

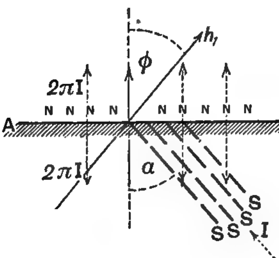


Fig. 44.—Boundary Conditions

This forms a component of the *polar* field at this part of the magnet; the remaining components of the polar field arise from the more distant parts of the magnet, and *cross the surface without change* of direction. The impressed field also crosses the surface without change. Hence if the second portion of the polar field and the impressed field are combined into a single force h_1 inclined at ϕ with the normal, the component *parallel* to surface is—

$$h_1 \sin \phi \text{ on both sides.}$$

The local free magnetism produces no force parallel to the surface.

Again, the induction *normal* to the surface outside the magnet is—

$$h_1 \cos \phi + 2\pi\mathrm{I} \cos a;$$

and inside the magnet it is—

$$h_1 \cos \phi - 2\pi\mathrm{I} \cos a + 4\pi\mathrm{I} \cos a.$$

It has therefore the same value on both sides. Thus—

(i) The component of the magnetic *force parallel* to the surface is the same on both sides.

(ii) The component of the *induction normal* to the surface is the same on both sides; if the resultant inductions B_1, B_2 make angles θ_1 and θ_2 with the normal, we have—

$$\mathrm{B}_1 \cos \theta_1 = \mathrm{B}_2 \cos \theta_2 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

70. Tubes of Force—Flux of Magnetic Force.

Suppose a small closed curve of any shape marked out on the N. polar surface of a magnet. Suppose lines of force drawn from all points of this curve, and continued until they reach the S. polar surface. Lines so drawn enclose a "tube" of force. The width of such a tube varies from point to point, being greater in the weak parts of the field. There is a simple connection between the strength of the field and the width of the tube. Before dealing with this we may consider an analogous case in the mechanics of fluids.

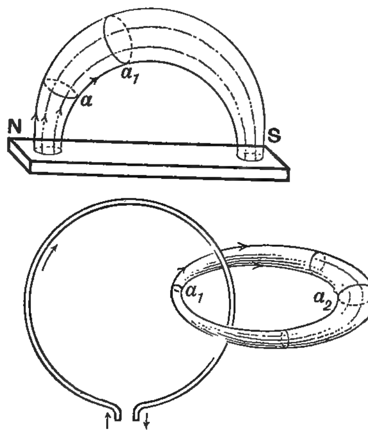


Fig. 45.—Magnetic Flux

Imagine a thin tube, say of glass, of the same shape as the tube of force, and suppose a steady flow of liquid is kept up through it. Now the quantity of liquid flowing per second must be the same through any cross-section of the tube. This quantity is equal to the velocity of the liquid at the section multiplied by the area of the section. Thus if a_1 and a_2 are the areas and v_1 and v_2 the velocities,

$$v_1 a_1 = v_2 a_2 = \text{a constant.}$$

The product va is called the *flux* of liquid, and is constant at all parts of the tube.

The velocity decreases as the tube widens, just as the field-strength does in a tube of magnetic force, and by analogy we make the following definition:—

The flux of magnetic force through any section of a tube of force is the product (field-strength at right angles to the section) × (area).

We usually consider sections normal to the tubes, but the flux is the same for an oblique section, provided we take the component of the force perpendicular to the plane of section. To prove this, suppose the normal section to be of area A; the oblique section is of larger area A′. If the forces perpendicular to these sections are F and F′, and the angle between them θ. Then—

$$F' = F \cos\theta \text{ and } A' = \frac{A}{\cos\theta}.$$

$$\therefore F'A' = FA \quad \ldots\ldots\ldots\ldots (3)$$

The flux is therefore the same for the oblique as for the normal section.

The flux of magnetic force is the same at all sections of a tube of force; or—

$$F_1 a_1 = F_2 a_2 \quad \ldots\ldots\ldots\ldots (4)$$

This relation may be proved from the law of inverse squares. It is analogous to the constancy of liquid flow considered above. (See Appendix.)

The chains of molecules forming the lines of magnetization may also be grouped into tubes. In a ring circularly magnetized the tubes are closed, but in a bar magnet they end on the polar surfaces.

The flux of magnetization may be defined as (strength of equivalent field) × (area of cross-section).

$$i.e. \quad 4\pi I \times a.$$

Assuming that the molecular magnets are all of equal strength and arranged in perfect chains, the flux of magnetization is constant throughout any one tube.[1]

71. Tubes of Induction.

The lines of induction may be grouped into tubes in like manner. The tubes of induction so formed possess an important property which the tubes of force and magnetization do not in general possess; the induction tubes are *always* closed,

[1] We omit here the consideration of volume distributions and non-uniform susceptibilities.

i.e. each tube forms a complete ring or circuit. A strict mathematical proof of this cannot be given here, but the following considerations indicate the general nature of the reasoning. We first make the following definition:—

The flux of induction through any surface is the product (induction normal to the surface) × (area).

Thus if P is the flux, $P = B \times a$. Then—

Outside the magnet, the flux of induction is the same as that of magnetic force, and is constant at all parts of any one tube. *Inside* the magnet the flux of induction is constant at

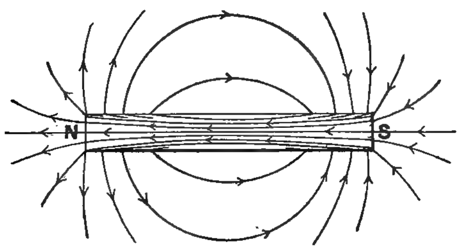


Fig. 46.—Lines of Induction

all points of a tube, for the component fluxes of force and magnetization are both continuous.

The flux is also constant *across the surface.*

To prove this, let a tube outside be taken of such a size and shape that its end exactly fits the end of an internal tube of induction where these meet on the surface. Let the common end of the two tubes have area A. Let the normal sections of the tubes have areas a_1, a_2; the inductions B_1 and B_2; and inclinations $_1$ and θ_2.

We have by (2)—

$$B_1 \cos \theta_1 = B_2 \cos \theta_2.$$

$$\therefore B_1 A \cos \theta_1 = B_2 A \cos \theta_2.$$

$$\therefore B_1 a_1 = B_2 a_2 \quad \text{.............................(5)}$$

Thus the flux is the same in both tubes, and we may consider one tube as a continuation of the other. Tubes of

induction, therefore, must be regarded as having no ends. Each tube is said to form a **perfect magnetic circuit**, from its analogy to a closed conducting electric circuit in which the flux of current is the same all round.

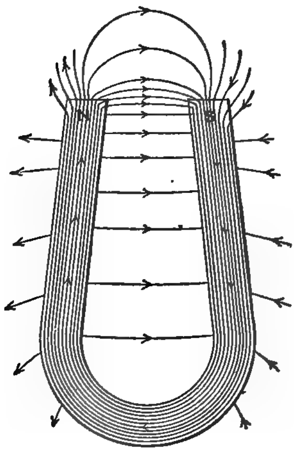


Fig. 47.—Lines of Induction

The lines of induction of a bar and horse-shoe magnet are shown in figs. 46, 47. Compare these carefully with the corresponding diagrams of lines of force (figs. 10, 11).

In fig. 47 many of the lines are not continued inside the magnet since they run too near the surface to be clearly shown. Remember that a complete line of *induction* has no ends, whilst a line of *force* may end on a polar surface.

72. Unit Tubes—Flux Density.

The term *magnetic flux* must always be understood to mean flux of *induction* unless the contrary is expressly stated.

Let a tube of induction be split up at any point into a number of smaller tubes of equal cross-section, and let the number be equal to the flux (Ba) at this section. Then if the tubes are continued their number will at *any* section represent the flux, since Ba is constant.

Also, the number passing through *unit* area will at any section represent the *induction* there.

Tubes so drawn are termed *unit tubes.*

It is more usual, however, to speak of *lines* of induction, and the induction at any point is expressed in *lines per square centimetre.* Used in this quantitative sense, "line" may be considered synonymous with "tube".

The total magnetic flux through any area is

expressed by the total number of unit tubes (or lines) through that area.

Flux density, or number of lines per unit area, represents the **induction normal to the area.**

73. Units of Induction and Flux.

In electro-magnetic measure, the unit of induction is the same as the unit of magnetic force, and is termed the *gauss*. The unit of magnetic flux is an induction of one gauss extended over an area of 1 square centimetre, and is termed the *maxwell*. Denoting magnetic flux by the symbol P, we have—

$$P = B \times a \qquad (6)$$

$$\text{or (maxwells)} = \text{(gausses)} \times \text{(square cms.)}.$$

The number of maxwells is equal to the total number of lines of induction, and the number of gausses is the number of lines per square cm.

74. Experimental Method of Studying Induction.

It is shown from the laws of induced currents that when the total flux of induction through a coil changes by P lines, there is a discharge of electricity round the coil given by—

$$Q = \frac{P \times n}{10^8 \times R} \qquad (7)$$

where Q is the total quantity of electricity discharged, n the number of turns on the coil, and R the resistance of the circuit. Q may be measured with a ballistic galvanometer, n and R may also be measured, and we thus obtain a means of calculating the total change of flux P. This formula is applied in the practical measurement of induction. We shall describe two methods of making the test.

Anchor Ring Method (fig. 48).

An anchor ring is first *uniformly* wound with a stout magnetizing coil—the primary P. A second coil of a large number of turns of fine wire is wound over this, but the winding need not be uniform. This coil forms the secondary S, and is arranged in

series with a ballistic galvanometer BG. The primary coil is connected in series with an ammeter AM, a rheostat VR, battery B, and reversing switch RS. (In the figure the primary coil is shown on one side only for the sake of clearness.)

The experiment is usually carried out by a "method of reversals". A small current is started in the primary circuit, and its value read on the ammeter. The current is then reversed, and the throw of the ballistic galvanometer is noted. The constant of the galvanometer being known from independent experiments (Chap. XXX), the quantity Q can be calculated, and from this the value

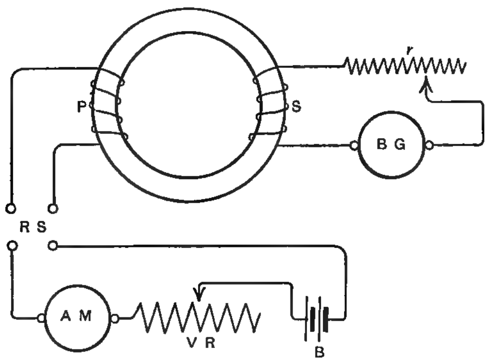


Fig. 48.—Ring Method

of P can be obtained. The induction B is obtained by dividing $\frac{1}{2}$P by the cross-sectional area of the ring. (We take $\frac{1}{2}$P since the algebraic *change* of flux during the reversal is twice the flux in either direction.)

The current is then increased and the observations repeated. By continuing the observations a cycle of values may be obtained.

The magnetizing force corresponding to each value of the current is obtained as in the magnetometer experiment (Art. 58) from the formula—

$$H = 4\pi n_1 C/10 \qquad (8)$$

where n_1 is the number of turns *per cm.* in the primary.

The Bar and Yoke Method.

The ring method is the most perfect method of making the inductive test, but is inconvenient when a number of specimens have to be dealt with, owing to the labour involved in winding

the coils. An alternative method due to Hopkinson allows the use of rods instead of rings, and the coils are arranged once for all. The instrument is shown in fig. 49. There is a thick rectangular mass of soft iron termed the *yoke.* The primary coil is a straight solenoid almost filling the central space of the rectangle. The solenoid is made in two lengths, which nearly meet at the centre of the rectangle. The secondary coil is mounted on a small reel, and can be pushed down into the gap left between the halves of the primary coil. When in this position the rod under test can be threaded through the reel; it is passed through a hole bored in the end of the yoke, and through the primary coil. The end of

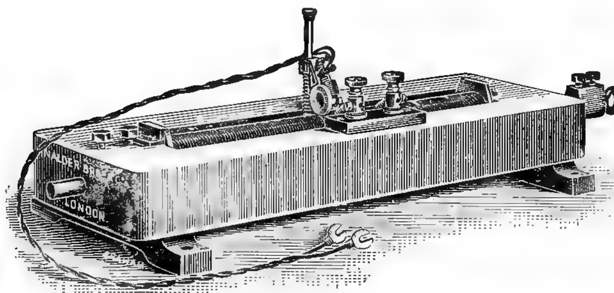


Fig. 49.—Bar and Yoke

the rod within the coil butts against a soft-iron stop (an equal rod) fitted through the opposite end of the yoke. (In the figure the rod under test is on the right and the stop on the left.) When the rod is withdrawn the secondary coil is pulled out of the gap by a spring.

The primary coil is arranged in series with a battery, variable resistance, and ammeter, as in the ring method. The secondary coil is connected to a ballistic galvanometer. This coil is pushed down between the primary coils, and the rod under test passed through it, so holding it in position. A current is now started in the primary, and its value is read on the ammeter. A flux of induction is produced in the rod, and if the rod is withdrawn, the secondary cuts all the lines of induction when the spring makes it jump out of the gap. The values of B and H are calculated as in the ring method from Equations (7) and (8).

The object of the yoke is to form a good magnetic circuit,

and the rods are made to accurately fit the holes in the yoke-ends for the same purpose.

Magnetometer Method.

The values of B for given values of H can be obtained from the results of the magnetometer experiment (Art. 58). For, since $B = H + 4\pi I$, if H and I are known, B can be calculated.

75. Curves for Induction and Force.

The curves showing the relation between B and H for a cycle of values are similar to those for magnetization (fig. 40), but the curve does not become quite horizontal when the specimen is saturated. After the saturation point is passed, an increase in the magnetizing force makes the same increase in the induction. Since the value of B may be several thousand times that of H, it is necessary to plot B to a much smaller scale than H; and the increase in B after saturation point is passed is scarcely noticeable in actual curves.

(For practical purposes H may be neglected in comparison with $4\pi I$, and then—

$$B = 4\pi I = 12{\cdot}57 \times I.)$$

76. Permeability.

When we commence with an unmagnetized specimen of iron or other ferromagnetic material, the value of the induction may be considered to depend on the value of H *only*, during the increase in H. In these circumstances we may make the following definition:—

> **The ratio of the magnetic induction to the magnetic force (when the induction entirely depends on the force) is termed the permeability of the material.**

In the case of substances showing no hysteresis the restriction stated in the parenthesis is unnecessary. See also note on Susceptibility (Art. 65).

Permeability is usually denoted by the symbol μ. Thus—

$$\mu = \frac{B}{H} \text{ or } B = \mu H \qquad \text{(9)}$$

Dividing equation on both sides by H we have—

$$\frac{B}{H} = 1 + 4\pi\frac{I}{H},$$

i.e. $$\mu = 1 + 4\pi k \ldots\ldots\ldots\ldots\ldots \text{(10)}$$

The permeability of a non-magnetic substance is unity.

The permeability of ferromagnetic substances is not constant. It is nearly constant for weak fields; increases rapidly when the magnetizing force reaches a certain value; and finally diminishes. This follows from Eqn. 10 and what has been said about susceptibility (see Arts. 63, 64).

Para- and Diamagnetic Substances.—Under the influence of powerful magnetic fields all substances exhibit magnetic

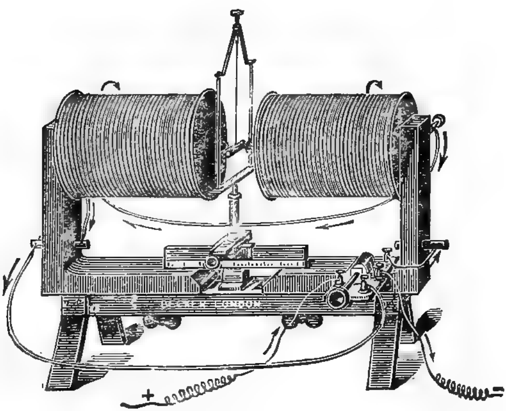


Fig. 50.—Diamagnetism

properties, but except in the case of compounds of the magnetizable substances—iron, nickel, and cobalt—the effects observed are excessively weak.

Further, many substances exhibit a magnetic property which is the reverse of that observed in iron. A bar of bismuth suspended between the poles of a powerful magnet sets at right angles to the line joining the poles (fig. 50),

whereas a bar of iron sets along this line. Substances which, like bismuth, are repelled from the poles of a magnet are termed *diamagnetic*, other substances which are attracted to the poles are termed *paramagnetic*, or simply *magnetic* substances.

The phenomenon of diamagnetism shows that the permeability of some substances is less than that of a vacuum, *i.e.* less than unity. The value for bismuth—the most strongly diamagnetic substance — as measured by specially devised methods is $\mu = \cdot 999824$. Other diamagnetic substances are antimony, phosphorus, gold, silver, copper, lead, zinc, tin, tellurium, sulphur, mercury, glass, water, and most liquids except solutions of salts of magnetizable metals.

The few strongly magnetic substances which are capable of permanent magnetization are sometimes termed, for distinction, *ferromagnetic*.

77. Examples.

1. An anchor ring 2 cm. thick is circularly magnetized. On cutting a radial gap it is found that the magnetic force in the gap is 3 kilogausses. Find the intensity of magnetization, and the magnetic flux in the ring (circular cross-section).

We have for the force in the gap $F = 4\pi I$.

$$\therefore\ 3000 = 12{\cdot}57 \times I$$
$$I = 238{\cdot}6 \text{ C.G.S. units.}$$

$$\begin{aligned}\text{The magnetic flux} &= \text{induction} \times \text{area of cross-section} \\ &= 4\pi I \times \pi r^2 \\ &= 3000 \times 3{\cdot}142 = 9426 \text{ maxwells or } 9{\cdot}426 \text{ kilolines.}\end{aligned}$$

2. Show that the magnetic force in a magnetized mass is the force in a long narrow "tunnel" or tube-shaped cavity in the metal in the direction of magnetization.

The force in the cavity is the resultant of (1) the polar force, (2) the impressed force, (3) force due to free magnetism on the ends of the tunnel. When the tunnel is very long in comparison with its diameter, the value of (3) is very small and may be neglected. Hence the field is the resultant of (1) and (2), and is therefore the magnetic force (H).

Note.—The force in such a tunnel is frequently taken as the *definition* of the magnetic force in the magnetized material.

3. Show that the magnetic force in a crevasse, acting at right angles to the surfaces of the latter, is equal to the induction at right angles to these surfaces.

The field due to the free magnetism on the faces of the crevasse is at right angles to these faces and represents the magnetization in this direction. Hence adding this to the components of the polar and impressed fields, we get the total field effective in current induction (B).

Note.—The component force in such a crevasse perpendicular to the faces is frequently taken as a *definition* of the induction-component in this direction.

4. A permanent simple bar magnet is placed inside a solenoid. If the magnetization at the centre of the magnet is 500 C.G.S. and the polar force at this point 200 lines per sq. cm., what is the induction at the centre if the current in the solenoid produces a field of 300 lines per sq. cm. and tends to preserve the magnetization?

$$\begin{aligned} B &= H + 4\pi I \\ &= 300 - 200 + 4\pi \times 500 \\ &= 6385 \text{ lines per sq. cm.} \end{aligned}$$

QUESTIONS

1. Explain what is meant by a line of magnetic induction. Give sketches of the lines of magnetic induction and those of magnetic force due to a horse-shoe magnet, both inside and outside the magnet. (1903.)

2. Define the magnetic induction B, the magnetic force H, and the intensity of magnetization I, and give the relation between these quantities. (1902.)

3. What is meant by the statement that the permeability of iron is (say) 800? Is the permeability of a given specimen of iron constant? If not, on what does its variation depend? (1903.)

CHAPTER VII

DISTRIBUTION—ELECTRO-MAGNETS

78. Distribution of Induced Magnetism.

The distribution of magnetism on the surface of a permanent magnet follows no definite law. The same is true of the so-called "induced" magnetism of a piece of iron if the iron possesses appreciable coercive power. But if we suppose that the coercive power is negligible, and hysteresis suppressed, say, by continual vibration, the induced magnetism will depend on the existing magnetic force, and not on the magnetic history of the specimen.

Materials like very soft iron, in which coercive power is negligible, may be conveniently termed "magnetically soft". (In dealing with the properties of such materials, we shall suppose that there is sufficient vibration to prevent the occurrence of remanent magnetism.)

When a magnetically soft material is placed in a field of magnetic force, the two following laws are obeyed:—

I. The directions of the magnetic force H, the magnetization I, and the induction B all coincide.

II. The intensities of I and B depend on the magnitude of the resultant magnetic force H at the point considered.

The polar field at each point in the iron must act in such a direction that, when it is compounded with the impressed field, the resultant force obeys laws I and II. The consequence is that the lines of magnetization or molecular chains assume a definite arrangement or distribution, depending on the following conditions:—

(*a*) **The distribution of force in the impressed field;**

(*b*) **The nature, temperature, etc., of the material;**

(c) The shape and position of the mass with respect to the impressed field.

The definite arrangement of the lines of magnetization of course entails definite distributions in the fields of induction and force.

We shall now consider some examples of the above general rules.

79. Soft Iron in the Field of a Permanent Magnet.

Here the magnetic force *in the iron* is the difference between (*a*) the impressed force, and (*b*) the demagnetizing field due to the induced poles. The latter field must be weaker than the former; for if not, Rule I would not be satisfied, and the resultant force would be opposed to the magnetization. Hence the magnetization induced must always be so weak that the polar force inside the iron is less than the impressed force.

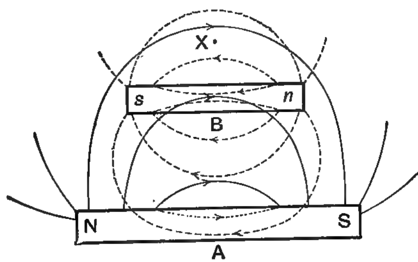
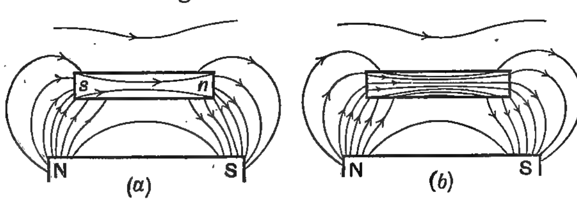


Fig. 51.—Component Fields of Force

Now, when the impressed field is that of a permanent magnet, it remains the stronger *outside* the iron also, for in the region



Resultant Field of Force

Resultant Field of Induction

Fig. 52

considered it is more uniform than that of the induced magnet. Hence the general direction or "drift" of the resultant field agrees at all points with that of the impressed field.

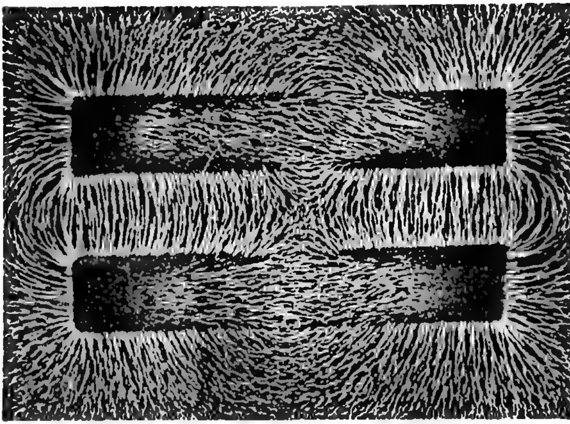


Fig. 53 (*a*).—Equal Permanent Magnets

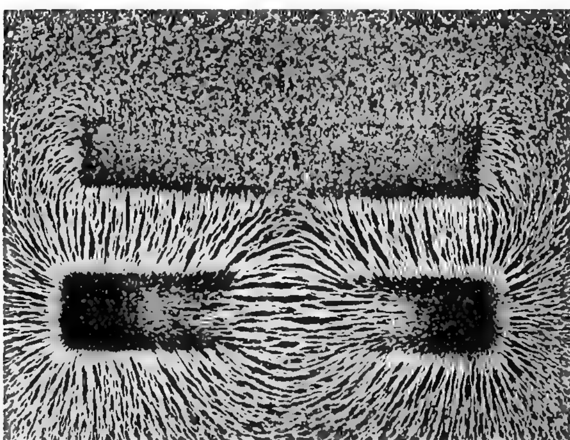


Fig. 53 (*b*).—Magnet and Soft-Iron Bar

EXAMPLE.—Suppose a bar of iron B placed parallel with a magnet A. The component and resultant fields are shown in figs. 51, 52 (*a*). The field of A at the point X is towards the right, whilst the field of B at the same point is towards the left,

Hence the resultant field acts towards the right, and its general "drift" is towards the right at all points near the iron.

The resultant field of induction is shown in fig. 52 (*b*).

A similar field is obtained with two permanent magnets when one is much weaker than the other. But if A and B are two magnets of nearly equal strength, the field of B at X overcomes that of A at the same point, and the resultant field has the form shown in figs. 13, 53 (*a*). We may easily confirm these conclusions with a compass-needle. If filings figures are obtained, the filings do not assume a definite arrangement in the region of X, in the case of the iron bar, owing to the excessively weak field there (fig. 53 (*b*)).

The lines of resultant magnetic *force* have the same directions as the lines of induction, but there is a sudden change in their number where they pass into the iron. This is because we must omit the component $4\pi I$ from the induction to find the magnetic force. Compare (*a*) and (*b*), fig. 52.

Notice also that the polar field of B opposes the polar (demagnetizing) field of A inside the latter. Hence the flux of induction from A is increased by the presence of B.

80. Soft Iron in the Field of a Current.

As in the case just considered, the polar field due to the induced magnetism cannot exceed the impressed field due to

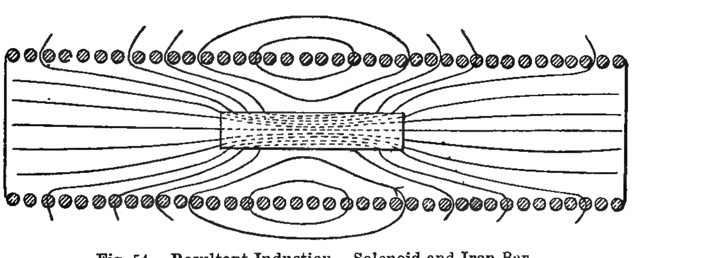


Fig. 54.—Resultant Induction. Solenoid and Iron Bar

the current if points *within* the iron are considered; but it may do so outside the iron. A simple example is that of a short iron rod placed inside a ring solenoid. Within the coil the im-

pressed force due to the current is greater than the polar force due to the induced magnetism. Outside the coil the current produces no field, and the polar force due to the iron forms the resultant. The resultant field of induction is shown in fig. 54.

81. Influence of Position on Direction and Intensity of Induction.

We shall consider some particular cases.

1. **Rod placed at right angles to the field.** This is shown in fig. 55 (*a*). The inducing field acts towards the right, and is supposed uniform. This rotates the molecules, the result being to induce N. free magnetism on the right-hand face and S. free magnetism on the left. The polar force acts towards the left, and, on account of the small distance between the right and left faces, it is comparatively strong. Hence the progress of magnetization will cease at an early stage: a weak magnetization produces enough backward polar force to nearly neutralize the impressed field. (The latter is supposed of *moderate* value. Very strong impressed fields would of course saturate the iron.) The direction of magnetization is perpendicular to the rod.

2. **Rod placed parallel with the field.** The difference in this case is that the free magnetism is produced only near the ends of the rod. The backward polar force is therefore comparatively weak, and a stronger degree of magnetization must be reached before the resultant force and magnetization are in equilibrium (*i.e.* correspond on the normal curve).

3. **Rod placed obliquely across the field.** If the molecules assumed the direction of the impressed field, there would be N. and S. magnetism along the lateral faces of the rod, as in case (1). The backward polar force h', tending to take the shortest path between these faces, will be at right angles to the rod. The resultant of the polar force and impressed force h must be found from the parallelogram law, and evidently lies more nearly parallel to the rod than the impressed field. But (Rule I) the magnetization cannot have

a different direction from the resultant force. Hence the molecules will turn round so as to lie more nearly parallel with the rod. This at once alters the distribution of free magnetism, shifting it nearer the ends, and in consequence the direction of h' is altered. The molecules finally take up a direction inclined to the length of the rod, such that the resultant of h and h' (*i.e.* H) lies in this direction also. The

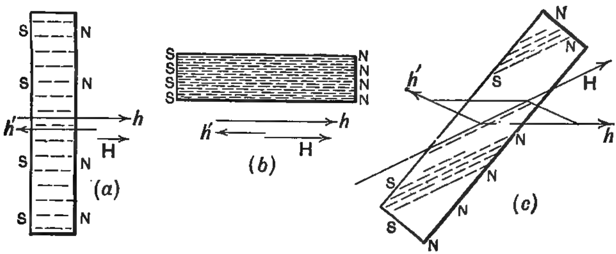


Fig. 55.—Influence of Position

intensity of magnetization is intermediate between that of cases (1) and (2).

Similar reasoning may be applied to other particular arrangements of the iron, and it will be found that the following general statements apply:—

> III. **The direction of resultant magnetization tends to follow the line of greatest length in the iron.**
>
> IV. **The intensity of magnetization (average) is greatest when the greatest length of the specimen is parallel to the impressed field.**

The extent to which the "tendency" mentioned in III is followed depends on the shape of the normal curve and the strength of the impressed field.

The appearance of the resultant lines of induction suggests the idea of their being "drawn into the iron" or finding "an easier path" through it. This notion is convenient for remembering the general shape of the lines. (But compare Art. 80.)

82. Magnetic Shielding.

The above principles may be applied in arranging a mass of soft iron so as to partly screen a region from magnetic influence. One example of this is shown in fig. 53 (*b*), the region on one side of the iron being protected from the magnet. The following experiment illustrates another case:—

Obtain three soft-iron rings about $2{\cdot}5 \times 1{\cdot}5$ cm. section and 10 cm. diameter. Pile them together so as to form a cylindrical box. Place a small vibration magnetometer within the rings. Support a bar magnet level with the middle ring, pointing towards the needle, and count the number of oscillations per minute. Now remove the rings, one at a time, without disturbing the magnet or needle, and find the number of oscillations per minute after each removal. It will be found greater each time. The results of an experiment are given below.

No. of Rings.	No. of Oscillations per Minute.	n^2.	Relative Forces.
3	11	121	1
2	14	196	1·6
1	17	289	2·38
0	28	784	6·48

Thus the space inside the ring is to a large extent screened from the action of the magnet. The extent of the screening depends on the quality and thickness of the iron and strength of field applied. A very thick, soft-iron box forms a very effective screen.

Fig. 56 shows the shielding effect of cylinders of various thicknesses.[1] The four quadrants must be taken separately. Individual lines may be traced by the aid of the attached numbers. The lines are, however, not drawn where they practically coincide with the outline of the cylinders.

The dotted lines in the same figure show the lines of induction for the conjugate case, namely, *tubular cavities* in a

[1] See paper by Prof. H. Du Bois, *Electrician*, Vol. XL.

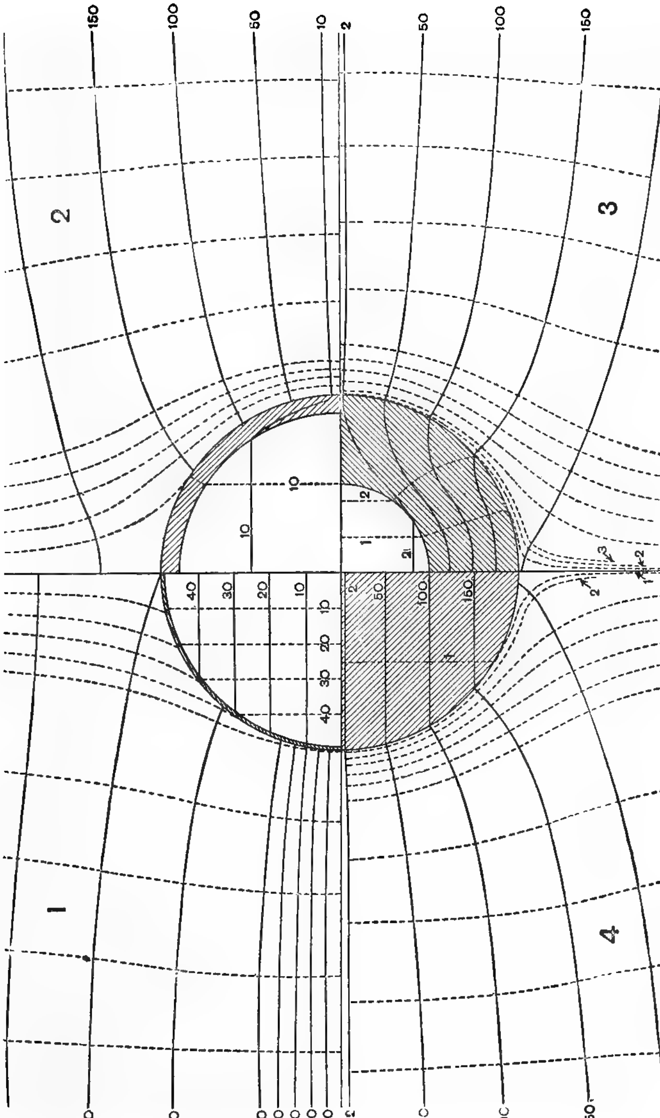


Fig. 56

mass of iron, each cavity having an iron core (except No. 4). Observe that the lines tend to remain in the iron, avoiding the cavity. The dotted lines in the first case, or the continuous lines in the second, are equipotential curves. (See Chap. XXIX.)

83. Magnetization by the Earth's Field.

The earth's field, although weak, is capable of inducing a considerable degree of magnetization in iron, owing to the high susceptibility of this substance. The direction and intensity of the induced magnetization is best arrived at by dealing with total force of the field acting along the lines of dip, and applying the principles III and IV above.

The effects are most evident with long rods. Place an iron poker in any position at right angles to the lines of dip and tap it. Test it with a compass-needle, and you will find that it is quite neutral. Now hold the poker along the lines of dip and tap it again. It will now be strongly magnetized.

Most long masses of iron, such as the pillars and girders used in the construction of buildings, will be found strongly magnetized. In the construction of ships also a large amount of iron is used, and the compass-needle is appreciably affected. The effects of the permanent and induced magnetism are compensated by small magnets and masses of soft iron placed near the compass.

84. Refraction of Lines of Induction.

Since the directions of induction and force coincide in the iron, their inclinations to the normal will be the same. Let θ_1 be the inclination of the line of induction to the normal just outside the iron, and θ_2 the inclination to the normal just inside. Then, by the results at the end of Art. 69,

$$\text{(i)}\quad H_1 \sin\theta_1 = H_2 \sin\theta_2.$$
$$\text{(ii)}\quad B_1 \cos\theta_1 = B_2 \cos\theta_2.$$

Therefore, by division, $$\frac{H_1}{B_1}\tan\theta_1 = \frac{H_2}{B_2}\tan\theta_2.$$

$$\text{But } H_1 = B_1, \text{ and } B_2 = \mu H_2.$$

$$\text{Hence } \tan\theta_1 = \frac{\tan\theta_2}{\mu};$$

$$\text{that is, } \frac{\tan\theta_2}{\tan\theta_1} = \mu \quad\text{.........................(1)}$$

The value of μ is greater than unity in magnetic substances. Hence—

V. The line of induction is bent away from the normal on entering the iron. (The line is refracted.)

Also, since μ has a very large value in iron, θ_1 is usually very small, and therefore at most points

The line of induction in air meets the surface of iron nearly at right angles.

85. Mechanical Action on Induced Magnets.

The mechanical forces which the field exerts on a magnet have already been considered in Art 20. But *when the direction of magnetization depends on the impressed field*, some further conclusions may be drawn which do not in general apply to permanent magnets.

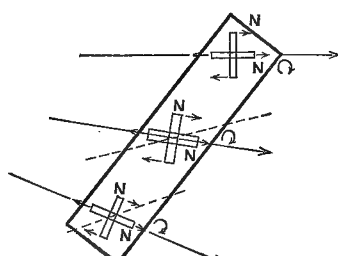


Fig. 57.—Mechanical Effect

Suppose an iron rod placed obliquely in a uniform or nearly uniform field (fig. 57). Let the long arrow represent the direction of the field, and the dotted line the magnetization. (Compare Art. 81.) Resolve the magnetization into components parallel to the impressed field, and at right angles to it, respectively. These components may be represented by two sets of magnetized particles or "molecules". Picture the particles of each set as compass-needles. The forces acting on these are shown by the small arrows in the figure.

The molecules may be supposed for the time being rigidly fixed in the bar. This is because they are prevented by the polar force from assuming the direction of the impressed field. The polar force does not, however, exert any resultant force *on the bar as a whole*, since the forces acting on the molecules from this cause are simply the mutual actions of different parts of the same body. The total action on the bar is therefore found by considering the influence of the impressed field *only*. The molecules may be supposed fixed in the direction obtained by the method of Art. 81.

You will see from the figure that the following rules hold:—

VI. The action on the component perpendicular to the impressed field is a couple tending to turn the bar into the axial position.

This applies whether the field is uniform or not.

VII. The action on the component parallel to the impressed field consists of two opposite forces. If the field is uniform, these are in equilibrium. If the field is not uniform, the particle is urged towards the stronger part of the field.

The whole system of such forces can be reduced to a couple and a force acting at the centre of suspension (centre of gravity) of the body. The couple helps that due to VI.

Example.—A small strip of iron is placed obliquely across the axis (produced) of a bar magnet. The couple VI tends to make it set along the axis. The system of forces VII tends to make all parts of the strip move towards the magnet along the lines of force; but this tendency is stronger at the end nearer the magnet. There will, therefore, be another couple tending to make the strip point along the axis. There is also a force urging the strip bodily towards the magnet.

If the iron is in the form of a small sphere, the couples vanish and the force (VII) only remains. A small iron ball placed on the bisector of a bar magnet tends to move along the bisector, i.e. *across* the lines of force, towards the magnet.

Similar reasoning may be applied to **diamagnetic** bodies. It will be found that Rule VI is unaltered, but VII must read, "the particle is urged from stronger to weaker portions of the field". In actual fields, which are never quite uniform, the couple due to the latter cause overbalances the weak couple due to VI, and a rod of the substance sets transversely to the field. This explanation involves the supposition of a magnetization opposed to the field, with some other assumptions, and can only be regarded as provisional.

86. Calculation of Dependent Magnetization and Induction.

The influence of the form of a mass of iron on the intensity and direction of magnetization has been considered in a general way in Art. 81. In a few cases the effect of demagnetizing force (h') may be subjected to calculation. When the polar field is directly opposed to the impressed field (h), we have—

$$\mathrm{H} = h - h'.$$

If the value of h' is uniform, it may be expressed as a fraction of the magnetization to which it is due, or as a fraction of $4\pi\mathrm{I}$. This fraction is termed the **demagnetizing factor.** Denoting it by z, we have—

$$\text{Effective inducing force} = \mathrm{H} = h - z \, . \, 4\pi\mathrm{I}.$$

The calculation of z involves higher mathematics, and can only be carried out in a few cases. We shall, in the following examples, merely quote the values necessary:—

1. Sphere of iron placed in a uniform field.

A uniform magnetization is produced. The free magnetism due to this gives rise to a uniform polar field within the sphere. Hence H, I, and B are all uniform inside the sphere.

In this case it can be shown that $z = \frac{1}{3}$.

Thus,
$$\mathrm{H} = h - \tfrac{1}{3} \, . \, 4\pi\mathrm{I}$$
$$= h - \tfrac{4}{3}\pi k\mathrm{H}. \quad (k = \text{susceptibility.})$$

Therefore,
$$\mathrm{H} = \frac{3h}{3 + 4\pi k} = \frac{3h}{\mu + 2}. \quad (\text{For } \mu = 1 + 4\pi k.)$$

Also, $$I = kH = \frac{3kh}{3 + 4\pi k};$$

and $$B = \mu H = \frac{3\mu h}{\mu + 2}.$$

Thus, taking an iron sphere, and assuming $\mu = 1200$, the effective field, keeping the iron magnetized, is only $\frac{1}{400}$th of the impressed field. Although the iron has this high permeability, the induction

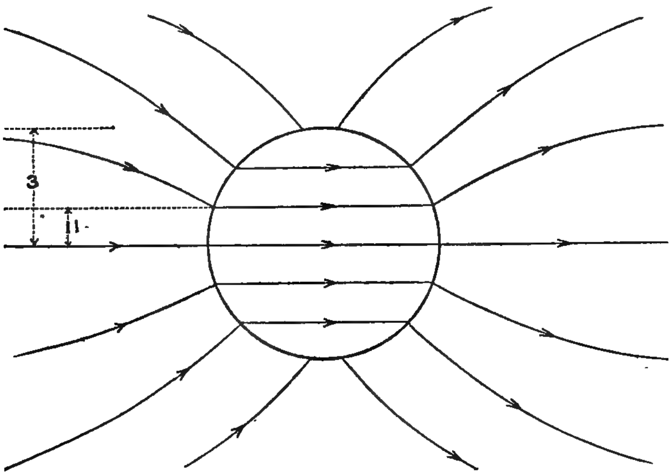


Fig. 58.—Sphere in Uniform Field. Induction Lines

in it is only about three times as great as the same impressed field would produce in a perfectly non-magnetic material; for

$$B = \frac{3\mu h}{\mu + 2} = \frac{3h}{1 + \frac{2}{\mu}}$$

$$= 3h \text{ nearly.} \quad \text{(See fig. 58.)}$$

This illustrates the importance of considering the polar force.

In a rod of circular section placed at right angles across the field, B is only twice h. (See fig. 56, quadrant 4.)

2. Anchor-ring with a narrow gap.

If the ring is placed in a uniform solenoid as in fig. 38, the magnetization produced is practically uniform. Assuming the

free magnetism only on the faces of the gap, it can be shown that the value of z is approximately given by—

$$z = \frac{d + r - \sqrt{(d^2 + r^2)}}{l},$$

where d = width of the gap, r = radius of cross-section, and l = length of the core.

EXAMPLE.—If the radius of the ring = 20 cm., the radius of section = 4 cm., and the mean width of the gap = ·35 cm. (= 1° angular width), find the impressed field required to produce a magnetization of 600.

Substituting in the formula, we find z = ·0027,

$$\text{H} = h - 20{\cdot}36.$$

But, from the normal curve, for I = 600, we have H = 2.

$$\therefore\ 2 = h - 20{\cdot}36.$$
$$h = 22{\cdot}36.$$

Hence nearly all the force (h) due to the current is neutralized by the backward force (h') due to the free poles at the gap. The surplus (= 2) produces the magnetization.

3. Complete Anchor-ring.

If there were no gap in the ring, we should have the full effect of the current. The normal curve shows that for H = 22 the value of I would be 1100. Thus in the preceding example a gap 1° wide lowers the magnetization by nearly 50 per cent.

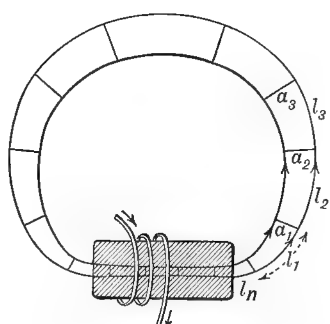


Fig. 59.—Magnetic Circuit

87. Equation for a Magnetic Circuit.

When the magnetization is entirely *dependent* on a current, a relation exists between the flux and the current which can be expressed in the form of an equation. This equation is of considerable practical importance.

Imagine partitions placed across the tube of induction so as to divide it into a number of short lengths, such that the values of B and H are constant throughout each length (fig. 59).

Let $l_1, l_2, \ldots\ldots l_n$ be the lengths of the sections;
$B_1, B_2, \ldots\ldots B_n$ „ inductions;
$H_1, H_2, \ldots\ldots H_n$ „ forces.

Similarly, let other quantities involved, *e.g.* the cross-sectional area a and the permeability μ, be distinguished by corresponding subscripts.

If the flux in the tube is P, we have—

$$P = B_1 a_1 = B_2 a_2 = \ldots\ldots = B_n a_n.$$

Thus, for one length—

$$\frac{P}{a_1} = B_1.$$

Therefore,

$$\frac{P}{a_1 \mu_1} = \frac{B_1}{\mu_1} = H_1:$$

and

$$\frac{P l_1}{a_1 \mu_1} = H_1 l_1.$$

H_1 is the resultant force along the tube, and is the algebraic sum of the impressed force h due to the current, and the polar force h'.

Thus,

$$\frac{P l_1}{a_1 \mu_1} = (h_1 + h_1') l_1.$$

Also,

$$\frac{P l_2}{a_2 \mu_2} = (h_2 + h_2') l_2;$$

and similarly for the other lengths; for the last—

$$\frac{P l_n}{a_n \mu_n} = (h_n + h_n') l_n.$$

Hence, by addition—

$$P\left(\frac{l_1}{a_1\mu_1} + \frac{l_2}{a_2\mu_2} + \ldots\ldots + \frac{l_n}{a_n\mu_n}\right)$$
$$= (h_1 l_1 + h_2 l_2 + \ldots\ldots + h_n l_n) + (h_1' l_1 + h_2' l_2 + \ldots\ldots + h_n' l_n).$$

Now the quantity in the last bracket is due entirely to polar magnetism, and it may be shown that when the sum is taken round the *whole* tube this quantity vanishes. (Chap. XXIX.).

The expression

$$h_1 l_1 + h_2 l_2 + \ldots\ldots + h_n l_n$$

involves the *impressed* forces only, and is called the **magneto-motive force** of the tube of induction. It can be shown equal to $4\pi n\mathrm{C}/10$, where C is the current in amperes. (See Chap. XXIX.)

Thus, for the whole tube, we have the relation—

$$\mathrm{P}\left(\frac{l_1}{a_1\mu_1}+\frac{l_2}{a_2\mu_2}+\ldots\ldots\ldots\ldots+\frac{l_n}{a_n\mu_n}\right)=\frac{4\pi n\mathrm{C}}{10},\ldots\ldots\ldots(2)$$

which is the equation required.

The quantity within the brackets is termed the **reluctance** of the magnetic circuit. The greater the reluctance for a given magnetic flux, the greater is the magneto-motive force required to produce that flux. Each term of the form $\frac{l}{a\mu}$ may be called the reluctance of the portion of the circuit to which it refers.

In air $\mu = 1$, but in iron it varies from 400 to 3000. Hence the reluctance of a portion of a tube of induction in air is from 400 to 3000 times the reluctance of an equal portion in iron. It follows that even if the tube is almost entirely in iron the reluctance of the portion in the short air gap is generally greater than the reluctance of all the remainder.

The reciprocal of μ is called the *reluctivity* of the substance. The reluctivity of iron is very small (·0025 to ·00033), and that of air is unity.

Eqn. (2) may be expressed—

$$\begin{gathered}(\text{Magnetic Flux}) \times (\text{Corresponding Reluctance})\\ = \text{Magneto-motive Force.}\end{gathered}$$

Observe that the reluctance of the tube of induction is *not* a constant quantity. This arises chiefly from the circumstance that the permeability is not constant, but varies with the flux density. Variations in permeability cause also variations in the size and shape of the induction tubes in most cases. (To realize this last statement consider what effect an increase of μ would have in Ex. 3, Art. 81.) The shape of the tubes is also dependent on the *arrangement* of the n coils of the magnetizing circuit, a matter which the equation takes no account of.

The form of Eqn. (2) is not always convenient for calculation. We may write H for $P/a\mu$. Thus—

$$H_1l_1 + H_2l_2 + \ldots\ldots + H_nl_n = \frac{4\pi nC}{10}, \ldots\ldots\ldots\ldots (3)$$

which is generally more convenient. If P is given, we first obtain B from the equation $P = Ba$. The value of H is then obtained from the normal curve.

88. **Comparison of Methods.**

In Art. 86 we briefly noticed how the magnetic force required to produce a given magnetization (and therefore induction) could be calculated in special cases. Equation (3) gives us an alternative method. Thus in Ex. 2, Art. 86, we require to know what impressed force will give $I = 600$.

We have $I = 600$. $H = 2$. $\therefore B = 7544$.

$$\therefore \text{ H in iron } (= H_1 \text{ say}) = 2.$$
$$\text{H in gap } (= H_2 \text{ say}) = 7544.$$

Hence, by Eqn. (3)—

$$2 \times (2 \,.\, \pi \,.\, 20 - \cdot35) + 7544 \times \cdot35 = \frac{4\pi nC}{10}.$$

If the coil is uniformly wound, h is uniform, and

$$\frac{4\pi nC}{10} = h \times (\text{circumference})$$
$$= h \times 2 \,.\, \pi \,.\, 20.$$

Thus, $\quad h \times 40\pi = 2(40\pi - \cdot35) + 7544 \times \cdot35,$

or, $\quad h = 23.$

The difference between this result and the value obtained in Art. 86 is due to the approximate value of z taken there. The chief difficulty in connection with the method of Art. 86 is the determination of the "demagnetizing" coefficient z, which depends on the distribution of polar force. This difficulty is overcome in the method of Art. 87 by taking complete magnetic circuits or tubes of induction. The terms depending on polar force then vanish. But a corresponding difficulty occurs, namely, the determination of the shape of the tubes of

induction. This is to be expected, since the methods are radically the same. We may say, therefore, that—

1. The "polar force" method (Art. 86) is employed when it is easier to determine the distribution of free magnetism than the complete shape of the induction tubes. This occurs in the case of spheres, ellipsoids, long rods, tubes of circular section, etc., placed in uniform fields.

2. The "induction (or magnetic circuit)" method (Art. 87) is employed when it is easier to determine the shape of the tubes of induction. This occurs when complete or nearly complete iron circuits are linked with the magnetizing coils.

In practical applications the second method is much more important than the first.

ELECTRO-MAGNETS

89. Forms of Electro-magnet.

The chief advantages of electro-magnets over permanent magnets are:—

(1) They can give rise to very strong fields.
(2) They may be quickly magnetized or demagnetized.
(3) Their strength may be readily varied by means of the current.

The form taken by the iron core varies according as the magnet is required to satisfy condition (1) or (2). Fig. 60 shows an arrangement for producing a strong field. The pole pieces are made conical. The tendency of the lines of induction to remain as much as possible in the iron causes a great flux density in the narrow gap left, a value of 50,000 C.G.S. units being obtainable. Examples of the second class are afforded by electric bells and many telegraphic instruments. The cores consist of short straight rods. There is thus a considerable polar force, which helps to produce a rapid demagnetization.

90. Electro-magnet Coils.

The coils used to carry the current also differ in the two cases. The effectiveness of a coil or the magneto-motive force

is given by $4\pi nC/10$, or $1{\cdot}257\, nC$. The product nC is known as the number of *ampere-turns*. The magnetic effect may be increased by increasing this product, but there are practical limitations to increasing *both* n and C, owing to heating effects.

If the magnet is to produce a strong field, the current may be derived from, say, a battery of accumulators connected directly with the coils. An increase in the length of wire

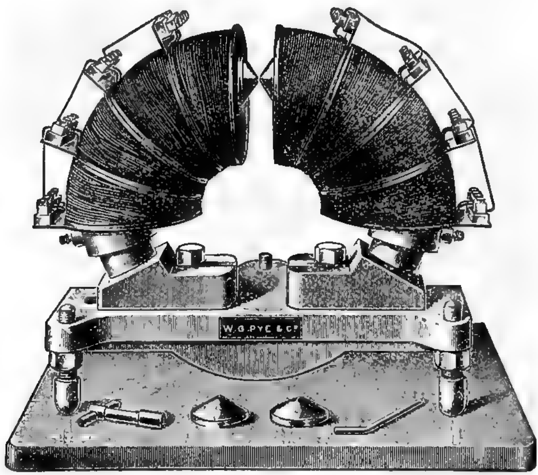


Fig. 60.—Electromagnet for Producing an Intense Field

then produces a corresponding decrease of current. Hence the product nC will not increase with n unless thicker wire is used when the number of turns is augmented. But if this is done, a large number of turns produces a bulky coil. The heat produced by the current cannot then readily escape from the circuit. Unless special ventilation is provided, the coil may become very hot. Hence, for magnets such as that shown in fig. 60, a comparatively small number of turns of very thick wire is used.

On the other hand, if the magnet is to be used on a long circuit, as is often the case with a telegraph instrument, an increase in the number of turns does not add much resistance

in proportion to that already in the circuit. Further, the current used is too weak to produce much heating effect. The magnetizing coils, therefore, are provided with a large number of turns of fine wire.

91. Calculation of Ampere-Turns.

The following property of induction lines may be regarded as proved by experiment:—

> VIII. "**When a complete or almost complete circuit of iron is linked with a magnetizing coil, the lines of induction remain in the iron, following all the bends of the latter until they reach the gap.**"

The shape of the lines of induction can consequently in such cases be assumed, and the equation of the magnetic circuit can be applied. This result is important in connection with the design of electro-magnets, particularly such as are used in dynamos, motors, and other electro-magnetic machinery. We shall illustrate this by some simple examples:—

An anchor-ring 60 cm. in circumference and 4 sq. cm. cross-sectional area has a radial gap ·5 cm. wide. Find the number of ampere-turns required to produce a flux of 44 kilolines in the gap. The lines of induction may be here supposed circular.

Induction (flux density) in the gap $= \frac{44000}{4} = 11000.$

Induction (flux density) in the iron $= \frac{44000}{4} = 11000.$

In the gap, $H_1 = B_1 = 11000$.

In the iron, H_2 is less than B_2, and must be found from the normal curve. Suppose this gives $H_2 = 10$ for $B_2 = 11000$, $l_1 = {\cdot}5$, $l_2 = 60 - {\cdot}5 = 59{\cdot}5$. Then, by equation (3)—

$$11000 \times {\cdot}5 + 10 \times 59{\cdot}5 = \frac{4\pi n C}{10},$$

$$6095 = 1{\cdot}257\ Cn,$$

$${\cdot}8 \times 6095 = nC.$$

$$4876 = nC = \text{ampere-turns required.}$$

For a current of 5 amperes we require 975 turns.

92. Leakage Coefficient.

Rule VIII is only approximately true in most cases. Some lines depart from the shape of the core and leak out "sideways" from the iron before reaching the gap or other point for which the calculation is made. These are called leakage lines, and their total number the leakage flux. (Fig. 61.)

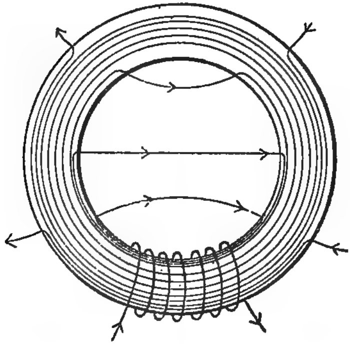


Fig. 61.—Leakage Lines

Hence, to produce a given flux P lines in the gap, the number produced in the iron must be greater than P, say P′. This number may be found by multiplying P by a factor greater than unity. This factor is termed the *leakage coefficient* (v). Thus—

$$P' = vP,$$

$$\text{or} \quad v = \frac{P'}{P} \quad \text{.........................(4)}$$

The leakage coefficient depends on the size of the gap, the distribution of the windings, and other conditions. Its value in important cases is known with sufficient accuracy from experience.

Example.—If the leakage coefficient in the example above is 1·2, the number of ampere-turns required will be

$$1 \cdot 2 \times 4876 = 5851.$$

The leakage from the ring becomes very small when the coils are wound uniformly over its entire surface instead of collected at one part.

Example.

An electro-magnet consists of two upright bars of section 11 cm. square, a horizontal yoke 13 cm. square, and two pole pieces 11 cm. square. The uprights are 50 cm. long, the yoke 30 cm., and each

pole piece 14 cm., an air gap 1 cm. wide being left between the poles. Given a table of values of B and H, find the ampere-turns necessary to produce a field of density 13000 lines per sq. cm., allowing a leakage coefficient 1·3.

B	H
12100	10
16900	45

Flux in the gap $= 13000 \times 11^2$.

Flux in pole pieces and iron circuit $= 13000 \times 1{\cdot}3 \times 11^2$
$= 2044900$.

Induction (flux density in gap) $= 13000$.

$$\text{Induction in pole pieces} = \frac{2044900}{121} = 16900.$$

$$\text{Induction in uprights} = \frac{2044900}{121} = 16900.$$

$$\text{Induction in yoke} = \frac{2044900}{169} = 12100.$$

$\therefore$ H in gap $= 13000$.
in pole pieces $= 45$.
in uprights $= 45$.
in yoke $= 10$.

$\therefore$ magneto-motive force $= (13000 \times 1) + 2(50 \times 45) + 2(14 \times 45) + 10 \times 30$.
$= 19060$.

$\therefore$ ampere-turns $= 19060 \div 1{\cdot}257 = 15163$.

QUESTIONS

1. Explain why an iron plate placed parallel to the lines of force in a field of moderate strength is more strongly magnetized than when it is placed at right angles to the lines. Why is this not true for very strong fields?

2. A small sphere of iron placed near a magnet is not in general urged along the lines of force. Give reasons for this, and discuss some particular cases.

3. An anchor-ring has a cross-sectional area of 3 sq. cm., a mean radius of 10 cm., and a radial gap 2 mm. wide. Find the number of ampere-turns required to produce a flux of 27,000 in the gap, allowing a leakage coefficient 1·33. Given also—

$$B = 9000 \text{ when } H = 5,$$
$$B = 12000 \text{ when } H = 10.$$

Also find the additional ampere-turns required owing to the leakage.

PART II.—STATIC ELECTRICITY

CHAPTER VIII

FUNDAMENTAL PHENOMENA

93. Electrification by Friction.

When a piece of amber is rubbed with fur, flannel, or similar material, it acquires the property of attracting any small particles, such as pieces of paper, cork, or bran. The existence of this property was known to the ancients, who mention in their writings that amber and jet will when rubbed attract light particles. But these were the only two substances recorded as possessing this property until, about the year 1600, it was discovered by Gilbert that many other materials possess a similar power, *e.g.* sulphur, sealing-wax, and resin. It is now known that any substance will exhibit the property in some degree when rubbed with a dissimilar substance.

A body which has acquired the property of attracting light particles of any material is said to be *electrified* or to be in a condition of *electrification.* Since the attraction observed is sometimes weak and sometimes strong, we attribute the difference to the degree of electrification, which is thus regarded as measurable.

94. Electroscopes.

It is evident that we use light particles in the above experiments simply because the force is a comparatively weak one. An instrument arranged to show the existence of electrification is called an electroscope. One form, devised by Weinhold, consists of a hollow ball of aluminum suspended at the end of a light rod, as shown in fig. 62.

Gilbert used balanced straws as electroscopes. A small ball of elder pith, gilded, and suspended by a long thread, forms a *pith-ball electroscope.* By means of any such simple electroscope we can detect electrification produced in many substances. Besides those mentioned above, shellac, vulcanite, or ebonite rubbed with flannel readily become electrified. Brown paper drawn between pieces of woollen cloth, and foreign note-paper rubbed with india-rubber also serve excellently if they are first made hot and dry. It is, in fact, essential to success in all these experiments that the substances used be quite dry, and it is therefore necessary to warm those which are hygroscopic. An open fire is best for this purpose, but whatever means is adopted the heating should be by *radiation.*

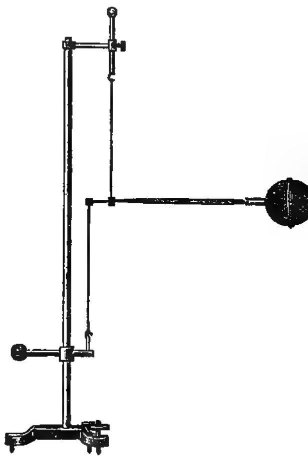


Fig. 62.—Weinhold's Electroscope

An effective arrangement consists of a sand-bath 20 cm. × 30 cm. supported over a board covered with a large sheet of tin-plate. The sand-bath is heated by a large bunsen, and the flannels, rods, etc., are laid on the board. If turned over occasionally the rubbers become thoroughly dried by the downward radiation.

Crystalline bodies such as quartz (rock crystal), gems (diamond, etc.), may also be electrified. Glass, especially lead-glass, is easily electrified by rubbing it with silk, if it is first made quite hot. The surface of glass ordinarily carries a film of moisture.

95. **Two Kinds of Electrification.**

Electrify an ebonite rod by rubbing it with fur. Suspend it in a stirrup hung by unspun thread. Bring a second ebonite rod, similarly electrified by rubbing with fur, near the first one. Re-

pulsion occurs. Repeat the experiment with two lead-glass rods, well warmed and rubbed with old silk. Repulsion occurs again. Now suspend the electrified ebonite rod and bring the electrified glass rod near. This time attraction takes place.

From a number of similar experiments with other materials we may show that it is possible to arrange electrified bodies into two groups represented by the glass and ebonite respectively; bodies in the same group repel each other, whilst those in one group attract those in the other.

1. *Ebonite* rubbed with collodion.	1. *Ebonite* rubbed with fur or flannel.
2. *Foreign note-paper* rubbed with india-rubber.	2. *Shellac* rubbed with fur or flannel.
3. *Glass* rubbed with silk.	3. *Sealing-wax* rubbed with fur or flannel.
	4. *Amber* rubbed with flannel.
	5. *Resin* rubbed with flannel.
	6. *Brown paper* rubbed with fur or wool.

We therefore conclude that—

(1) There are two kinds of electrification;

(2) Similarly electrified bodies repel each other, and dissimilarly electrified bodies attract each other.

It is usual to distinguish the two kinds of electrification by the terms *positive* and *negative.* Bodies mentioned in the left-hand column above are positively electrified. Notice that a given substance may fall in with one group or the other according to the nature of the rubber used. We therefore see that—

(3) Bodies are not absolutely positive or negative; the kind of electrification depends on the nature of the rubber as well as on the body rubbed.

The result in (2) enables us easily to detect the kind of electrification on a body.

Electrify an ebonite rod with fur and suspend it. Bring the body to be tested near the rubbed end of the ebonite. Repulsion indicates a negative electrification. Attraction indicates a positive electrification, or possibly a neutral body. In the event of attraction occurring it is necessary to repeat the test with a glass rod rubbed with silk. In all cases bring the body to be tested *gradually* near the suspended rod, and notice the *first* effect produced.

In accordance with Newton's Third Law, the attraction or repulsion is in all cases *mutual;* that is, the force is exerted equally on each body.

If two bodies positively and negatively electrified respectively are brought near the same object they tend to neutralize each other's effects. When the effects are exactly neutralized we say that the positive and negative electrifications are exactly equal.

96. Conduction.

(i) If you hold a metal rod in your hand and attempt to electrify it in the ordinary way, you will not succeed. But if you first mount the metal on a vulcanite handle, and then brush it lightly with fur (being careful not to rub the handle), you will find that it becomes strongly electrified. A metal, therefore, can be electrified, but it is necessary to mount it in a special way. The same is true of other substances, for example, carbon and wood.

(ii) Mount a brass rod a foot long on a vulcanite handle. Brush the rod near the end with a small fur brush (a gold-beater's tip serves the purpose well). Now test the rod with a pith-ball pendulum. You will find that *all* parts of the rod are electrified. Next take a long rod of sealing-wax and treat it similarly. Only the part rubbed will be found electrified; and a similar result is obtained with ebonite, shellac, and other materials.

(iii) Support two metal rods of equal size on ebonite handles. Electrify one as above. Test it with a Weinhold pendulum, holding it at a definite distance. Next allow the rods to touch each other. Each rod will now deflect the pendulum, but the deflection is much less than before.

From (ii) we conclude that the condition of electrification can extend from one part of a metal to another; and from experiments similar to (iii) we conclude that the spreading of

the electrified condition is due to an actual *transference* of something to which the condition of electrification is due. We therefore adopt the theory that electrification is due to a "substance" or agent which we term *electricity*, and that electricity is a measurable physical quantity, subject to the law of addition.

The transference of electricity from one part of a metal to another is termed *conduction*. Substances which allow this transference are termed *conductors*: those like sealing-wax, vulcanite, etc., which do not allow the electricity to flow through them, are called *non-conductors*. The electricity in conductors must be considered to act as a "fluid". The electrified body is often said to possess an *electric charge*, or *charge of electricity*.

Earth-connection.—We can now see the necessity for the vulcanite support in (i) above. Metals, wood, and the human body are all conductors. Hence if a piece of electrified metal is allowed to touch the hand, table, walls of the room, or surrounding objects generally, its charge of electricity will flow over all these and become "lost". This spreading of the charge alone would account for the disappearance of the electrification, but we shall see later that the process is not merely a spreading, for the charge at the same time becomes neutralized. It is convenient, however, for practical purposes to regard the charge as escaping to "earth", and the process is therefore often called "earthing" the conductor.

97. Insulators.

In order to prevent the charge on a conductor from becoming earthed or flowing to other conductors, it is necessary to provide non-conducting supports. The conductor is then said to be *insulated*. Since this is the chief use of non-conductors, they are usually termed *insulators*.

Two or more conductors in contact form, for the purposes of static electricity, a single conductor. The transference of electricity from one conductor to another is usually accompanied by an "electric spark".

98. Tests to Distinguish Conductors from Insulators.

A substance which can be held in the hand and electrified by rubbing is, of course, an insulator. When this method is impracticable the following simple tests may be applied:—

Mount a brass rod on an ebonite handle and electrify it by lightly brushing it with fur. See that it attracts a pith-ball pendulum. Now test the body, if solid, by holding one end between the fingers and allowing the other end to touch the charged metal. Test the latter again with the pendulum; if the metal is discharged, then the body tested is a conductor. The surface of the solid must be scraped (or otherwise rendered quite clean) and dried, before making the test. Liquids may be tried by soaking a silk thread with them. Hold one end of the silk in the fingers and allow the other to touch the metal. Dry air is a good insulator, otherwise bodies could not retain their charges. The same is true of other gases.

Insulators		Conductors
Quartz.	Resins.	Metals.
Sulphur.	India-rubber.	Carbon.
Ebonite.	Silk.	Wood.
Paraffin wax.	Wool.	The human body.
Mica.	Glass.	Water.
		Acids.
Oils, turpentine, petroleum.		
Gases.		

99. Forms of Insulated Conductors.

The insulated conductors used in static experiments often consist of spheres, or thin discs, of brass, varying from 5 to 10 cm. diameter. Elongated conductors are formed of wooden cylinders with rounded ends smoothly covered with tin-foil. If the conductors are required to retain strong charges it is necessary to avoid sharp points or jagged edges. All portions of the conductor must be smoothly rounded and free from corrosion. The best supports for general purposes consist of rough ebonite rod about half an inch thick. Lengths of 8 in.

should be mounted on wooden bases weighted with lead, and provided with a small brass head, to which the conductor may be attached by a screw. The support must be kept away from flames and heating apparatus, as it becomes softened by heat; and it is unnecessary to warm the ebonite even in wet weather. If the surface becomes deteriorated through handling, its insulating power may be at once restored by scraping it with a knife. When not in use ebonite should be kept in a dark place.

A block of paraffin wax freshly pared also forms an excellent and handy support. Sulphur is the best substance to use from the point of view of obtaining good insulation, but it is too brittle to be used except in small stout blocks. These may be cast in paper moulds. The sulphur must be heated until it just melts, and then poured into the mould. If allowed to thicken during the melting, the insulation is impaired. Quartz fibres may also be used where a fine insulating suspension is required.

100. Simultaneous Production of the Two Electrifications.

Make a flannel cap to fit closely on the end of a polished vulcanite rod. Attach a silk thread to the flannel. Now by means of the thread pull the flannel round, keeping it on the end of the rod. Without removing the cap, test the arrangement with a pith-ball pendulum; there is no effect. Remove the flannel by means of the silk thread, and test the rod and flannel separately, using a suspended electrified rod of ebonite for this purpose. The vulcanite is found negatively charged and the flannel positively charged.

We conclude therefore that—

(i) The rubber becomes electrified as well as the body rubbed.

(ii) The charges are unlike in kind and equal in amount.

Similar experiments may be made with glass and silk, sealing-wax and fur, etc.

In most experiments the rubber offers a fairly large surface to the hand, and therefore loses its electrification even if made of insulating material. We must, therefore, insulate the rubber.

It follows from (ii) that if we know the kind of electrification produced on one of the bodies, that on the other may be at once inferred. We may arrange substances in a series such that any substance mentioned becomes positively charged if rubbed with one occurring later in the series, *e.g.* fur, wool, ivory, quartz, glass, silk, sulphur, ebonite, india-rubber, collodion. The results are, however, uncertain if the substances come close together in the list.

This is known as the *frictional order.*

101. Electric Induction.

(i) Support an elongated conductor on an insulating stand and in a horizontal position. Place a pith-ball pendulum near one end of the conductor, so that the ball is close to the conductor. Hold a strongly-charged stick of ebonite near the other end of the conductor. The pith-ball is at once strongly attracted. The conductor therefore becomes charged, but not by conduction, because it is separated from the ebonite by the non-conducting air-space. The charge on the conductor is merely due to the influence of the charge on the ebonite. The process of *developing* a charge in a conductor *by influence* is termed *electric induction.* The charges produced in this way are termed *induced.*

(ii) Mount two metal cylinders on insulating supports, and place them in contact to form one long cylinder. Hold a charged rod near (not too near) one end, and whilst it is there separate the conductors. If each is now tested with a suspended ebonite rod it will be found that the two conductors are oppositely charged. The induced charge on the nearer portion is of opposite nature to the inducing charge, that on the farther portion is of like nature. If the inducing body is removed before separating the cylinders, the induced charges exactly neutralize each other, and are therefore equal.

The production of induced charges is explained by supposing that *two kinds* of electricity exist in equal amounts in a neutral body. When a charge is brought near, one kind of electricity is attracted and the other kind repelled, so pro-

ducing an excess of one kind at one end of the conductor and an excess of the other kind at the other end. Adopting this theory, we see that the *charge* existing at any part of a body is the *excess* of one kind of electricity at that point.

102. Free and Bound Charge.

If whilst the inducing body is held near the conductor we connect the latter with earth for a moment, and lastly remove the inducing body, we find that the conductor has acquired a charge of one kind only, of *opposite* sign to the inducing charge. The repelled charge, therefore, has escaped to earth, whilst the attracted charge remains. These results are conveniently expressed by saying that the attracted charge is *bound* by the attracting or inducing charge, whilst the repelled charge is *free*, and escapes when a channel is opened for it.

Preservation of Induced Charges.—We see from the above experiments that we may preserve the induced charge on a conductor in two ways—

(i) **by dividing the conductor into two parts (equal or unequal) before removing the inducing body;**

(ii) **by earth connecting the conductor momentarily before removing the inducing body.**

The latter method yields *a strong charge of opposite kind to the inducing charge.*

103. Attraction Accompanies Induction.

If a + charged body is brought near a pith-ball hung by a *silk* thread, negative electrification is induced on the near side and positive on the farther side of the ball. The former charge causes the pith-ball to be attracted by the inducing body, whilst the latter charge causes repulsion. The negative induced charge is a little nearer, however, and therefore there is a weak resultant attraction.

If the ball is hung by a cotton thread it is earth-connected, and has a strong induced charge of one kind only. This is

unlike to the inducing charge, and there is therefore a strong attraction.

A small ball of ebonite or other insulator, *initially quite neutral*, is feebly attracted when a charged body is brought near. The charges induced on insulators are extremely feeble compared with those induced on conductors under like conditions.

104. Gold-leaf Electroscope.

One form of this instrument is shown in fig. 63. A brass rod has a small brass plate about 1 cm. square attached to its lower end. The rod is fitted through an ebonite plug, and carries at its upper end a circular disc or induction-plate. The plug fits a hole in the top of a wooden box the front of which is removed and replaced by a glass window. Two strips of gold-leaf each 5 cm. long and 1 cm. broad are attached to the plate at the lower end of the rod. A circular scale at the back of the case provides for the reading of the angular divergence of the leaves.

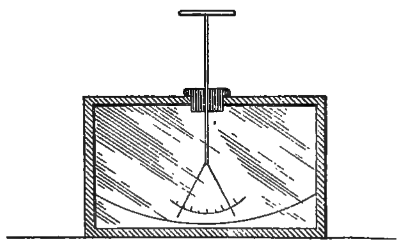


Fig. 63.—Electroscope—"Induction" Form

The chief advantage of the gold-leaf electroscope is that it may be used for rough quantitative measurements. For example, if experiment (iii), Art. 96, is repeated with this form of electroscope, the rods being held in similar positions with respect to the disc, the equality of the division of charge may be shown.

The instrument should be charged, as a rule, by induction. To give the leaves a positive charge bring a negatively charged body near, release the free charge from the leaves by touching the disc momentarily, and lastly remove the charged rod.

The case being of wood or metal, protects the leaves from the *direct* influence of all external bodies.

Strongly-charged bodies should not be brought close to the electroscope, as the repulsion between the leaves may become strong enough to tear them. For testing such bodies a *proof-plane* may be used. This is a small metal disc (about 3 cm. in diameter) mounted at the end of a thin rod of ebonite or other insulator. It is first made to touch the conductor to be tested, and acquires a portion of the latter's charge by conduction. The proof-plane is now made to transfer its charge to the electroscope; the leaves diverge. A negatively-charged ebonite rod is now brought gradually near from a distance. If the first effect is a collapse it indicates a positive charge; if a greater divergence, a negative charge.

We shall often use g.l.e. as a contraction for "gold-leaf electroscope".

105. Tests of Conducting Power.

Support a strip of the substance to be tested on two insulating uprights. Join one end of the strip to the electroscope and charge the latter. Notice that the divergence of the leaves is steady. Now connect the other end of the strip to earth and observe the leaves carefully. If they collapse instantly, the substance is a conductor; if they remain divergent, it is an insulator.

But it is found that many substances cause the leaves to collapse, though not instantly. The divergence gradually decreases at a rate depending on the substance used. Such substances form an intermediate class known as *partial* conductors. With the methods adopted in current electricity we may show that the good or bad conductivity of a substance is a distinction merely in respect of *time*. Good conductors will allow the charge to escape quickly, whilst bad conductors allow it to escape slowly. The former are said to offer little resistance to the flow, whilst the latter offer great resistance. Perfect conductors and perfect insulators are unknown. Substances can be arranged in order of their conducting power, and differences can be observed even between good conductors. (But in the experiments of *static* electricity we cannot observe any difference in the behaviour of the different metals, and it is

immaterial, therefore, what metal we make the conductors of.) Examples of partial conductors are dry wood, paper, string, cardboard, and cotton.

106. Seat of Electrification on Conductors.

(i) Mount two metal balls of equal size, one being "solid" and the other hollow. Charge one of them, and then place the other in contact with it. Now hold each ball in turn at the same distance from the disc of a gold-leaf electroscope. You will find that they produce equal amounts of divergence. The charge has been equally shared.

(ii) Place a deep metal can on a block of paraffin wax and charge it. Touch the inner surface of the can with a proof-plane, then bring the latter out and test with an electroscope. There is no effect. Apply the proof-plane to the outer surface and test again. A charge is found in this case.

These experiments show that

The charge of a conductor resides on the outer surface only.

Other experiments which prove this important result are Biot's experiment, where all the charge is picked off a sphere by removing two hemispherical covers; and Faraday's "butterfly net" experiment, which shows that when a conductor is pulled inside out the charge passes at once to the new outer surface.

The statement does not hold true in the case of a hollow conductor if there are *charged* bodies suspended within it.

107. Property of a Hollow Conductor.

If we bring a charged conductor in contact with a neutral one, some of the charge is transferred to the latter, but some of the charge is retained also by the first conductor.

An important exception to the last statement occurs in the case of a hollow conductor when the contact is made with its *inner* surface.

If a charged conductor is placed entirely within a hollow conductor and caused to touch the inner surface, the whole of the charge is

transferred to the latter, and passes to the outer surface.

This holds whether the hollow conductor possesses a charge initially or not.

108. Electroscope for Comparing Charges.

If an electroscope is provided with a deep vessel of thin brass (or preferably aluminium) in place of the ordinary disc, it may be used to test the equality of charges on bodies of different shapes, and for other purposes which we shall explain later. Each body to be tested is made to give up its charge in turn by contact with the inner surface of the receiver, the electroscope being discharged between the successive tests.

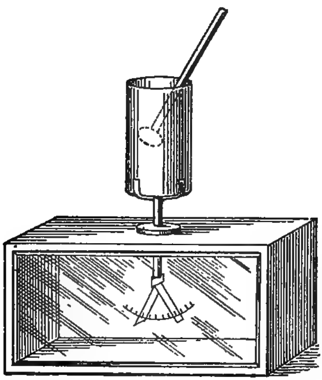


Fig. 64.—"Quantity" Form of Electroscope

Equality of charge on bodies of different shapes cannot be tested by simply holding them near or in contact with the disc of a g.l.e. A difference in shape or size in the conductors will cause some of the charge to be differently placed with respect to the electroscope, so that equal charges may not produce equal divergence.

We shall refer to the above as the *receiver* or *quantity* form of electroscope (fig. 64).

109. Distribution.

The amount of charge on unit area (1 sq. cm.) of the surface of a conductor at any point is called the *surface density* of the charge at that point. The *distribution* of the charge denotes the *relative* surface densities at all parts of the surface.

To compare surface densities.—The conductor used

should be hollow, and proof-planes of equal area constructed to fit the parts to be tested.

Example.—Insulate a metal cube at a distance from the table by suspending it with silk thread. Prepare three proof-planes each of area 3 sq. cm. to fit the middle of the face of the cube, the middle of one edge, and the corner respectively (fig. 65). Charge the cube, and hold the first proof-plane in contact with the middle of a face of the cube. Now hold the proof-plane within the receiver of a quantity electroscope, but without contact. (This, for a reason explained in Art. 128, produces the same divergence as if the charge were actually transferred to the receiver.) Note the divergence. Test the middle of the edge and the corner similarly with the other proof-planes. The densities may be recorded in terms of the divergences obtained.

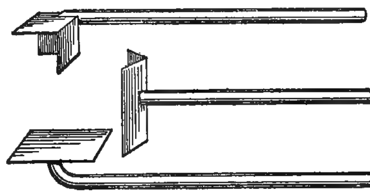


Fig. 65.—Proof-Planes

Theoretically the charge removed by each proof-plane should be restored (by contact with the inner surface of the cube) before making the next test. If the proof-planes are large this should be done.

The results of experiments on conductors of various shapes point to the following rule:—

The density is greatest on those portions of the surface which are most sharply rounded.

(The conductor is here supposed *isolated*; *i.e.* removed from the influence of all neighbouring conductors or insulators.)

110. Action of Points.

When the rounding of the surface of a conductor is very sharp, as at the point of a needle, the density becomes so great that the air surrounding the point becomes electrified by contact and is immediately repelled. Unelectrified air takes its place and is repelled in turn. This goes on until so much

electricity is carried off the conductor that the remaining density, even at the point, is too small to electrify the air. During the discharge there is an "electric wind", which always blows *from* the point for either kind of charge.

111. Transference of Charge from Insulators.

When two conductors are brought in contact, any charge they may possess spreads over both. But if a charged insulator is brought into contact with a neutral conductor the charge can only be transferred from the point of contact. This is well illustrated by the following experiment:—

Give an ebonite rod a *weak* charge. Hold it in contact with the disc of a g.l.e. The leaves diverge, but on removal of the rod they completely collapse. The divergence was due to the induced charge. But if the rod is stroked on the edge of the disc, the charge is as it were "scraped" off the rod, and the leaves remain divergent when the latter is removed. Again, if the rod is strongly charged, especially if it yields a noticeable spark, a fair charge is transferred without the stroking.

In consequence of the possibility of little charge being transferred by contact, it is generally better in charging a conductor from an insulator to *use the process of induction and earthing.*

The property of points may be applied to virtually transfer the charge from an insulator to a conductor. Attach the eye end of a needle to an insulated conductor by a particle of bees'-wax. Move a charged ebonite rod backwards and forwards at right angles to the needle, but close to the point. It will be found that in this way the rod loses most of its charge, whilst the conductor gains a charge of the same kind. If the rod is negative, it induces a positive charge on the point and a negative charge on the farther portion of the conductor. The positive charge collects densely on the point, and this is aided by the attraction of the negative on the rod. A positively-electrified wind blows from the point and over the surface of the ebonite. Hence the conductor loses positive electricity and is left with a negative charge.

112. Frictional Machines.

The above principles are applied in the construction of machines to obtain a constant supply of charge by frictional methods. A standard form (fig. 66) consists of a glass plate about 45 cm. diameter, mounted on a horizontal axle so that it can be turned by a handle. It passes between two pairs of silk rubbers, attached to the top and bottom of the frame carrying the axle. Rows of spikes forming a "comb" are placed on each side of the plate, at the ends of a horizontal diameter, and are connected to a large insulated brass rod forming the prime conductor. The glass becomes charged positively by passing between the rubbers, and, through the action of the points this charge is virtually transferred to the prime conductor. These machines are now entirely superseded by induction machines. (See Chap. XVII.)

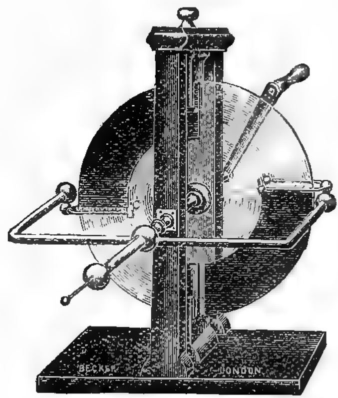


Fig. 66.—Plate Frictional Machine

113. Sources of Electrification.

Although we speak of "frictional" electrification, it is probable that friction has very little to do with the development of charges. The best effects are often produced by rubbing the objects as lightly and *briskly* as possible. The contact of dissimilar substances is an essential part of the process. There are many other ways of producing electrification. When cork is pressed on metals or india-rubber it becomes electrified. Certain crystals, such as mica, when cleaved become electrified. Sugar becomes electrified when crushed, and in the dark may be seen to emit light. Sulphur also becomes electrified when crushed. Evaporation and crystallization are also accompanied by the development of electric charges. A crystal of tour-

maline when heated or cooled acquires an "electric polarity". It becomes + charged at one end and − at the other during a rise of temperature, but the polarity is reversed when the crystal begins to cool. Tourmaline and other crystals possessing a similar property are termed *pyro-electric* substances. Besides these and many other interesting means of developing electric charges, there are some to be specially considered on account of their practical importance, namely, voltaic cells, thermo-electric junctions, and generators depending on electro-magnetic induction. These will be considered later.

In all cases the production or disappearance of one kind of charge is accompanied by the production or disappearance respectively of an equal amount of the opposite kind.

114. One-Fluid Theory of Electricity.

In the explanation of induction (Art. 101) we have assumed that the two kinds of electricity are displaced in opposite directions in the conductor; that is, we have assumed that both kinds are *fluid*. But the results may be equally well explained by supposing that only one kind is fluid. This may be seen from fig. 67. This represents diagrammatically an elongated conductor divided into imaginary compartments each containing an equal number of particles of + and − electricity (*a*). Suppose a positively-charged body brought near the left-hand end. Adopting the two-fluid theory, we may suppose the whole of the positive electricity to move one division towards the right, and the negative one division towards the left. This leaves a charge or excess of two particles at each end (*b*). On the one-fluid theory we may suppose that the negative particles remain fixed, whilst the positive particles are repelled through two divisions (*c*); or that the positive particles remain fixed whilst the negative particles are attracted through two divisions (*d*).

In all three cases the induced charges and forces are equally well explained, but the one-fluid theory has been consistently used in connection with current electricity; and the whole

language of the subject is bound up with the supposition that the positive electricity moves, as in (*c*).

In what follows we shall therefore assume as our **working theory**:—

> (i) **that the positive electricity alone acts as a fluid, the negative being, as it were, inseparable from the particles of matter;**
>
> (ii) **that a neutral body contains equal amounts of the two kinds.**

It follows that when positive electricity flows out of a neutral body it leaves an excess of negative electricity or a

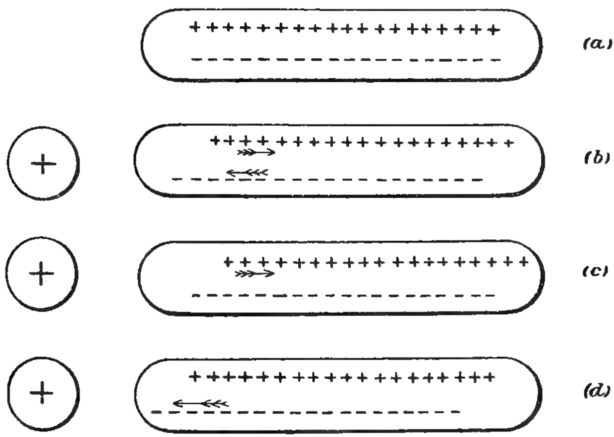


Fig. 67.—Two-fluid and One-fluid Theories

negative charge. The term "electricity" in such expressions as "a current of electricity" must be understood to refer to the *positive* kind.

The normal theory stated in (i) and (ii) may be adopted in static electricity, and throughout in current electricity when we are dealing with solid conductors. In some cases, however, we lay aside the normal theory and adopt one of the others. The chief cases are:—

(1) *In Static Electricity.*—In the explanation of many experimental results we assume that the negative electricity flows, because this often provides a more natural and convenient explanation. For example, if we "scrape" a negatively-charged rod on an insulated conductor the latter becomes charged negatively, and we naturally attribute this to a transference of negative electricity from the rod to the conductor. But, of course, we may *when necessary* interpret this in terms of the normal theory by saying that positive electricity passes from the conductor to the rod.

(2) *In Current Electricity.*—Where we have to deal with conduction in electrolytes or gases the + and − charges are supposed to be *carried* by moving particles or "ions".

Figs. 90 (*a*) and (*b*) show diagrammatically the inductive displacement of positive electricity by a positive inducing charge.

115. Comparison of Theories.

The various theories which have at different times been proposed agree in that they account for electric forces by the attractions or repulsions between the particles of two different "substances". In the two-fluid theory of Symmer and Du Fay these substances were supposed to be distinct from matter in that they were without weight. They were supposed to exist in equal quantities in every neutral body. The one-fluid theory propounded by Franklin replaced one of the electricities by *matter*, and reserved the term *electricity* for the other substance. (The application of the terms *positive* and *negative* to charges is due to Franklin.) According to Franklin's view, the particles of electricity repel each other, but attract particles of matter. An extension of this theory due to Æpinus made the additional supposition that the particles of matter repel each other, thus restoring the symmetry of forces observed with the two-fluid theory. The law of gravitation was accounted for by supposing the attractions a little stronger than the repulsions.

A theory developed in recent years differs from these in

supposing that matter *consists* of the two electricities. An atom of matter is regarded as being composed of a "sphere" of positive electricity together with a swarm of "particles" of negative electricity—termed "corpuscles" by Prof. J. J. Thomson and "electrons" by Larmor and Lorenz. The theory thus resembles a one-fluid theory applied to atoms, and corresponds to that of Franklin or Æpinus according to the forces assumed to exist between the positive and negative parts of the atom.

At the present time we must assume that there are two kinds of electricity, the positive being bound to atoms, whilst some of the negative particles are removable from the atoms, and, in solid conductors, are capable of independent motion. The negative electricity is thus the fluid portion, so that, according to modern views, the third supposition (*d*) would be the correct one. The working theory stated above only differs from this in that, to conform with long-established custom we take the positive as fluid. In some parts of the subject however, for example, in the phenomena of discharge through gases, it is necessary to lay aside the established notion of the positive electricity being fluid, and to suppose the positive current replaced by an equal flow of negative electricity in the reverse direction.

CHAPTER IX

ELECTRIC FORCE

116. Forces between Point Charges.

The forces of attraction and repulsion occurring between two point charges of electricity are subject to the following laws:—

I. Like kinds of electricity repel each other, unlike kinds attract, the forces being always equal and oppositely directed.

II. The force between two charges of electricity is inversely proportional to the square of their distance apart.

The simplest expression satisfying the first law is—

$$f \propto ee',$$

where f is the force and e, e' are defined as quantities of electricity. A negative value of f denotes attraction, and a positive value repulsion, provided the proper signs are attached to the numerical values of e.

The second law may be expressed—

$$f \propto \frac{1}{d^2},$$

and is the **law of inverse squares** applied to electricity. The two relations may be expressed in one, thus—

$$f \propto \frac{ee'}{d^2}.$$

We must now show that these laws are true experimentally. The first law has been already dealt with in Chap. VIII. The law of inverse squares cannot be proved *directly*, since it refers to *point* charges, but indirect proofs may be given. The best proof is that originally devised by Cavendish, and improved by Maxwell. The principle of the Cavendish experiment is as follows.

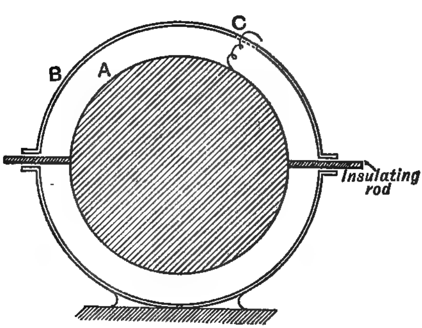


Fig. 68.—The Cavendish Experiment

Let A and B (fig. 68) be two concentric metal spheres. The inner sphere is supported on an insulating rod. The outer sphere or shell B is made in halves, and insulated on a frame

(not shown in the figure). A small hole is left in the shell at C, through which a loose wire connecting the two spheres may be withdrawn with the aid of a silk thread.

Suppose now that a charge is given to the outer sphere. We may show by calculation that if the law of inverse squares is true, the charge will produce no force within the shell, and therefore no force within the inner sphere. But if the law of distance is $f \propto 1/d^n$, where n is greater or less than 2, there will be a radial force within the shell. If n is less than 2, then the force on positive electricity will act outwards; if n is greater than 2, inwards. In the former case some of the positive electricity of A will be displaced outwards and along the connecting-wire to B, since the conductors cannot prevent a flow of electricity; this would leave A negatively charged. In the second case the positive electricity is displaced inwards, some flows along the connecting-wire to A, which therefore acquires a positive charge.

The test is made by first placing the wire so as to connect the spheres, and then giving B a charge. The wire is now withdrawn by means of the silk, and the outer hemispheres are removed. A is then tested with a sensitive electroscope.

It is found that A is quite neutral, so proving that $n = 2$. In the modification introduced by Maxwell to ensure great accuracy the outer sphere was not removed, but the inner one was placed in connection with a very sensitive electrometer by a wire passing through the hole. The special advantage of this is that the insulation of the inner sphere is entirely within the outer shell, the two spheres being separated by a ring of ebonite. (The effect of a leaky rod in the original form of apparatus will be understood after reading the section on hollow conductors.)

Maxwell's results showed that n could not differ from 2 by more than $\pm$ ·000046. We may also notice that the assumption that the law of distance is that of *some inverse power* is supported by the experimental fact that the distribution of charge is always the same on bodies which are geometrically similar.

Coulomb obtained an approximately direct proof of the law by an application of the torsion balance. The principle of this

instrument has already been explained in Chap. III. For measuring electric forces the form illustrated in fig. 69 may be used. The lower end of the wire is attached to a light insulating rod which carries a small ball at one end and a damping vane at the other.

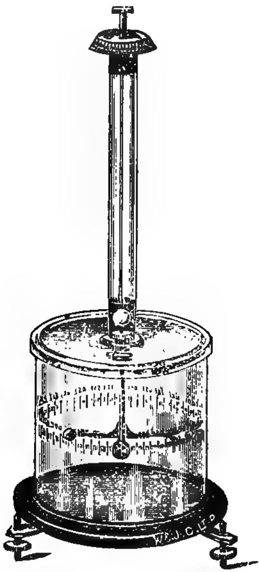


Fig. 69.—Coulomb's Torsion Balance

A second insulated ball can be introduced through the cover, so that it rests lightly in contact with the first. The torsion head is adjusted so that in this position there is no twist on the wire. When the balls are similarly charged they repel each other, and a torsion δ is produced. By means of the torsion head the balls can be made to approach each other. To reduce the deflection to (say) $\frac{\delta}{2}$ we require a movement of the head $= A°$.

The force of repulsion will (for small angles) be proportional to the angle of torsion. Thus—

$$\text{Ratio of forces} = \frac{\delta}{A + \frac{1}{2}\delta}.$$

Coulomb found that this ratio was very nearly 1:4, so confirming the law of inverse squares.

Owing to inductive effects on the case, leakage, and the considerable size of the balls in comparison with their distance apart, the results obtained are very rough approximations.

117. Unit Quantity of Electricity.

The C.G.S. electrostatic unit of charge is such that it repels an equal charge at a distance of 1 cm. in air with a force of 1 dyne.

From the law of force we have—

$$f = (\text{const.}) \times \frac{ee'}{d^2}.$$

The constant depends on the units adopted. Thus, if f is in dynes, d in cm., and e in electrostatic units—

$$f = \frac{ee'}{d^2} \quad \cdots\cdots\cdots\cdots (1)$$

This formula requires modification when the medium is other than air.

Example.—If two charges repel each other with a force of 8 dynes at a distance of 6 cm., find the magnitude of each charge, one being 3 times as great as the other.

$$\text{Here} \quad 8 = \frac{3e \times e}{36}. \quad \therefore e^2 = 96$$
$$e = 4\sqrt{6} \text{ e.s. units.}$$

No special name is given to the electro-static unit of electricity.

Since the centimetre-gram-second system of units is always adopted in electrostatic measure, we shall generally omit the letters c.g.s. in speaking of e.s. units. For practical units of charge, etc., see Chap. XV.

Remember that the above law applies only to point charges, *i.e.* charges existing on bodies of very small dimensions compared with their distance apart.

In ordinary cases the charges are spread over conductors of considerable area, and the equation (1) cannot be directly applied. The equation would not apply, for example, to the attraction exerted between two oppositely-charged parallel plates. In such cases as this, the force may be found by making use of the properties of tubes of force (Chap. XIV).

See also the remarks made in connection with magnetism (Arts. 18, 19).

118. Electric Field.

This term is applied to any region in which a body is subjected to forces due to electrification.

The field may be mapped out by lines of force. These indicate at each point the direction in which a very small positively-charged body would be urged by the influence of

other electric charges. As in the case of the magnetic field, it is usual to mark the lines with arrows. A small negatively-charged body would move against the arrows. The shape of the lines may be calculated for special cases, and their general shape may be ascertained by considering how a small positively charged body would move under the influence of the various charges on the electrified bodies.

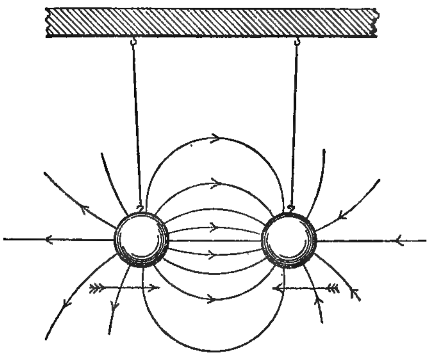


Fig. 70.—Attraction

Figs. 70, 71 show the form of the lines in the case of spheres having equal charges. The charges have opposite signs in fig. 70, and like signs in fig. 71.

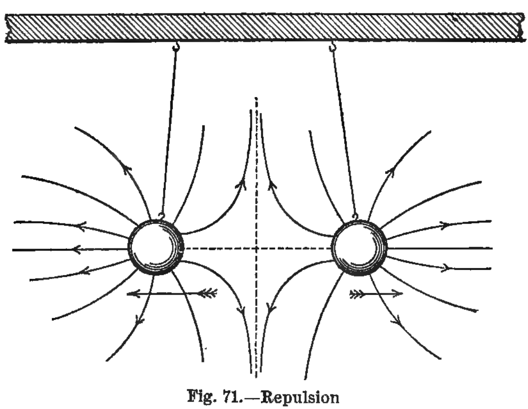


Fig. 71.—Repulsion

The student should now read Arts. 18 and 19 again, as the theory of "action in a medium" applies equally to electric action. A pith-ball suspended in a vacuum vessel can be

attracted by an electrified rod. When the leaves of an electroscope diverge, it is the medium (the ether) in contact with the leaves which causes them to move.

119. Electric Force.

The force exerted on a small charged body placed in the field depends on—

(i) **the intensity or strength of the field;**
(ii) **the amount of charge acted on;**

provided that the charge acted on is too small to disturb the distribution of the other charges.

Calling the point test charge e, and the field intensity F, we have—

$$f = \mathrm{F} \times e, \quad \text{(2)}$$

where f is the mechanical force exerted on the small body.

The intensity F is characterized by direction as well as magnitude. The term *electric force* is therefore frequently used. Thus—

(mechanical force) = (electric force) × (charge).

It may be shown from Eqn. (1) that the electric force as defined by (2) is a constant quantity. Hence, if we put $e = 1$, the corresponding value of f is a measure of this constant. Thus—

The measure of the electric force at any point of the field is the force which would be exerted on a unit charge placed at the point,

if this charge produced no change in the distribution of the other electrification.

If the charge is expressed in e.s. units and the mechanical force in dynes, the electric force is in e.s. units also. The practical unit of electric force will be mentioned in connection with potential.

The same kind of distinction must be made between electric force and mechanical force as was made in the case of magnetism.

EXAMPLE.—Find the electric force at a point where a force of 50 dynes is exerted on a particle of lead shot carrying a charge of ·04 e.s. units.

$$F = \frac{f}{e} = \frac{50}{\cdot 04}$$
$$= 1250 \text{ dynes per e.s. unit.}$$

120. Property of a Conductor.

(i) Insulate a cylinder made of fine wire-gauze. Place a pith-ball pendulum inside the cylinder, and cover the latter with a lid of conducting material or wire-gauze. Bring a charged rod near the outside of the cylinder. You will find that the pith-ball is not attracted.

(ii) Faraday had a room built in the form of a 12-ft. cube. The framework was made of wood and covered with copper wire, paper, and bands of tin-foil to make it a good conductor. The room was insulated and connected to an electrical machine. Faraday took electrometers, lighted candles, and other tests of electrical states into the room, but could not detect the slightest effect on them, although the outer surface of the cube was powerfully charged and gave off large sparks and brushes. (See *Experimental Researches*, 1173, 1174.)

The charge on the outer surface of a hollow conductor, therefore, produces no electric force within the conductor. It is evident that this must also be true if the conductor is filled up so that it becomes solid throughout. Hence we have the following important result:—

There is no resultant electric force within the material of conducting bodies when the electricity is static.

Note.—This law and the other laws of electrostatics deduced from it do not apply when the static charges are maintained by an e.m.f. generated in the conductor. This case is dealt with more fully in Chaps. XV and XXVII.

If an electric force is produced within a conductor, a flow of electricity is set up which continues until the new distribution produces no resultant force within the conductor.

In this respect a conductor acts towards electricity some-

what as a material fluid acts towards bodies immersed in it. A fluid like water cannot prevent the motion of solid bodies through it. A force, however feeble, will always cause a floating body to move, and in the same way electric force, *however weak*, will always produce motion of electricity in a conductor. A conductor, therefore, whatever its resistance, **cannot prevent** motion of electricity under the influence of electric force. This is an essential axiom of electrostatics.

121. Lines of Force Terminate on Conductors Normally.

It follows at once from the result of the preceding article that lines of electric force must suddenly terminate at the surface of a conductor. Further, the termination must be in a normal direction, that is, perpendicular to the surface of the conductor. This may be proved as follows:—

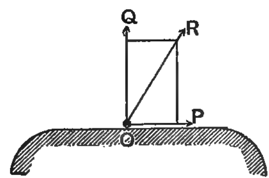


Fig. 72

If the termination is not normal, let OR be its direction (fig. 72). Resolve OR into components, OP parallel to the surface, and OQ at right angles to it. Now the force OP acting on a particle of electricity at O will produce movement, since the conductor cannot prevent the slipping of the electricity however weak the force may be. Hence motion of electricity will be set up, and will continue until the *resultant* force is everywhere normal to the surface.

Note.—It is necessary to warn the student here, that this statement refers only to resultant force. The ultimate components of the field radiating from every particle of electricity or from every point charge must be regarded as penetrating all substances indifferently in straight lines. This must be kept in mind in such reasoning as that applied in the Cavendish experiment.

The lines of force do not, as a rule, terminate at right angles to the surface of a wire through which a steady current is maintained.

122. Dielectrics.

Lines of electric force can pass through solid and liquid insulators as well as through air or a vacuum. If an electroscope is placed within a dry glass vessel, a charged body outside makes the leaves diverge widely. When we wish to refer to an insulator in this connection, it is usual to speak of it as a "dielectric", a term which we owe to Faraday. We

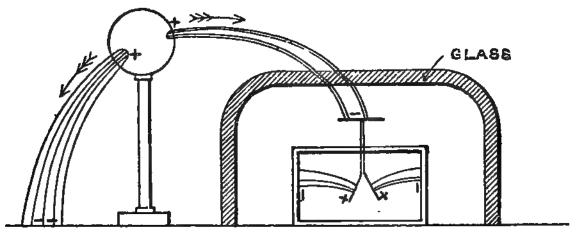


Fig. 73.—Property of Dielectric

use the term insulator when referring to the fact that the substance will not conduct *electricity*, and the term dielectric when we refer to the fact that the substance will transmit or conduct *lines of force.*

123. Properties of Resultant Lines of Force.

We may enumerate here the following properties. The *resultant* lines—

(1) show at each point the direction of force on a very small + charged body;

(2) end on the charged surfaces of conductors or insulators;

(3) never meet nor cross each other;

(4) can exist in solid and liquid dielectrics;

(5) terminate in a normal direction on conducting surfaces;

(6) cannot have the positive and negative ends on the same conductor.

With reference to (5) and (6) see also Art. 120, *Note.*

The third property follows at once from the fact that there can only be one direction for the *resultant* force at any point.

124. Fields of Point Charges—Neutral Points.

The fields of unequal point charges may be traced generally as explained in Art. 118. It is well to first determine the neutral points, or points of no resultant field.

EXAMPLE 1.—Two point charges are placed 30 cm. apart. If the charges are +12 and +3 units respectively, find the position of the neutral point.

Let A and B (fig. 74) be the charges and N the neutral point.

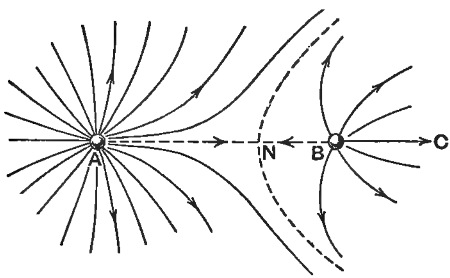


Fig. 74.—Neutral Point

Let AN $= x$; then BN $= 30 - x$. If a unit + charge is placed at N the repulsion of A is equal to the repulsion of B.

$$\therefore \frac{12 \times 1}{x^2} = \frac{3 \times 1}{(30 - x)^2},$$

$$\frac{2}{x} = \frac{1}{30 - x},$$

$$x = 20.$$

The general form of the lines is shown in the figure. The lines do not cross at the neutral point (compare the fifth property mentioned above), since the force vanishes when the lines are just on the point of meeting.

EXAMPLE 2.—If the charges are +36 and −3 respectively, find the neutral point.

When the charges are unlike, the neutral point lies on AB

produced and on the side of the weaker charge. From the data given, if a unit positive charge is placed at N (fig. 75) and $BN = x$,

Repulsion due to A $= \frac{36}{(30 + x)^2}$; attraction due to B $= \frac{-3}{x^2}$.

Thus numerically—

$$\frac{36}{(30 + x)^2} = \frac{3}{x^2};$$

$$\frac{\sqrt{12}}{30 + x} = \frac{1}{x}. \quad \therefore x = \frac{30}{2\sqrt{3} - 1} = 12{\cdot}2 \text{ nearly.}$$

The lines of force proceeding from the stronger charge may be divided into two groups—those which reach the weaker

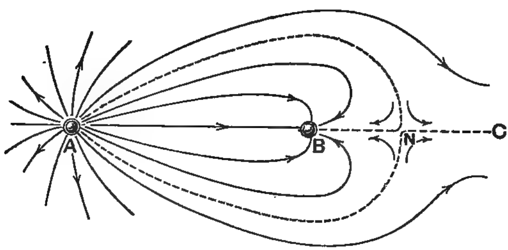


Fig. 75.—Neutral Point

charge and those which extend to infinity. The dotted line shown in the diagram divides these groups.

125. Surface Density.

According to the definition of surface density (Art. 109), this quantity must be expressed in units of charge per unit area. The electrostatic unit is therefore 1 e.s. unit of charge per sq. cm.

The average surface density over an irregular conductor is the total charge divided by the total area, or—

$$\sigma = \frac{Q}{A}.$$

If the charge on a sphere is uniformly distributed, then the

density at any point can be found from this equation. The area of the sphere is $4\pi r^2$ where r is the radius. Thus—

$$\sigma = \frac{Q}{4\pi r^2}.$$

126. Electric Force Near a Conducting Surface.

When the charge on a sphere is *uniformly* distributed, it acts with respect to all points *outside* the sphere as if it were collected at the centre.

We may therefore make all calculations in this case by the rules for *point* charges. The force at a distance d from the centre of the sphere is—

$$F = \frac{Q}{d^2}.$$

If the point is *just outside* the sphere, $d = r$ practically, and therefore—

$$F = \frac{Q}{r^2},$$
$$= 4\pi\sigma.$$

The force *just inside* is zero. Thus in crossing the surface there is a sudden change of electric force equal to 4π times the surface density of the charge at the point of crossing. Since exactly the same conditions would hold in crossing the surface of any charged conductor, the result is true for a conductor of any shape, but it may be formally proved as follows:—

Let AB be a small portion of the surface of a conductor. The electric force just inside or just outside the conductor at AB may be regarded as made up of two components—

(*a*) The force due to the charge (q) on AB;
(*b*) The force due to all the other charges in the field.

The first component is normal to the surface, and acts in opposite directions on the two sides with a strength equal to $2\pi\sigma$. (See Appendix.)

But the *resultant* force just inside the conductor is zero. Hence, inside the conductor, the second component must just

neutralize the first. Thus, calling the second component F_2 and the first F_1, we have—

$$F_1 - F_2 = 0.$$

$$\text{But} \quad F_1 = 2\pi\sigma,$$

$$\therefore \quad F_2 = 2\pi\sigma.$$

Now the force due to the distant charges does not alter appreciably either in magnitude or direction in crossing the surface. Hence just outside the conductor at AB, F_1 and F_2 help each other, and—

$$\begin{aligned} F &= F_1 + F_2, \\ &= 2\pi\sigma + 2\pi\sigma, \\ &= 4\pi\sigma. \end{aligned}$$

127. Examples.

1. A small pith-ball weighing one decigram, suspended by a silk fibre and charged with positive electricity, is repelled when a charged glass rod is brought near it. If the direction of the electric field of the glass rod near the ball is horizontal, and its magnitude equal to 20 C.G.S. electrostatic units when the deflection of the fibre is 45°, what is the charge on the ball? (1906.)

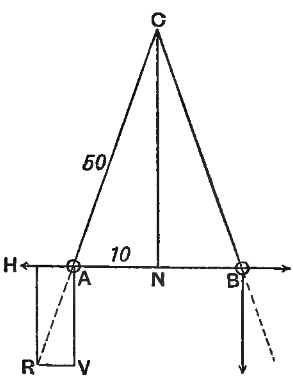


Fig. 76

Make a diagram of the arrangement and mark the forces. These are the weight w of the pith-ball, the repulsion f, and the tension on the fibre. The parallelogram of forces is a square, hence

$$\begin{aligned} w &= f \\ &= \text{electric force} \times \text{charge} \\ &= 20 \times e. \end{aligned}$$

But $w = \cdot 1$ gram's wt. $= \cdot 1 \times 981$ dynes. Thus—

$$\begin{aligned} 98 \cdot 1 &= 20 \times e, \\ e &= 4 \cdot 905 \text{ C.G.S. e.s. units.} \end{aligned}$$

2. Two small spherical pith-balls, each one decigram in weight, are suspended from a point by threads 50 cm. long, and are equally

charged so as to repel each other to a distance of 20 cm. Find the charge on each in electrostatic units ($g = 980$). (1901.)

Make a sketch of the arrangement, and draw the parallelogram of forces for one ball. The two threads and the line joining the balls form an isosceles triangle (fig. 76). We have by similar triangles—

$$\frac{AH}{AV} = \frac{RV}{AV} = \frac{AN}{NC}.$$

$AN = 10$, $NC = \sqrt{50^2 - 10^2} = 10\sqrt{24}$.

AH represents the repulsion, or $\frac{e^2}{20^2}$.

AV represents the weight of one ball, or $\cdot 1 \times 980$.

Thus by substitution—

$$\frac{e^2}{20^2 \times 98} = \frac{10}{10\sqrt{24}},$$

$$e^2 = \frac{20^2 \times 98}{\sqrt{24}} = 8000,$$

$$e = 89\cdot 4 \text{ e.s. C.G.S. units.}$$

3. The electric force close to the surface of a charged metal disc is 30 C.G.S. units. Find the surface density of the charge.

We have $F = 4\pi\sigma$.

$$\therefore \sigma = \frac{30}{12\cdot 57} = 2\cdot 38 \text{ units per sq. cm.}$$

EXERCISES

1. Describe a method of proving that the force between two small electrified bodies varies inversely as the square of the distance between them. (1895.)

2. Two small insulated conductors, unequally and oppositely electrified, are near each other. Show by a diagram the general course of the lines of force in the field, and give general reasons for the shape which you give to any one of the curved lines you draw. (1897.)

3. Define unit charge of electricity. Two charged conducting spheres repel each other with a force equal to the weight of a milligram when placed at a certain distance from each other. If the charge on one of the spheres is doubled and the distance between the spheres also doubled, what is the amount of repulsion? (1903.)

4. Draw carefully the lines of force due to charges 1 and -2 placed at a distance of 1 cm. apart. (1903.)

5. Give a careful freehand drawing of the lines of force due to a charge of 4 units of positive electricity at A and one of 1 unit of negative at B, if the distance between A and B is 2·5 cm. (1904.)

6. Give a careful drawing of lines of force due to a positive charge of 9 units at A and a negative charge of 1 unit at B. $AB = 1$ inch. (1905.)

7. Two equally charged spheres repel each other when their centres are half a metre apart with a force equal to the weight of 6 milligrams. What is the charge on each in electrostatic units? (1892.)

8. Find the strength of the electric field at a point just outside an isolated sphere of 10 cm. radius charged with 10 electrostatic units. (1903.)

CHAPTER X

INDUCED ELECTRIFICATION

128. The quantitative relation between the *induced* and *inducing* charges was first systematically investigated by Faraday in his series of "ice-pail" experiments.

Faraday's "Ice-pail" Experiment.

Let a deep metal can be insulated and joined to an electroscope (fig. 77). Lower a charged metal ball into the vessel, and when it is once *well* inside note carefully the divergence of the leaves. Now let the ball touch the inner surface of the can, and at the same time carefully watch the leaves. It will be found that they are undisturbed and remain steadily divergent when the ball is removed. The latter, if tested, will now be found quite neutral.

The experiment therefore proves that the charge on the ball exactly neutralizes the charge which it induces on the inner surface of the vessel, leaving the outer induced charge undisturbed.

When a charged body is introduced into a hollow conductor, the charges induced on the

inner and outer surfaces of the conductor are each equal to the inducing charge.

The experiment may be varied, and the result confirmed in various ways. For example, lower the ball into the can as before, then earth-connect the latter for a moment to remove the free charge on the outer surface, and lastly remove the ball. The inner induced charge then flows to the outside, and

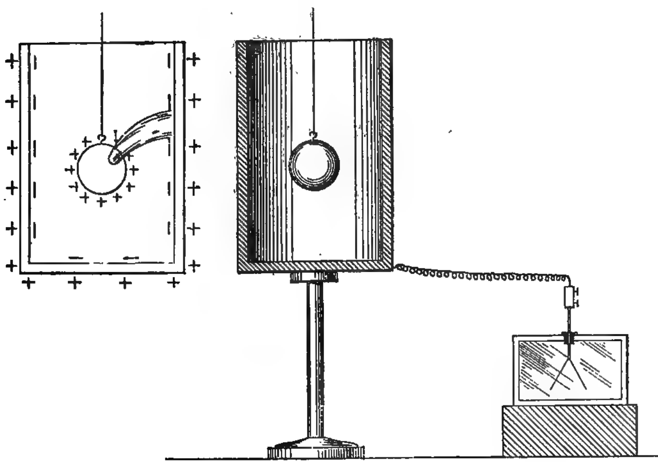


Fig. 77.—Faraday's Ice-pail Experiment

a divergence of the leaves is produced equal to the previous one, but with opposite electrification. If the ball (which has retained its original charge) is now allowed to touch the vessel, the whole arrangement becomes neutral.

129. **Tubes of Electric Induction.**

In the above arrangement, imagine a small area marked out on the surface of the ball, and let lines of force be drawn from all parts of the boundary of this area, and continued until they meet the inner surface of the vessel. In this way we trace out what is termed a *tube of electric induction.* There is a positive charge at one end of the tube, and a negative

charge at the other. Now we may imagine the whole space between the ball and the can to be occupied by similar tubes of induction. The total amount of positive charge at the positive ends of the tubes is, by the result of the ice-pail experiment, equal to the total amount of negative charge at the other ends. But since the electric action must be regarded as of the same *nature* in all the tubes, it follows that equal amounts of the charges must exist at the ends of any one tube.

This result is independent of the size of the vessel, and remains true if the tubes are "cut through" by a metal cylinder placed within the vessel so as to surround the charged body.

A tube of electric induction is a tube so drawn in the electric field that if it is intercepted by a conductor the charge induced on the latter will be the same at whatever part of the tube the conductor is placed.

Tubes of induction are also called *tubes of polarization.*

The term "tube of induction" is used in preference to the term "tube of force", because, as we shall see, in a tube even of uniform cross-section, passing through different dielectrics, the force may vary considerably but the induction is constant. The *direction* of the force (except in rare cases) coincides with the direction of the induction, so that the lines of force may be taken as a guide in drawing the induction tubes. It is possible also to draw tubes of force, but these will suddenly change in number at the surface of a dielectric.

A unit tube of induction may be defined as one having unit positive and negative charges respectively at its ends.

If the whole electric field is divided into *unit* tubes of induction, it is evident that the tubes will be very narrow where they end on dense charges. We may, therefore, conveniently represent the density of the charge by dividing the field into unit tubes.

We also make the following definition:—

The density of the electric induction at any point of the field is the number of unit tubes

of induction which pass at right angles through a unit area placed at the point.

Since the actions taking place in the tubes are regarded as of the same *nature* at all points, we have by the result obtained in Art. 126—

The electric force in air at any point $= 4\pi\sigma$, where σ is the density of the induction tubes.

130. Corresponding Charges.

The charges on the various objects in the electric field may be connected by sets of induction tubes. The equal and opposite charges at the ends of any *set* of tubes may be termed *corresponding charges.*

For example, if a charged sphere is insulated above the table, the tubes of induction proceeding from it to surrounding objects find their way mostly to the table. The greater portion of the charge on the sphere has its corresponding charge on the table, the remainder having its corresponding charge on the walls, floor, ceiling, etc.

Earth-connection.—We can now better understand the effect of earthing a conductor. If the sphere just mentioned is joined to earth, the corresponding charges neutralize each other through the connecting-wire. If any example of earth-connection is carefully studied it will be found that the disappearance of the free charge is always accompanied by its neutralization. Hence—

The free charge is that portion of the electrification which has a virtually corresponding electrification on earth-connected conductors.

We say "virtually" corresponding, because the tubes do not always proceed direct to the earth-connected bodies. A conductor placed in the path of the tubes will cut them in two, and therefore the immediately corresponding charge would exist on such a conductor. But the tubes ending on one side of the conductor virtually recommence on the other side, and

the existence of this conductor has no influence on the result of the earth-connection.

131. **Properties of Induction Tubes.**

The main properties are the following:—

> (1) **The direction of the induction tube at any point coincides with the direction of the line of force at that point** (with some unimportant exceptions).

See, therefore, Properties of Lines of Force, Art. 123.

> (2) **Equal charges of unlike sign exist at the respective ends of each tube.**
>
> (3) **The tubes tend to contract lengthwise and expand sideways.**
>
> (4) **The tubes cross the surface of an uncharged dielectric without change of induction value.**

The third property enables us to understand how attraction and repulsion are produced by actions taking place in the medium. Referring to figs. 70, 71, we see that if the spheres have unlike charges they are pulled together by the tension in the tubes, whereas if they are similarly charged they are pulled apart, so producing the appearance of repulsion. The leaves of a gold-leaf electroscope are pulled apart by the tension of the tubes ending on their outer surfaces.

There is no antagonism between such explanations as this and the simpler one usually adopted, where we suppose the charges on the leaves to repel each other directly. In the latter explanation we consider merely the *component* forces arising from a distribution of point charges over the leaves: in the method described above we consider the *resultant* condition of the medium in contact with the two surfaces of the leaves. For general purposes the method of "components" is more convenient.

If one portion of a conducting surface is charged, there must be *some* charge on all portions of the same surface, though in places this may be of too small a density to admit of experimental detec-

tion. The density is only zero along the neutral lines separating positive and negative induced charges.

132. Increase of Induced Charge during Earthing.

Consider the typical example of induction shown in fig. 78. The induced charges are shown along with the general drift of the induction tubes. If we earth the conductor the positive charge at N, being free, flows to earth. The tubes between N and the table—the group marked C—therefore disappear.

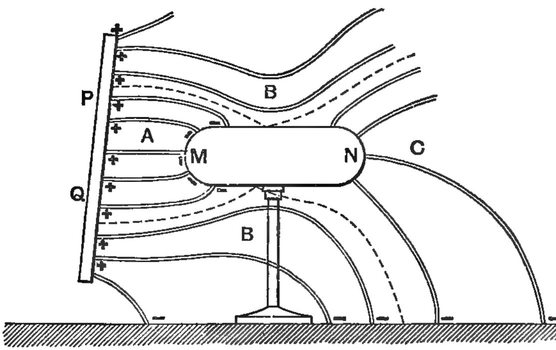


Fig. 78.—Induction

But the action does not stop at this point. When the tubes C have disappeared, the group B is left without anything to balance its tendency to expand (fig. 79). The B tubes are therefore driven inwards to the conductor and break on its surface, each forming two separate tubes. The left-hand portion becomes an addition to the A set, and the right-hand one forms a new C tube which, following the example of its predecessors, vanishes. The process continues until the conductor is covered with tubes of the A set completely (fig. 80).

The process may also be described thus. When the free charge escapes from N, this portion becomes neutral. The charged rod therefore induces fresh charges on this neutral portion. The new free charge then escapes, leaving another portion neutral, and more induction takes place. And so on,

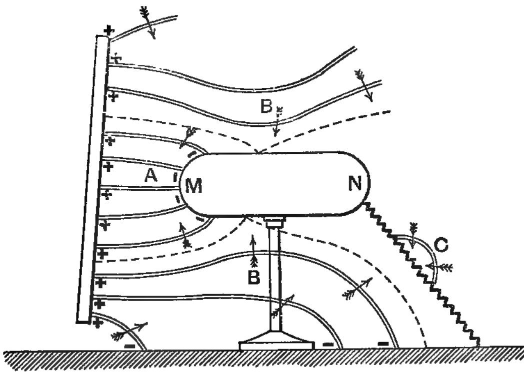


Fig. 79.—Earthing Process—Intermediate

until the whole of the conductor is covered with the bound charge (fig. 80).

The process, of course, does not take place in steps, but rapidly and continuously during the moment of earthing.

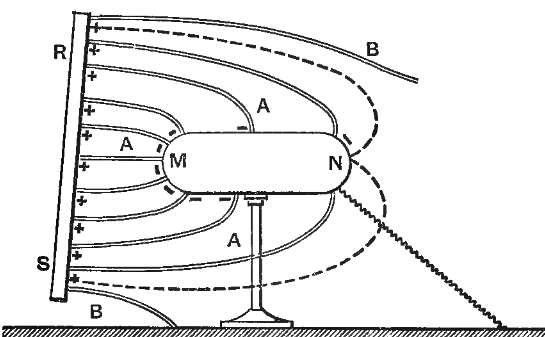


Fig. 80.—Earthing Process—Final State

This process of further induction takes place at the moment of earthing in *all* cases where we charge a body by induction and then earth it, but the *extent* to which it occurs varies greatly according to circumstances. The following cases may be considered as the extremes:—

(1) Conditions as in the ice-pail experiment. Here there is practically no further induction during earthing. The repelled induced charge escapes to earth simply, leaving the outside of the vessel nearly neutral.

(2) Conditions of ice-pail experiment reversed, *i.e.* inducing charge on the hollow conductor and the insulated conductor lowered inside. There are practically no initial induced charges on the latter. But when the earth-connection is made by means of a wire introduced through the opening in the vessel, the further induction causes a relatively large induced charge (see Chap. XII).

Other typical examples of increased induced charges are given in Arts. 133, 134, and Chap. XII.

The extra induction causes a concentration of the *inducing* charge on the side nearest the body under induction. In the limiting case (2) above, the charge is attracted to the inner side of the vessel.

133. **Corresponding Induced and Inducing Charges.**

When a conductor surrounds a body *on all sides*, as in the ice-pail experiment, the induced charge corresponds to the whole of the inducing charge. But if this condition is not fulfilled the induced charge corresponds to a portion only of the inducing charge, and is therefore less than the latter in total amount.

(1) In the example, fig. 78, the tubes from the rod pass mostly to the table. The electrification on PQ is equal to that on the corresponding surface at M.

(2) After the earth-connection the induced charge is greater and equal to that on the surface RS, but is evidently still less than the total inducing charge (fig. 80).

(3) If the inducing body is a metal plate A placed near and parallel to a thin plate B, the induced charge on the latter is equal to about one-half of A's charge. But if B is earth-connected its free charge disappears, and *further induction* goes on until the induced charge is nearly equal to A's. The charges collect almost entirely on the nearer faces of the plates.

In the ice-pail experiment, as usually made, the induced and inducing charges are not quite equal, since some of the induc-

tion tubes escape through the open top. The can should therefore be a fairly deep one.

134. Electrophorus.

This is a convenient arrangement for obtaining a succession of charges from a single initial frictional charge by induction.

Fig. 81.—Electrophorus

The usual form is shown in figs. 81, 82. G is a generating plate of polished ebonite; S a brass plate called the "sole", which forms a good conducting surface in contact with the lower side of the ebonite. C is a collecting plate of metal with an insulating handle of ebonite.

Mode of Use and Theory.—(i) C is removed, and the upper surface of the ebonite is rubbed with fur, so producing a negative charge on it. The induction tubes pass both upwards and downwards from this charged surface. Most of the tubes, say 90 per cent, extend through the ebonite to the sole, on which a positive charge is induced. The remaining 10 per cent pass to surrounding objects.

(ii) C is now replaced, and since it only intercepts the upper tubes, *small* charges, each roughly 10 per cent of the negative inducing charge, are developed on its upper and lower surfaces.

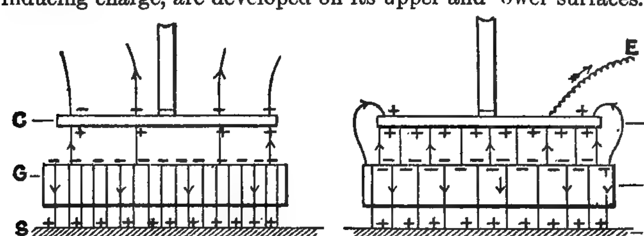


Fig. 82.—Electrophorus—Theory

(iii) C is next earth-connected. The free charge escapes, *further induction takes place,* until C is charged all over with positive electricity. The negative charge on G now lies between two earth-

connected conductors S and C. Its induction is divided between them, C taking the larger share because it is the nearer. The induced charge on C may now be 60 or 70 per cent of the inducing charge on the ebonite, and collects almost entirely on the under surface of C.

(iv) If the collecting plate is now disconnected from earth and removed, it carries away its strong positive charge, and a spark may be easily drawn from it. The generating plate and sole return to their initial state (i).

It is evident that the process may be repeated many times, until, in fact, the charge on G has leaked away.

The existence of the further induction in step (iii) may be easily demonstrated as follows:—Lay a small pith-ball on the collecting plate and proceed to charge the latter. Unless the pith-ball is very small, it will be undisturbed at stage (ii) of the experiment, also when the plate is touched. But when the collecting-plate is lifted, the pith-ball jumps right off.

Notice carefully that the collecting plate acquires no appreciable charge from the ebonite. This is because it does not come in close contact with the negative charge in the surface of the ebonite.

135. The Replenisher.

A convenient form of apparatus for increasing the charge on a conductor by induction is the *replenisher* devised by Lord Kelvin. The instrument is shown in figs. 83, 83A.

A and B are two curved pieces of metal termed the *inductors*: one of these, A, is joined to the charged conductor; the other, B, is joined to earth. C and D are two narrow brass strips mounted at the ends of a rod of ebonite. The latter is attached to a vertical spindle, provided at the top with a milled head. In this way the *carriers* C and D may be rapidly rotated. Two spring tongues

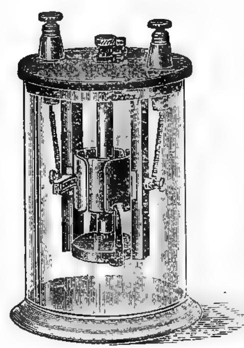


Fig. 83.—Replenisher

project through slits in A and B, and come into contact with the carriers as the latter revolve. The tongues are attached to the inductors. The carriers when near the edges of the inductors are momentarily placed in connection by means of two contact springs XY joined by a wire.

In this last position, if A is positively charged, a negative charge is induced at X and C, and the repelled positive charge flows to Y and D. On further rotation in the clockwise direction C becomes earthed by contact with the spring tongue of B.

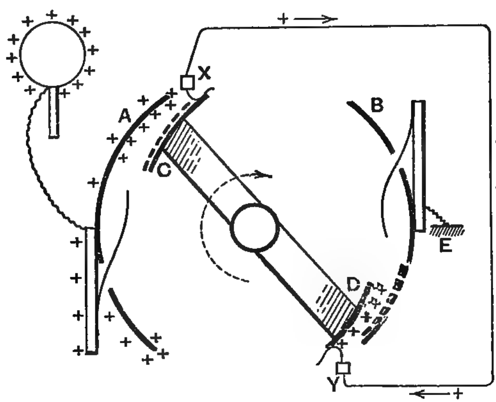


Fig. 83A.—Replenisher Theory

At the same time D comes in contact with the spring tongue of A. Since A is curved, it *acts partially as a hollow conductor.* D, therefore, gives up most of its positive charge. (Compare Art. 107.) The extra charge thus acquired by A is shared with the conductor connected to A. The process is repeated at every revolution, and the charge of the conductor slowly increases.

The instrument is mostly used to replenish from time to time the charge which disappears from a conductor through leakage.

136. Effect of Induction on a Charged Conductor.

If the conductor subjected to inductive influence is already charged, we may trace the effects produced as in the following examples.

(i) Suppose an insulated metal ball charged positively, and let this be gradually brought near a rod similarly charged. The charge is repelled more and more to the farther side of the ball until the density at the point nearest the rod becomes zero, *i.e.* if the rod is charged strongly. A still nearer approach causes the development of negative and positive charges on the nearer side of the ball, the negative being bound.

If the ball is connected to earth at any stage of the experiment, it is simplest to suppose that the earthing process takes place in two steps. First, the repelled free charge escapes entirely to earth. Secondly, the surface of the conductor thus left neutral becomes charged by the ordinary process of induction and earthing. (See Art. 132.)

(ii) The case where the charge already possessed is of opposite nature to the inducing charge may be dealt with similarly. For example, let a g.l.e. be charged negatively. Its charge is all free and partly on the leaves, partly on the rod and disc. Bring up a positively-charged rod. The charge is gradually attracted from the leaves to the cap as the rod approaches. When all the charge is bound, the leaves are practically neutral and collapsed. A still nearer approach of the rod causes further induction, the new negative being bound on the cap, and the new positive being repelled to the leaves, which diverge again.

Notice that the leaves (*a*) decrease in divergence, (*b*) collapse, (*c*) rediverge with the opposite charge.

If the electroscope is earthed at any stage of the process the free charge is first earthed, and the neutral portion so left is subjected to the ordinary process of induction and earthing. The leaves, of course, collapse.

In both (i) and (ii) the extra charge acquired by the earthing process tends to collect on that portion of the conductor nearest the inducing body. In the electroscope experiment it collects on the cap.

137. **Charging by Contact.**

Take a conductor A, *well* insulated, and proceed to charge it with an electrophorus. At the first approach of the collecting plate C a spark passes, perhaps 1 cm. long. At the second

approach of C (after recharging) there is a second spark, but shorter than the first, say $\frac{1}{2}$ cm. On the third approach the spark is still smaller, and so on until after a few repetitions no further charge can be transferred between C and A. But a good spark may be obtained by presenting the knuckle to A or C. There is therefore a limit to the charge which may be transferred from a given source to a conductor. This may be explained as follows:—

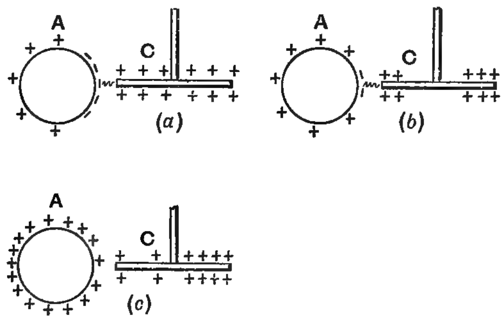


Fig. 84.—Charging by Contact

Suppose C positive. The first spark occurs between the induced negative on the near side of A and an equal amount of C's positive charge. On the second approach, the charge on C must repel A's charge to the farther side before it can induce a negative charge. The negative charge induced is therefore smaller, and the spark occurs at less distance. At the third approach there is more charge on A to be repelled before negative can be induced. Hence a still smaller negative charge and shorter spark are produced. This goes on until C *just fails to induce a negative charge* by the time it comes into contact with A. Remember that a spark always occurs between *unlike* charges.

CHAPTER XI

POTENTIAL

138. *Electric Potential* is a subject which, although quite as important as those of current, charge, force, etc., is not so easily understood when it is approached for the first time.

This is mainly on account of its purely mathematical nature and definition. A knowledge of its meaning can be acquired most directly by considering the work done on a small charge when this is moved through the electric field. We shall therefore first consider some examples of the principle of work.

139. Work Done by a Force.

A force is said to do work when the body on which it acts moves along the line in which the force or an effective component is applied.

The measure of the work done is defined as the product—

(effective force) × (distance).

Thus we may write—

$$W = f \times s \quad (1)$$

The unit of work, therefore, depends on the units of force and length. If f is in lbs. and s in feet, the work done is in ft.-lbs. If f is in grams and s in cm., the work is expressed in gram-centimetres. Again, if we adopt C.G.S. units, f is in dynes, s in centimetres, and the work done is expressed in cm.-dynes. This last unit has received a special name, the *erg*.

The erg is a very small unit of work. It may be imagined roughly as the work done in lifting a small pin's head through a height of half an inch. Owing to the smallness of the erg a larger practical unit is adopted, the *joule*.

The erg is the C.G.S. unit of work, and is the work done by a force of one dyne acting through a distance of one centimetre.

The joule is the practical unit of work, and is equal to 10 million ergs.

[The practical unit used in mechanical engineering is the foot-lb.]

EXAMPLES.—1. An engine moves a mass of 100 tons for half an hour at a uniform speed of 30 miles an hour. The force exerted

by the engine is $\frac{1}{150}$ of the weight of the moving mass. Find the work done.

$$\begin{aligned} \text{Force} &= \tfrac{1}{150} \times 100 \times 2240 \text{ lbs. weight.} \\ \text{Distance} &= 30 \times \tfrac{1}{2} \times 5280 \text{ feet.} \\ \text{Work} &= 118{,}272{,}000 \text{ foot-lbs.} \end{aligned}$$

2. A weight of 5 kilograms is lifted to a height of 2 metres. Find the work done.

$$W = 5 \times 2 = 10 \text{ kilogram-metres.}$$

If we wish to express this in ergs or in joules, we have—

$$\begin{aligned} 1 \text{ kilogram} &= 1000 \text{ grams, } 1 \text{ gram wt.} = 980 \text{ dynes.} \\ \text{Force} &= 5 \times 1000 \times 980 \text{ dynes.} \\ \text{Distance} &= 2 \times 100 \text{ centimetres.} \\ \text{Work} &= 980{,}000{,}000 \text{ ergs} \\ &= 98 \text{ joules.} \end{aligned}$$

In equation (1), if the force is applied in the direction of motion, as in the case of a locomotive hauling a train along a straight line, f is simply the force applied. But in many cases the force under consideration does not act in the direction of motion. This is due to the influence of other forces applied to the body at the same time. In such instances we must consider the *effective* part of the force. Resolve the force into components parallel to the direction of motion and perpendicular to it respectively; the former is the effective part of the force.

EXAMPLE.—A barge moves along a canal under the combined influence of a pull exerted by a horse on the bank and the reaction on the rudder. If the rope makes 30° with the direction of the canal, and the pull on the rope is 30 kilograms wt., find the work done when the barge moves 300 metres.

Resolving the force parallel and perpendicular to the motion, we have—

$$\begin{aligned} \text{Effective force} &= 30 \times \cos 30^\circ = \frac{30\sqrt{3}}{2} \text{ kilograms wt.} \\ \text{Distance} &= 300 \text{ metres.} \\ \text{Work} &= 15 \times \sqrt{3} \times 300 \text{ kilogram-metres} \\ &= 7794 \text{ kilogram-metres} \\ &= 76381 \text{ joules.} \end{aligned}$$

140. Positive and Negative Work.

When the force considered assists the motion, it is said to do positive work. But in some cases the force we are dealing with may oppose the motion produced by other forces. It is then said to do negative work. For example, if we lift a weight from the ground, the force we exert does positive work, but the force of gravity which opposes the motion at the same time does negative work.

A force does no work unless the body acted on moves. If you press your hand on the table, you exert force but do no work.

Again—

A force does no work when it acts at right angles to the direction of motion.

This is evident, for in this case the force considered neither helps nor hinders the motion produced by other agencies.

141. Work Done by Electric Forces.

Imagine any small body, say a pith-ball or particle of lead-shot, insulated and electrified with a small positive charge. If we introduce this into the electric field, it is urged in the direction of the lines of force. Hence if we allow the pith-ball to move in the positive direction of the lines (*i.e.* with the arrows), the electric force will *help* the motion and do positive work. But if we move the pith-ball in the reverse direction, the electric force will *hinder* the motion and do negative work. Again, if we move the ball at right angles to the lines of force, no work is done by the electric forces, because in this case they neither assist the motion nor oppose it. If the motion takes place obliquely across the lines, work is done, and is positive or negative according as the component of the electric force along the line of motion helps or opposes the movement.

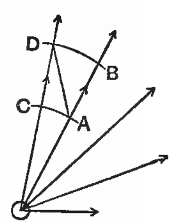


Fig. 85

Consider the field represented in fig. 85. If a small + charge

is moved from A to B or A to D, the electric field helps the motion and does positive work. It does negative work if the motion is from B to A or from D to A; but it does no work if the motion is from A to C or from C to A.

142. Calculation of Work for Point Charges.

We must now consider the following problem:—

"To find the work done by the electric forces when a small charge e' moves from a point A near a charge e to another point B at greater distance".

Let the charge e be situated at O (fig. 86), and let the charge e', which we shall call the *test charge*, be placed at A. Since

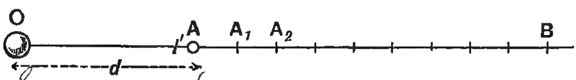


Fig. 86

the force exerted on e' falls off rapidly as it is moved farther from e, it is necessary to divide the whole distance between A and B into small sections; over each of these the force may be considered to decrease regularly. Let AA_1, A_1A_2, etc., be successive short lengths each $= x$ cm. Let $OA = d$, $OB = d_1$.

$$\text{Force exerted on } e' \text{ at A} = \frac{ee'}{d^2},$$

$$\text{,,} \qquad \text{,,} \quad e' \text{ at } A_1 = \frac{ee'}{(d+x)^2}.$$

Hence the average force between A and A_1

$$= \frac{ee'}{x}\left(\frac{1}{d} - \frac{1}{d+x}\right). \quad \text{(See Appendix.)}$$

Hence the work done between A and A_1

$$= ee'\left(\frac{1}{d} - \frac{1}{d+x}\right).$$

Similarly, the work done between A_1 and A_2

$$= ee'\left(\frac{1}{d+x} - \frac{1}{d+2x}\right);$$

and so on for the other sections.

Thus the total work done on e' in moving it from A to B

$$= ee'\left\{\left(\frac{1}{d} - \frac{1}{d+x}\right) + \left(\frac{1}{d+x} - \frac{1}{d+2x}\right) + \ldots\ldots\ldots + \left(\frac{1}{d_1 - x} - \frac{1}{d_1}\right)\right\};$$

or $$W = ee'\left\{\frac{1}{d} - \frac{1}{d_1}\right\}.$$

We have supposed the test charge moved along a radial line from O.

But if the test charge moves along **any curved path beginning at A and ending at B**, the amount of work will be the same.

This is an important point in the present calculation, and may be proved as follows:—

Let ANMB be the curved path (fig. 87). With O as centre, and OA_1, OA_2, etc., as radii, draw circles to cut the curve. The

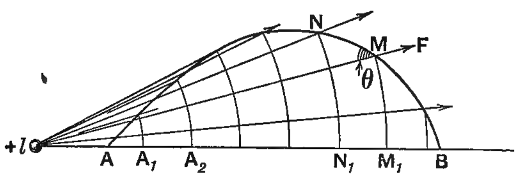


Fig. 87.—Indifference of Path

latter is thus divided into a number of short lengths. Join the points where the circles meet the curve to the point O by radial lines. Let NM be one of the sections, l its length, θ its inclination to the radial line through M, F the average *radial* force between M and N.

Then—

Effective force along NM $= F \cos \theta$.
Work done between N and M $= F \cos \theta \times l$,
$= Fx$.

But since the force is the same at equal distances from the point charge at O, F is also the force at N_1. Therefore—

Work done from N to M = work done from N_1 to M_1.

Similar reasoning applies to all the short lengths into which the curve is divided. Hence (taking account of the fact that the work along some parts of a path may be negative) we have—

Work done between A and B along the curved path = work done between A and B along the radial path

$$= ee'\left\{\frac{1}{d} - \frac{1}{d_1}\right\} \qquad \text{(2)}$$

Hence for given charges the work done between A and B **depends only on the positions of A and B** with respect to the charge e.

If B is supposed removed to an infinite distance, $1/d_1 = 0$. Thus, if e' starts at a distance d from e, and moves off to infinity, the work done by the electric forces is—

$$W = \frac{ee'}{d} \qquad \text{(3)}$$

143. Potential.

Let us now suppose that the test charge is a *unit positive* charge. The work done in moving this to infinity is by (3) equal to e/d. This is a quantity which depends, for a given charge, only on the *position* of the starting-point, and is called the potential of the point.

> **The potential at any point is the work which would be done by the electric forces if a unit positive charge were moved from that point to infinity.**

Potential is denoted by V. Thus for a point charge, putting $e' = 1$ in (3), we have—

$$V = \frac{e}{d} \qquad \text{(4)}$$

Note.—The potential may also be defined as the work done *against* the electric forces if a unit charge is brought up *from* infinity to the point considered.

EXAMPLE.—The potential at a point 16 cm. from a point charge of 40 positive units is—

$$V = \frac{40}{16} = 2{\cdot}5 \text{ ergs per unit charge.}$$

Notice the unit in which the potential is expressed. Since potential is the work done in moving a unit charge, it is expressed in *ergs per unit charge.* The work done in moving any charge e' to infinity is from (3) and (4).

$$W = Ve' \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Since the work done is independent of the path chosen, it follows that—

The difference of the potentials of two points is the amount of work done by the electric forces when a unit positive charge moves from one point to the other.

The work will be positive or negative according to the direction of transference.

The expression "difference of potentials" is usually abbreviated into "potential-difference or P.D.".

EXAMPLE.—To find the work done by electric forces when a charge of -20 units is removed from A, at a distance of 12 cm., to B, at a distance of 18 cm. from a charge of $+72$ units.

$$\text{Potential at A} = V_A = \frac{72}{12} = 6.$$

$$\text{Potential at B} = V_B = \frac{72}{18} = 4.$$

$$\text{Potential difference} = V_A - V_B = +2.$$

$$\text{Work done in removing } -20 \text{ units} = 2 \times -20 = -40 \text{ ergs.}$$

This work is negative because the electric attraction resists the movement.

144. Potential Due to a System of Charged Bodies.

We have so far considered only the potential due to a single point charge.

If there are a number of small electrified bodies, the potential produced by the whole system is found by adding the potentials due to the separate charges, algebraically.

If the charges e_1, e_2, e_3, etc., are situated at distances d_1, d_2, d_3, etc., respectively from the point considered, then—

$$V = \frac{e_1}{d_2} + \frac{e_2}{d_2} + \frac{e_3}{d_3} + \ldots\ldots\ldots + \frac{e_n}{d_n} \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

EXAMPLES.—1. Charges of 20 and -10 are placed at the ends of the base of an isosceles triangle. Find the potential at the apex, the slant height being 16 cm.

$$V \text{ (at apex)} = \frac{20}{16} + \frac{-10}{16} = \frac{5}{8} \text{ erg/unit.}$$

2. Charges of -6, 12, 18, 9 are placed at the corners A, B, C, D respectively of a square, side = 10 cm. Find the potential at the middle point of the square.

Distance of the middle point from the corners $= \frac{10}{\sqrt{2}}$.

$$\therefore V = \frac{-6\sqrt{2}}{10} + \frac{12\sqrt{2}}{10} + \frac{18\sqrt{2}}{10} + \frac{9\sqrt{2}}{10}.$$

$$= \frac{33\sqrt{2}}{10} = 4{\cdot}66 \text{ ergs per unit charge.}$$

3. A right-angled triangle has sides 4 cm. and 3 cm. Charges of 16 and -12 are placed at the corners where these sides respectively meet the hypotenuse. Find the potential at the right angle.

$$V = \frac{16}{4} - \frac{12}{3} = 0.$$

In this case the amount of positive work done on the test charge by the repulsion of the positive charge is exactly neutralized by the negative work done by the attraction of the negative charge.

It will be evident from the above examples of Eqn. (6) that if all the charges in the field are positive, the potential at all points will be positive; if all the charges are negative, the potential at all points will be negative; but if some of the charges are positive and others negative, the potential may be positive, negative, or zero, according to the position of the point in the field.

In most cases the charges are not collected on very small bodies, but are spread over the surfaces of ordinary conductors or insulators. The same principles, however, will

apply. We must imagine that the whole charge is divided into a large number of portions or "elements", each small enough to be considered as a point charge. The potential is then found according to Eqn. (6).

145. **Property of a Conductor.**

We have seen (Art. 121) that the lines of force end on the surface of a *conductor* at right angles. Suppose, now, that a small charged body is placed just outside the conductor, and moved all round the latter along a path which keeps close to the surface of the conductor. The movement will thus be always at right angles to the electric force, and no work will be done. Hence there is no difference of potential between any two points just outside a conductor's surface. Further, since there is no electric force in the material of the conductor, there can be no difference of potential within the conductor. Hence we have an important result—

> I. **All parts of a conductor are at the same potential.**

There are no exceptions to this (other than that stated in Art. 120, *Note*) in **static** electricity. The statement also applies to any number of conductors in connection or contact.

The earth is a conductor, but we have, of course, no means of determining its potential. Hence we have no means of determining the *absolute* potential of a body. We can only consider what potential the charges in the field would produce, if the earth's potential were left out of account. Further, owing to the size of the earth, no charges which can be produced artificially can affect its potential. We therefore take the earth's potential as a standard of reference, and for convenience consider it as of **zero** value. A conductor joined to earth forms part of the earth for the time being, and must come to the earth's potential. Thus in **static** electricity—

> II. **An earth-connected conductor has always a zero potential.**

146. Variation of Potential in the Field.

The reference of potential measurements to the earth as a standard finds a simple analogy in the measurement of levels. The height of a mountain or the depth of a mine may be expressed as a number of feet above or below sea-level, the latter being the most convenient surface of reference. In the mathematical sense, a depth below sea-level could be expressed as a *negative* height. In consequence of this analogy, it has become customary to speak of a positive potential as being *higher* than the earth's potential, and a negative potential is said to be *lower* than the earth's. We thus adopt a scale of potential. It is necessary to bear in mind that a large *negative* potential is *lower* than a small one.

The changes of potential in the electric field are most readily traced from the lines of force. Suppose A and B are two points on a line of force, whose direction is from A to B. If we place a test charge at A and move it towards B, the electric forces do positive work. Hence the forces do less work when the test charge is taken from B to infinity than when it is taken from A to infinity, *i.e.* the potential at B is lower than the potential at A. Thus—

> **III. The potential falls continuously from the positive end of a line of force to the negative end.**

In other words, the potential falls in the direction of the arrows. It follows also that—

> **IV. The potential is uniform where there is no field of force.**

In tracing the changes of potential in the field, it is well to *commence at some point of known potential*, preferably an earth-connected conductor.

EXAMPLES.—In fig. 74: The potential at a distant point C is zero. It *rises* as we proceed *against* the line of force towards B. The conductor B is therefore at a positive potential. It falls as we go from B to N, and rises again as we approach A. The potential at N is positive, for this point is influenced by positive charges only.

In fig. 75: The potential at C is zero. It rises against the line of

force until we reach N, and falls again as we approach B along the line NB.

In fig. 88: E is at zero potential. The potential rises from E to D as we proceed through the field against the lines of force. The potential of D is therefore positive. At C the potential has the same value as at D, since it is uniform all through the conductor. (Notice carefully that the potential at C is positive, but the charge at the same place is negative.) The potential rises still further as we proceed from C to B. It is uniform all over and

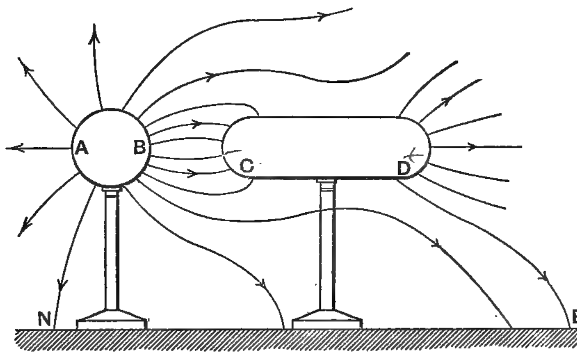


Fig 88

through AB. From A to N the potential falls, its value at N being zero.

In fig. 89: Since CD is earth-connected, it is at zero potential at all parts. The rise as we proceed against the lines from C to B makes the potential of B positive.

The student will find it a useful exercise to represent the above changes graphically. (For examples of potential graphs see figs. 94, 96, *et seq.*)

We must here warn the student against two errors which are not uncommonly made at this stage.

(1) Do not suppose that there is any necessary agreement in sign between the charge at any part of a body and the potential there. If the charge is, say, negative, the potential *may* be negative, but it is just as often positive or zero. For example (fig. 88), the potential at C is positive, but the charge

is negative. The negative potential, which the charge at C alone would produce, is overbalanced by the positive potential produced by the positive charge on AB. The *actual potential* at C is therefore *positive*. Again (fig. 89), the charge on CD produces a negative potential at C, whilst the charge on AB produces an equal positive potential at C. The *actual potential*

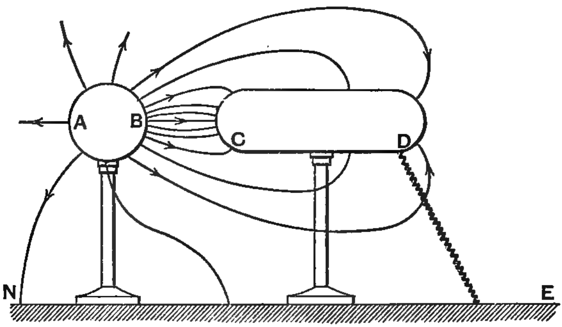


Fig 89

at C is therefore *zero*. Examples of this kind will repay careful reflection, and will enable the student to make a clear distinction between *charge* and *potential*.

(2) Do not suppose that a region free from electric force is necessarily at zero potential. It must be at a *uniform* potential, but this may be positive, negative, or zero.

147. Relation of Electric Force to Potential Difference.

Suppose two points A and B are situated a small distance apart on a line of force. If B is at the lower potential the force acts from A to B, and is practically uniform over the short distance. If F is the electric force, and AB = s, we have—

$$V_A - V_B = \text{work done on unit charge transferred from A to B}$$

$$= F \times s.$$

$$\therefore F = \frac{V_A - V_B}{s}.$$

Thus—

V. The electric force at any point is equal to the fall of potential per unit length of the line of force.

EXAMPLE.—The electric field between two parallel plates 3 cm. apart is uniform. If the P.D. of the plates is 15 electrostatic units, what is the electric force between the plates and the density of the charge on each plate?

$$F = \frac{15}{3} = 5 \text{ dynes per unit charge.}$$

But by Art. 126,

$$F = 4\pi\sigma.$$

$$\therefore \sigma = \frac{5}{4\pi} = {\cdot}39 \text{ e.s. unit per sq. cm.}$$

148. Flow of Electricity and Potential-Difference.

Since positive electricity tends to move in the direction of the electric force, it follows that the electricity tends to move from a point A to a neighbouring point B when B is at a lower potential than A.

VI. Positive electricity tends to move from points of higher to points of lower potential; that is, down the potential gradient.

If a small material particle such as a pith-ball is positively charged, it tends to move down the potential slope; if negatively charged, it tends to move up the potential slope.

If two conductors are electrically connected, they become virtually one conductor, and therefore acquire the same potential. In general a flow of electricity takes place at the moment of connection. The forces which produce this flow act according to the above law.

VII. When two conductors are connected, electricity flows from the conductor of higher potential to that of lower, until their potentials are equalized.

If the conductors are initially at the same potential, no flow takes place.

When we connect a conductor to earth, electricity flows *to* the earth if the potential is positive, and *from* the earth to the conductor if the potential of the latter is negative.

149. Remarks on Potential.

We have no real knowledge that potential represents any physical condition of a body. In the experiment with an insulated room, as made by Faraday, no effect is observed on the instruments placed within the chamber when the latter is highly charged, nor is any effect noticed by the experimentist in his own person. Hence, so far as experimental evidence goes, the objects within the room are in exactly the same state as when the room is uncharged. But (supposing the cube to have a large positive charge) all the interior objects are at a high positive potential, because work must be done in bringing a small charge from the earth to the interior of the cube. When we say that a body has a positive or negative potential, all we mean is that we could not proceed from that body to the earth without passing, *somewhere* on the journey, through an electric field; but this field need not necessarily extend up to the body considered. Thus potential refers rather to the *circumstances* in which the conductor is placed than to its electrical condition or state.

The rules enunciated above show, however, that these circumstances determine the direction of flow of electricity. It is therefore convenient to think of potential as if it were some condition of a body. Hence analogies have been drawn with the conditions which determine the direction of flow in the case of other physical quantities. The flow of heat from a hot body to a cold one may be compared with the flow of positive electricity from a body at higher potential to a body at lower potential. This analogy between potential and temperature has served a useful purpose in the mathematical theory of conduction. But in ordinary electric practice an analogy with fluid pressure has been found more serviceable, since electricity is regarded as a "fluid", and we shall consider this view more fully.

150. Potential of a Conductor—Practical View.

A large proportion of the problems with which the practical electrician is concerned have reference to the flow of electricity in solid conductors. It such cases it has been found useful to adopt a working notion of the potential of a conductor, based on the analogy between the flow of electricity and the flow of material fluids. Potential in the former case plays the same part as pressure does in the latter, and the potential of a conductor is therefore frequently spoken of as **electrical pressure.**

Suppose we have a metal reservoir provided with a stop-cock and containing compressed air. If we open the stop-cock, the excess of air rushes out and the pressure of the remaining air in the vessel becomes the same as that of the atmosphere. If, on the other hand, the reservoir were partially exhausted, then on opening the stop-cock the air would rush in until the pressure inside became atmospheric or "normal". Again, if two reservoirs containing air at different pressures are placed in connection, then, whatever the relative size of the reservoirs, air will be forced from that of higher pressure to that of lower until the pressures are equalized.

Now give an insulated conductor a positive charge, *i.e.* an *excess* of positive electricity. If we join the conductor to earth, the excess of positive electricity at once escapes, and the conductor becomes neutral. On the other hand, if the conductor has a negative charge, *i.e.* a *deficit* of positive electricity, then on connecting it to earth electricity flows from the earth into the conductor (keeping strictly to the normal theory) until the deficit is made up. Let us speak of the tendency of the positive electricity to escape as *electric pressure.* An excess of electricity on an isolated conductor raises its electric pressure above that of the earth, whilst a deficit of electricity brings its pressure below that of the earth. The potential in the former case is positive and in the latter negative. The potential of the conductor may therefore be regarded as its electric pressure with respect to the earth, or it may be described as the *tendency of the positive electricity to escape* from the conductor.

Substituting the idea of pressure for potential, we may state the rule for electric flow thus—

When two conductors are connected, positive electricity flows from the conductor of higher electric pressure to that of lower until the pressures are equalized.

The following examples will assist the student to realize the effect of *induction* on the electric pressure or potential of a conductor:—

Suppose that a sphere positively charged is brought near one end of a neutral conductor (fig. 90 (*a*)). Induction takes

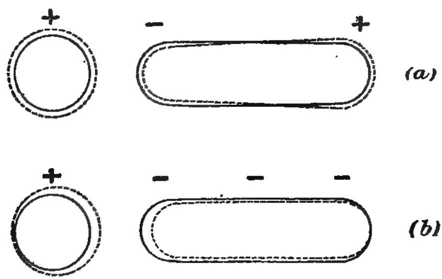


Fig. 90.—Inductive Displacement

place; according to the one-fluid theory, there is a displacement of the positive electricity in the conductor (indicated by the dotted line in the figure). This displacement is due to the repulsion exerted by the positive inducing charge. If we wish to adapt the pressure notion to this case, we must imagine that the inducing charge, in addition to displacing the electricity, produces in the latter a tendency to escape from *all parts* of the conductor. But we have called this tendency the electric pressure. Hence the electric pressure is increased, *i.e.* all parts of the conductor are brought to a positive potential through the influence of the charged sphere. If we connect the conductor to earth by attaching an earth-wire to it at *any* point, some of the positive electricity immediately rushes along the wire to earth. The flow to earth stops as soon as the negative charge left becomes sufficiently great

to neutralize the effect of the positive charge on the sphere. The distribution is indicated by the dotted line (fig. 90 (*b*)).

On the other hand, suppose the sphere to be negatively charged. It now attracts the whole body of positive electricity in the conductor. We must now imagine that the attracted electricity has a decreased tendency to escape; under the influence of the inducing charge, the electric pressure in the conductor is below normal, and the potential is negative at all parts. If any part of the conductor is connected to earth, positive electricity flows *from* the earth *into* the conductor. The flow continues until the positive charge acquired neutralizes the effect of the inducing negative charge, *i.e.* until the potential is zero.

In connection with this *working notion* of potential, the following points should be observed:—

(1) The normal theory only is used (Art. 114).

(2) The tendency to escape, or the pressure, exists in the electricity all through the conductor, not merely in the excess or deficit at the surface.

(3) The approach of a positively-charged body increases the tendency of the electricity to escape or raises the pressure.

(4) The approach of a negatively-charged body lowers the pressure.

151. Comparison of Potentials.

Theoretical.—In problems of a descriptive nature (which afford excellent practice in the study of potential), it is often necessary to ascertain the relative potentials of the conductors or to place them in order of magnitude. The student will do well to adopt at first the following method:—

First, make an outline diagram showing the induced and inducing charges, and draw the lines of force connecting the corresponding charges. Then, *starting at an earthed conductor* or some point having potential of known sign, trace the changes of potential through the field (Art. 146), making sure at each step that your result agrees with rules I and II (Art. 145).

You will only obtain a thorough grasp of the laws of potential by studying them from as many points of view as possible. When you have worked out any problem by the above method, and have determined the potentials of the *conductors*, you should think over your result in connection with the pressure notion (Art. 150).

As a check on results, it is helpful to make use of the idea of "free" and "bound" charges. The potential is zero when the whole charge is *bound*, and positive or negative according as the *free* charge is positive or negative. (The converse to the last statement is true except in the particular case considered in Art. 166, *Note*.)

To determine whether the potential of a conductor is raised or lowered by a movement or other operation, notice whether the field of force between that conductor and the earth is strengthened or weakened, and observe the direction of this field. In many cases we can obtain the result simply from (3) and (4), Art. 150.

Experimental.—*Electroscope as a Potential Indicator.*—A gold-leaf electroscope if connected by a long fine wire to the conductor tested, can be used for rough comparisons of potential. The case of an electroscope protects the leaves from external influence. The divergence, therefore, depends simply on the charge which the leaves carry, and therefore on the *difference* of potential between the leaves and the case. We may determine experimentally the relation between the divergence and the P.D.

Connect a small electroscope to a large insulated conductor. Gradually increase the charge on the system. Since the charge taken by a small electroscope may be neglected, the potential of the conductor must increase with the charge given to the system. The electroscope is joined to the conductor, and therefore its potential also is raised. It will be observed that an increase of P.D. is always accompanied by a greater divergence.

If the walls of the electroscope are earthed, the P.D. becomes *the potential* of the leaves with respect to the earth.

If an electroscope is provided with a circular scale by which the divergence may be recorded, the instrument may be calibrated to read directly in units of potential.

The electroscope, if used to test the potential of bodies charged by frictional methods, should be as small as practicable. Otherwise connection with the electroscope would alter the potential of the conductor tested. (For a similar reason, thermometers must be of small size compared with the volume of liquid to be tested.)

152. Other Kinds of Potential.

The use of the mathematical conception of potential is by no means confined to the study of electric force. In dealing with magnetism, gravitation, and the flow of liquids, we may define potentials just as in dealing with electric force.

Magnetic potential is the work done by magnetic forces when unit pole is transferred from the point considered to an infinite distance. (See Chap. XXIX.)

Velocity potential (when one exists) is a quantity whose rate of change in a given direction at any point is the velocity of the liquid at that point in the given direction.

153. Examples.

1. The charge and the potential of an isolated sphere are each numerically equal to 10. Draw as correctly as you can the equipotential surfaces for potentials 2, 4, 6, 8. (1902.)

An *equipotential surface* is one which passes through all consecutive points at the same potential. The surface of a conductor on which the charge is at rest is necessarily an equipotential surface. If the meaning of potential-difference is kept in mind, it will be evident that *equipotential surfaces cross the lines of force at right angles.*

In the example given, the lines of force are radial. The equipotential surfaces are therefore spheres concentric with the charged sphere.

Let r_1 be the radius of the charged sphere.

Then potential of sphere $= \dfrac{Q}{r_1}$.

$$i.e.\ 10 = \frac{10}{r_1},\ \text{or}\ r_1 = 1.$$

Let r_2, r_3, etc. be radii of surfaces for potentials 8, 6, 4, 2. Then—

$$8 = \frac{10}{r_2}, \quad \text{or} \quad r_2 = 1\tfrac{1}{4};$$

$$6 = \frac{10}{r_3}, \quad \text{or} \quad r_3 = 1\tfrac{2}{3};$$

$$4 = \frac{10}{r_4}, \quad \text{or} \quad r_4 = 2\tfrac{1}{2};$$

$$2 = \frac{10}{r_5}, \quad \text{or} \quad r_5 = 5.$$

Circles representing spheres of these radii may now be drawn.

2. Two insulated conducting spheres A and B are placed near but not in contact with each other, B being connected by a fine wire with the cap of a gold-leaf electroscope. State and explain the behaviour of the leaves of the electroscope when (1) A receives a positive charge, (2) the wire is removed, (3) the electroscope is momentarily touched, (4) the wire connection is restored. (1899.)

(1) A's charge produces induced charges on B, and brings B's potential to a positive value. The electroscope therefore gets a positive potential, therefore also a positive charge. This positive charge must be a part of the repelled charge on B. A question which now arises is: "Does all the repelled charge on B go to the electroscope?" To answer this we must remember that B's potential is positive, and in this case lines of force pass from B to the earth; *i.e.* B retains some of its free positive charge.

(The charge on the electroscope does not affect the potential of B, on account of the conducting walls of the instrument.)

(2) Since the wire carries a negligible charge, its removal produces no effect.

(3) The electroscope is discharged, and its potential becomes zero.

(4) The restoration of the wire joins B (at positive potential) with electroscope (at zero potential). More of B's free charge flows to electroscope, which is again brought to a positive potential. On connection of B with the gold-leaf electroscope its potential falls a little, and further induction goes on. Compare Art. 132.

The leaves of the electroscope (1) diverge +, (2) remain diverged, (3) collapse, (4) rediverge with +.

The student should draw a graph for the potential in each of these cases.

EXERCISES

1. How much energy is expended in carrying a charge of 50 units of electricity from a place where the potential is 20 to another where it is 30? What is meant by saying that the potential of a conductor is 20? (1894.)

2. An insulated conductor, placed at some distance from other conductors, receives a charge of positive electricity. Is its potential altered when another conductor, uninsulated and uncharged, is brought near to it? Give reasons for your answer. (1898.)

3. Two parallel insulated plates have equal and opposite charges of electricity; they are originally placed close together, and are then pulled farther apart. If the gold-leaf electroscope is connected with one of the plates, and its case with the other, what will be the effect on the electroscope of increasing the distance between the plates? (1901.)

4. Give a careful drawing of the equipotential surfaces due to a + charge of 4 units and a − charge of 1 unit at a distance 10 cm. apart. (1905.)

Note.—First draw the lines of force; then a second set of lines cutting these everywhere at right angles represents the equipotential surfaces.

5. Charges 1, 2, 3, and −4 units are placed at the corners of a square, taken in order. If the length of each side is 2 cm., find the potential at the middle point of the side joining 1 and 2. (1902.)

CHAPTER XII

CAPACITY—DISTRIBUTION—ENERGY

154. Electric Capacity.

The relation between the charge of an isolated conductor and its potential depends on the size and shape of the conductor. The charge required to produce a given potential generally increases with the size of the conductor. The relation between charge and potential is termed "coefficient of capacity", or for brevity, simply "capacity".

The coefficient of capacity of a given con-

ductor is the ratio of the charge on the conductor to the potential to which the conductor is raised by that charge.

$$\text{Thus the capacity } S = \frac{Q}{V}.$$

The laws relating to capacity are intimately connected with those relating to distribution. We shall therefore deal with the two subjects concurrently.

155. **Isolated Conductors.**

The term "isolated" is applied to conductors which are practically removed from the influence of all other bodies, *e.g.* a conductor mounted on a tall stand, or suspended in the middle of a large room.

If you double the charge on a conductor, the extra quantity will distribute itself in the same manner as the original quantity, and the density at each point becomes doubled. In general, if you increase the charge n times, the density at each point is increased n times. Hence we have the result—

I. **The surface density at a given point of an isolated conductor is proportional to the total charge.**

It follows that the *shape* of the lines of force will remain unaltered when the charge is strengthened or weakened. For the component forces (due to every small portion of the charge on the conductor) acting on a unit charge placed near the conductor, will *all* be increased n times when the total charge is increased n times. The resultant force at a given point is therefore increased n times, but unaltered in direction.

Further, since the force at every point increases in the same ratio, the work done in taking unit charge to infinity, or the potential, is also increased in this ratio.

Therefore for a given conductor V is proportional to Q; or

$$\frac{Q}{V} = \text{a constant.}$$

Thus the capacity of a given isolated conductor is constant.

The following hydrostatic analogy is useful. Suppose that two cylindrical vessels of indefinite height contain water. The quantity of water, Q, in each is proportional to the height or head of liquid. Thus—

$$Q \propto h.$$

To convert this into an equation we must multiply h by the area of the base; then—

$$Q = (\text{area of base}) \times h.$$

The area of the base may therefore be called the "coefficient of capacity" of the vessel. The greater the area the greater the volume of water required to produce a given head. If two such vessels are connected, the water acquires the same level in both, but the quantities are different (fig. 91). So also in the electrical case: if two conductors of different capacities are connected, the potentials become the same in both, but the charges are unequally shared.

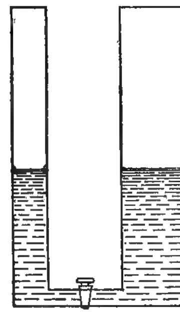


Fig. 91.—Hydrostatic Analogy

156. Calculation of Capacity.

The coefficient of capacity can be calculated by elementary mathematics only in the case of a sphere.

If the charge is uniformly distributed, we may suppose that it is collected into a point charge at the centre. The potential at a distance d from the centre is given by the equation—

$$V = \frac{Q}{d}.$$

Therefore at the surface of the conductor,

$$V = \frac{Q}{r}.$$

Thus,

$$S = \frac{Q}{V} = r \qquad \text{(1)}$$

The capacity of an isolated sphere (in air) is equal to its radius.

The capacity of a thin circular disc can be shown to equal $\frac{d}{\pi}$, where d = diameter of the disc.

Observe carefully that the capacity of a conductor is proportional to its *diameter* (not to its area).

157. Sharing of Charges.

When isolated conductors are brought into connection by thin wires, they acquire the same potential. Since the wire is generally of negligible capacity, we may assume that the whole of the charge is shared between the conductors.

Let the conductors have capacities S_1, S_2, S_3, etc., respectively. Let the total charge be Q, and let V be the common potential. If q_1, q_2, q_3, etc., are the respective shares of charge—

$$q_1 = S_1V,\ q_2 = S_2V, \text{ and } q_3 = S_3V, \text{ etc.}$$

The total capacity—

$$S = \frac{Q}{V} = \frac{q_1 + q_2 + q_3 + \text{etc.}}{V} = S_1 + S_2 + S_3 + \text{etc.} \ldots \quad (2)$$

i.e. the total capacity is the sum of the capacities of the separate conductors.

Also, $$\frac{q_1}{Q} = \frac{S_1V}{(S_1 + S_2 + S_3 + \text{etc.})V} = \frac{S_1}{S_1 + S_2 + S_3 + \text{etc.}} \ldots\ldots (3)$$

This result may be stated thus—

The charge taken by one conductor is the same fraction of the total charge as the capacity of that conductor is of the total capacity.

EXAMPLES.—1. Two isolated spheres of potentials 5 and 7, and diameters 3 and 2 respectively, are joined. Find the charge acquired by each.

Initial charges are $1{\cdot}5 \times 5$ and 1×7.

Total charge = $14{\cdot}5$.

$$\frac{q_1}{Q} = \frac{S_1}{S} = \frac{3}{3+2}. \quad \therefore q_1 = \tfrac{3}{5} \times 14{\cdot}5 = 8{\cdot}7,$$

and similarly $q_2 = \tfrac{2}{5} \times 14{\cdot}5 = 5{\cdot}8$.

2. Three isolated conductors have capacities 6, 8, 4. The first has a potential 30, and the others are neutral. Find the charge acquired by the third, and the common potential after they are connected.

$$q_1 = S_1 V_1 = 6 \times 30$$

$$V \text{ (after connection)} = \frac{Q}{S} = \frac{180}{6+8+4} = 10.$$

$$\text{Also } q_3 = S_3 V = 4 \times 10 = 40.$$

Surface density of shared charges.

Let two spheres have radii r_1, r_2, and when the spheres are electrified and joined let the charges be q_1, q_2 respectively.

Then $$V = \frac{q_1}{r_1} = \frac{q_2}{r_2}.$$

The surface densities are—

$$\sigma_1 = \frac{q_1}{4\pi r_1^2} \quad \text{and} \quad \sigma_2 = \frac{q_2}{4\pi r_2^2}.$$

$$\therefore \frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1} \qquad \text{(4)}$$

The surface densities are *inversely* as the radii.

If one sphere has twice the diameter of the other, it will receive twice the charge; but this, being spread over an area four times as great, will have only half the density.

158. **Groups of Conductors.**

When a conductor forms one of a group, it loses its individuality as regards electrical properties. These depend no longer solely on the size and shape of the conductor, but equally on the size, shape, relative position, and connections of those which surround it. The term *configuration* conveniently denotes these factors taken collectively. The configuration of a group of conductors is constant so long as they remain unmoved, and their connections are unaltered. The mere removal of a thin wire earth-connection would alter the electrical configuration.

If one of the conductors, A, of a system is charged, electrification is induced on all the neighbouring conductors, but if

A is removed, the induced charges at once disappear, provided the configuration has not otherwise been altered. It is convenient to distinguish between **induced** and **independent** charge. The latter is the charge which remains on a conductor when all neighbouring bodies are removed. A conductor which is in connection with the earth may have a very strong *induced* charge, but none of this can be considered *independent* so long as the conductor remains earth-connected. If the earth wire is removed the configuration is altered, and the induced charge at once becomes independent.

Let us suppose that only one conductor, A, has an independent charge. A little thought will show that if this is increased, say n times, the density of the induced charge is everywhere increased in the same ratio. This is equivalent to splitting up each of the original unit tubes of induction into n new unit tubes, *without altering their general shape.*

> **II. When only one conductor of a group possesses an independent charge, the density of the induced electrification at any given point is proportional to the independent charge.**

It follows by the same line of reasoning as was used in Art. 155, that the potential of the independently-charged conductor is proportional to its charge. The capacity is therefore constant so long as the configuration remains unaltered. (The capacity of a conductor which forms one of a group must be defined as the ratio Q/V when all the other conductors are free from *independent* charges.)

159. Variation of Capacity.

The capacity of a given conductor is least when the conductor is isolated. We may increase the capacity of a conductor in three ways, namely—

By bringing near to it

(1) **an earth-connected conductor;**

or (2) **an insulated conductor;**

or (3) **an uncharged dielectric.**

Let us first consider these from the point of view of the pressure notion. Let AB be the (+) conductor (fig. 88), and let an insulated conductor CD be brought near it. The charge induced at C reacts on the electricity in AB and tends to reduce the pressure; but at the same time the charge induced at D tends to counteract this effect. The charge at C has the greater effect, and the result is a lowering of the pressure in AB. Since the potential of AB is decreased without alteration of charge, the capacity Q/V is increased. If CD is now put to earth (fig. 89) it gets a stronger negative charge, and the positive at D disappears. Thus for a double reason there is greater lowering of potential in AB and consequent increase of capacity.

Next take the point of view afforded by lines of force. When CD is placed near AB the lines tend to collect between B and C, thus reducing the number between A and N. The weakening of the force between A and N means decrease of A's potential. Thus the capacity of AB is increased. By similar reasoning it is evident that the capacity of AB will be still further increased when CD is put to earth.

In actual measurements of the capacity of conductors, the fact that a table is an earth-connected conductor must not be lost sight of.

The effect of a dielectric on the tubes of electric induction (fig. 121) is similar to the effect of a piece of iron on tubes of magnetic induction, but very much weaker. Thus the tubes tending to run through the dielectric are drawn from the region between A and E. (The distortion of the lines is much exaggerated, for clearness, in the figure.)

The second rule of distribution (Art. 158, II) applies if an uncharged mass of dielectric forms one of the group.

160. Energy in the Electric Field.

The spark which usually accompanies the discharge of a conductor is evidence of the existence of energy in the field. The energy thus dissipated in the form of heat, sound, etc., is the equivalent of the work done in electrifying the conductor.

The work which must be expended in order to give a charge to an isolated conductor (or an independent charge to *one* conductor only of a group) may be calculated as follows:—

Imagine that the charge is brought up to the conductor (from the earth) in very small portions each equal to q, say on a very small carrier ball. Practically no work is done in bringing up the first quantity. A little work is done during the second approach, owing to the repulsion of the small charge which the conductor has acquired. As the charge accumulates, the work done during each successive approach of the carrier ball increases.

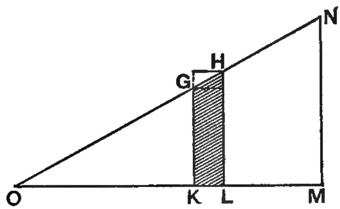


Fig. 92

The work may be represented graphically. Let abscissæ from O represent the successive charges acquired by the conductor, and let the corresponding ordinates represent potentials (fig. 92). Since the potential is proportional to the charge, the rise of potential is shown by a straight line ON. Let GK ($=V'$) be the potential already acquired at any stage of the process of electrification, and KL = additional charge q brought up by the next approach of the carrier ball. Then by definition of potential the work done in bringing up q is—

$$w = V'q.$$

This is represented by the *area* of the rectangle shaded. At the next step a little more work is done, represented by a similar rectangle; and so on throughout the process.

By the method of limits we see that when q is indefinitely small the total work done during the whole charging process is represented by the area of the triangle OMN, or—

$$\tfrac{1}{2}\text{OM} \times \text{MN}.$$

But OM represents Q′ and MN represents V;

$$\therefore \text{W} = \tfrac{1}{2}\text{QV} \quad \text{.......................(5)}$$

This may be described as—

"The product of the whole charge into the average potential during the charging process."

Substituting SV for Q in Eqn. (5) we have—

$$W = \tfrac{1}{2}SV^2 \qquad (6)$$

Substituting Q/S for V, we get—

$$W = \tfrac{1}{2}\frac{Q^2}{S} \qquad (7)$$

EXAMPLE.—A conductor capacity 5 is charged with 9 units of + electricity. It is joined by a fine wire to a sphere brought to potential −8 with a negative charge of 16 units. Find the energy dissipated during the sharing of electrification.

(i) Energy before connection	$= \tfrac{1}{2}\frac{81}{5} + \tfrac{1}{2}(-8)(-16),$
	$= 72{\cdot}1.$
(ii) Total charge to be shared	$= 9 - 16 = -7.$
Capacity of the sphere	$= \frac{-16}{-8} = 2.$
∴ total capacity	$= 5 + 2 = 7.$
∴ energy left after connection	$= \tfrac{1}{2}\frac{(-7)^2}{7} = 3{\cdot}5.$
(iii) Energy dissipated	$= 72{\cdot}1 - 3{\cdot}5,$
	$= 68{\cdot}6$ ergs.

Only a small portion of this appears as heat in the spark.

The energy of an electric charge is entirely stored in the tubes of induction.

For the calculation of energy per unit volume of the field, see Arts. 184, 209.

161. Groups with more than one Independent Charge.

By adopting the method of reasoning already given, the student should prove the following law of distribution.

III. The surface density at any point of a system of conductors is proportional to any one of the independent charges: provided that when any change is made in this charge, all the other independent charges are made to increase or decrease in the same ratio.

A similar rule applies to the potential at any point and the density of the induction tubes.

To arrive at an expression for the energy of the system, we must suppose as many carrier balls provided as there are independent charges. These must be supposed to bring up their charges *simultaneously* to their respective conductors. The charges brought up at each step must bear the same ratios to one another as the total charges which the conductors

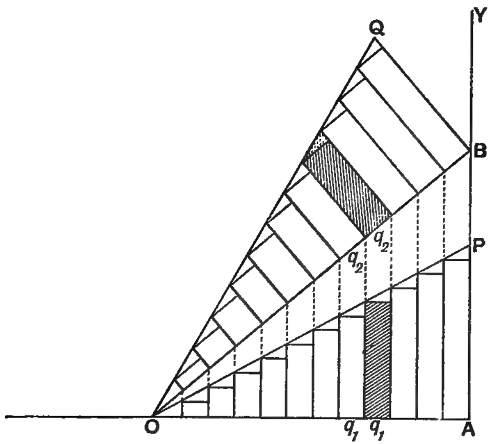


Fig. 93

respectively acquire. Thus for two independent charges if Q_1, Q_2 are the charges finally acquired, and q_1, q_2 the charges brought up at each step,

$$q_1 : q_2 = Q_1 : Q_2.$$

The work done may then be represented graphically. Thus, if there are two conductors, draw OA to represent Q_1, and AY of indefinite length at right angles to it (fig. 93). With O as centre, and radius OB representing Q_2, mark off B on AY. Join OB. Make AP $= V_1$ and BQ (at right angles to OB) $= V_2$. The progress of the double charging process is indicated by the intermediate dividing lines, these always being taken in

pairs, as shown by the dotted connecting lines. The total energy is represented by (area OAP + area OBQ).

The method may be applied to the general case of any number of conductors. In the figure there will be as many triangles representing work done as there are independent charges. Thus in general—

$$W = \tfrac{1}{2}Q_1V_1 + \tfrac{1}{2}Q_2V_2 + \dots\dots\dots\dots + \tfrac{1}{2}Q_nV_n.$$

Note.—This expression must be treated *as a whole.* Although the energy is expressed *in terms of* the potentials and charges of the separate conductors, we cannot say that the energy $\frac{1}{2}Q_1V_1$ *belongs* to the first conductor. V_1 depends on Q_2, Q_3, etc., as well as on Q_1.

162. Similar Conductors.

Two conductors are geometrically *similar* when every diameter of one bears the same ratio to the corresponding diameter in the other. Thus if a rod of circular section is three times as long as another, then to be geometrically similar it must also be three times as thick. All spheres are similar to one another, also all cubes.

IV. All similar isolated conductors have the same distribution of charge.

This is a consequence of the fact that the force varies according to an inverse power of the distance. With care it may be confirmed experimentally. Riess found that for cubes the densities at the middle of a face, middle of an edge, and corner were approximately proportional to 1, 2·5, and 4, respectively. On ellipsoids the densities at the extremities of the principal axes are proportional to the lengths of the axes.

Corollary.—The densities at corresponding points of similar equally charged conductors are inversely as the squares of the linear dimensions.

(This may easily be proved by dividing the surfaces of the conductors into small areas by *corresponding* lines.)

Combining this result with Rule I, we have—

$$\rho_1 : \rho_2 = \frac{Q_1}{l_1^2} : \frac{Q_2}{l_2^2},$$

where ρ_1, ρ_2 refer to corresponding points, and l_1, l_2 to corresponding diameters.

The statements of the present article will apply to geometrically similar groups of conductors, provided we increase or decrease all the independent charges in the same ratio.

EXAMPLE.—A cube A has an edge 3 times as long as a cube B, and has 4 times the charge. Compare the densities at the midpoints of the faces of the cubes.

$$\rho_1 : \rho_2 = \frac{Q_1}{Q_2} \cdot \frac{l_2^2}{l_1^2},$$
$$= \frac{4}{1} \cdot \frac{1}{9}.$$

The density on B is $2\frac{1}{4}$ times as great as the density on A.

HOLLOW CONDUCTORS

163. We shall now consider certain special properties possessed by hollow conducting bodies. Strictly, these properties apply only where the vessels, etc., are *completely closed.* Practically, it is sufficient that the opening be small in comparison with the diameters of the conductor, *e.g.* a hollow sphere with a small hole at the top, and a deep metal can. Further, the experiments may be carried out with vessels constructed of fine wire gauze.

The term "interior charges" will be applied to the electrification of the inner surface of the conductor or of any body situated in the interior space; and "exterior charges" to the electrification of the outer surface of the vessel and of bodies situated outside.

The rules to be considered fall naturally into three groups—

(1) Laws relating to amount of induced charge.
(2) ,, ,, distribution.
(3) ,, ,, potential.

164. **Induced Charges.**

By direct experiment with a proof plane we may prove the following rule:—

I. No charge is induced on the inner surface of a hollow conductor by a charge situated outside the conductor (fig. 94).

This is true whether the vessel is insulated or earthed, pro-

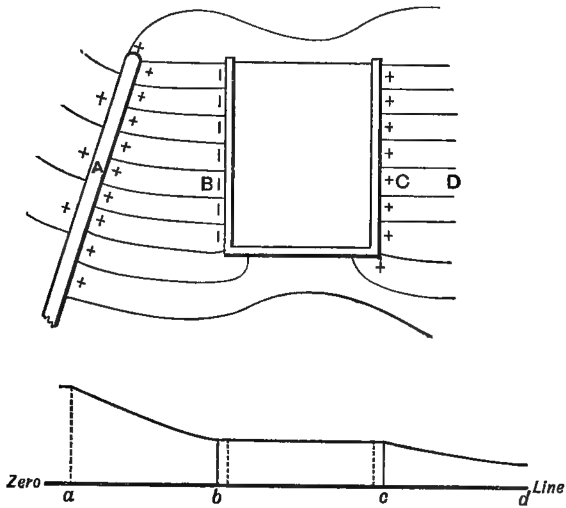


Fig. 94

vided that there are no conductors inside the vessel connected with objects outside or with the earth.

II. When charged bodies are situated within a hollow conductor the sum of the charges on the inner surface of the conductor is equal to the sum of the charges on the bodies introduced, but is of opposite sign.

Algebraic sum is here meant. If two objects are introduced with charges $+10$ and -14, then if the charge induced on one portion of the inner surface is -3, the charge induced on the remainder must be $+7$.

This is an extension of the result of Faraday's ice-pail

experiment. It may be proved by repeating the experiment, introducing two charged balls into the can, and allowing both to touch the inner surface of the vessel. The law may be briefly stated thus—

The algebraic sum of the interior charges is zero.

This is equally true for insulated and uninsulated conductors.

165. Distribution.

We may show experimentally by the use of a proof plane that the following rule applies:—

> **III. The distribution of charge on the outer surface of a hollow conductor is independent of the position of charged bodies (if any) inside.**

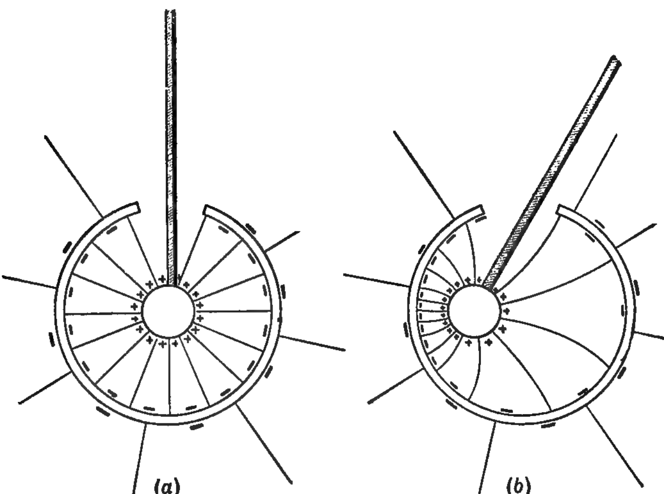


Fig. 95.--Independence of Internal and External Fields

The charges on the inner and outer surfaces may be of the same sign, or opposite.

Fig. 95 (a) shows a small sphere positively charged suspended concentrically within a spherical shell. The latter has

been earthed for a moment and then recharged negatively. Fig. 95 (*b*) shows the distribution after the small sphere has been displaced. The distribution inside is altered, but the *density and distribution outside are unaffected.*

Again, exterior charges produce no induced charges on the inner surface of a vessel (Rule I), and no electric force in the interior space (fig. 94). We have therefore the rule—

IV. The distribution of interior charges is unaffected by electrification on bodies outside the conductor, or on the outer surface of the conductor itself.

On this rule depends the use of the gold-leaf electroscope as a potential indicator, the case serving as the hollow conductor. Also in the quadrant electrometer the box formed by the opposite quadrants is a hollow conductor.

If the inner conductor is joined to some other conductor outside, Rules III and IV apply only to *distribution*, as stated; but if it is insulated from all bodies outside, the rules apply to absolute surface *density* as well.

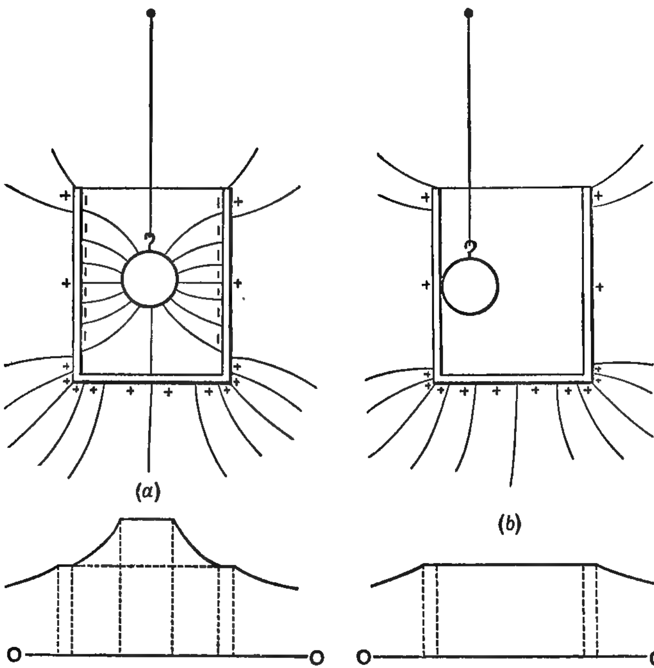
166. Potential.

In the absence of interior charges, there is no electric force within a hollow conductor. It follows, therefore, that there is no *difference* of potential in the interior.

V. The potential is uniform throughout the interior space of a hollow conductor containing no charged bodies.

This may also be tested experimentally. Charge an insulated can, or hold an electrified rod near it (as in fig. 94). Mount two small carrier balls on long, well-scraped ebonite handles. Place the balls in contact with each other well inside the can. Separate and remove them, being careful not to touch the can. It will be found that neither ball is charged. Hence, when the balls are inside the can, there is no inductive displacement of electricity from one ball to the other, and the potentials at interior points are therefore equal.

Note.—The presence of an *un*charged body inside the can (see fig. 98) makes no difference. The body, although quite neutral at all parts, has the same potential as the can. It is worth noticing that this is *the only instance where an object can have a positive or negative potential and yet be perfectly free from charge at all points.* In order to make use of the pressure notion in this case,



(a) (b)

Fig. 96

we must imagine that each portion of the charge on the can exerts on the electricity in the neutral object an influence which increases or decreases the tendency of the electricity to escape. (But since the forces due to the portions on opposite sides of the can balance, there is no displacement of the electricity.) When the conductor introduced into the can is connected with some conductor outside, the connecting wire disturbs the balance and electricity flows either to or from the object.

VI. The potential of the walls of a hollow conductor depends only on the exterior charges.

In Faraday's ice-pail experiment the neutralization of the interior charges produced no change in the divergence of the electroscope leaves, *i.e.* no change in the potential of the vessel (fig. 96 (a, b)). But if the exterior charge is neutralized the

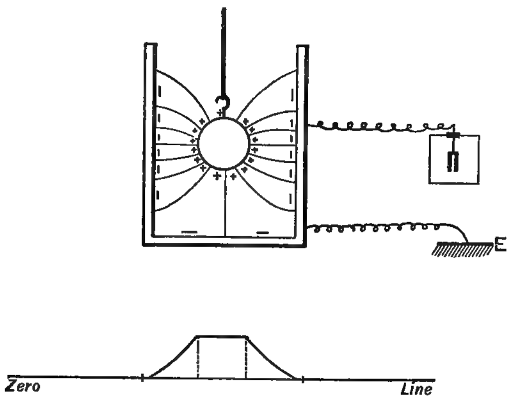


Fig. 97

leaves collapse, and the potential of the walls is zero, even if the interior contains charged bodies (fig. 97). It must be noticed that this rule applies to the "walls" of the conductor, not to the interior space.

Rule VI may also be proved from Rule III, applying the definition of potential as the "work done in the electric field".

VII The potential of the body introduced, relative to the walls of the vessel, depends only on the interior charges.

This follows at once from Rule IV, and the definition of potential-difference.

To find the potential of the body introduced into a hollow conductor, we may proceed thus—

(i) Find the potential of the body relative to the walls of the vessel, considering interior charges only.

(ii) Find the potential of the walls of the vessel, considering exterior charges only.
(iii) Take the algebraic sum of (i) and (ii).

In drawing the potential graph, we may regard the portion referring to the interior as a separate curve built on the remainder. If the interior charges remain undisturbed, an alteration of exterior charges produces no change in the shape of this part of the curve—merely raising or lowering it bodily. Compare figs. 96 (*a*) and 97.

167. Electrostatic Screens.

It is often necessary to shield delicate instruments from the disturbing effect of neighbouring charged bodies. This may be done by placing them inside a hollow conductor. If a gold-leaf electroscope is placed inside a can (fig. 94) it will be shielded from the influence of a charged rod outside.

An electroscope with glass walls is screened by tinfoil strips or wire gauze attached to the *inner* surface of the glass. An electrometer is provided with a brass case, or "bird-cage", which serves the same purpose.

A hollow conductor serves as a screen, whether insulated or earthed.

A metal plate serves as a screen *only when earth-connected.*

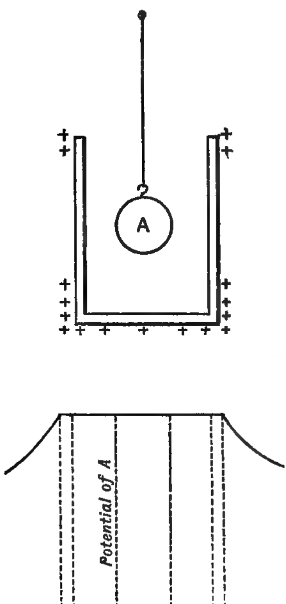


Fig. 98

168. Further Examples of Induction.

In the ice-pail experiment the charged ball induces charges on the vessel. In some experiments this process is reversed:

a charge is induced on the ball by the charge on the vessel *owing to a connection between the ball and some exterior conductor.*

The student should notice how the rules we have enunciated are applied to the solution of problems of this kind.

Let a can be insulated and charged positively. The interior space is thus raised to a positive uniform potential (Rule V). A

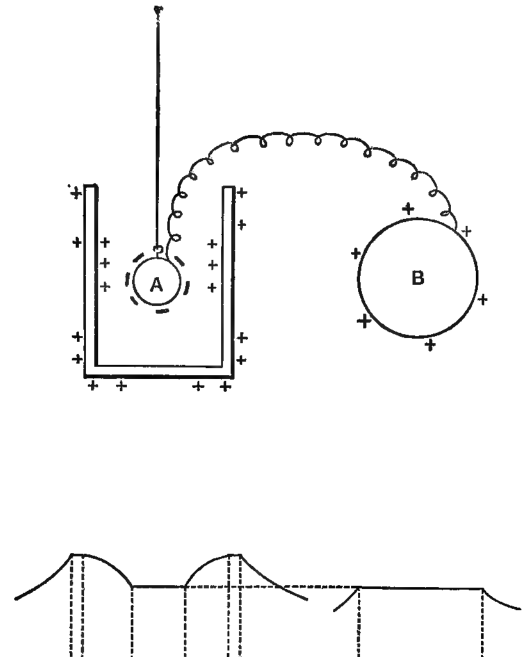


Fig. 99

metal ball A (fig. 98) lowered into the interior by a silk thread remains neutral at all points, but is raised to the same potential as the vessel (Rule V. *Note*).

Now suppose the ball connected to an exterior conductor B. The latter is at a lower potential than A initially; therefore electricity flows from A to B until the potentials are equalized. This flow leaves A with a negative charge. Therefore (Rule II) there

must be an equal positive charge on the inner surface of the vessel and this can only be drawn from the charge on the outside of the vessel, which is therefore diminished. Therefore (Rule VI) the potential of the vessel falls. It does not, however, fall so much as the potential of A, since there must be a drop of potential in passing from the positive charge on the inner surface of the vessel

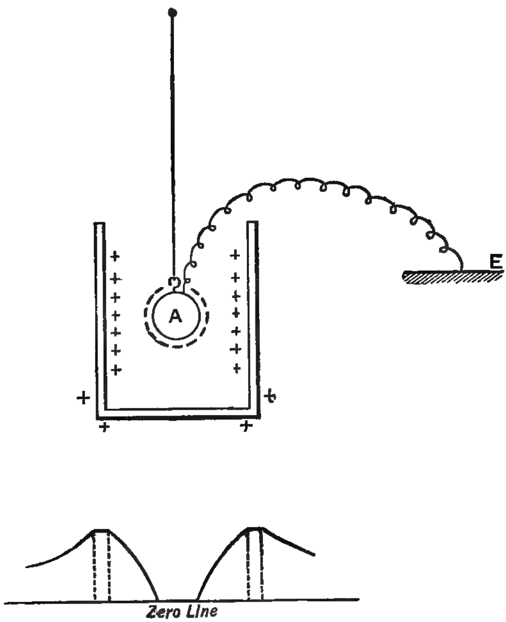


Fig. 100

to the negative charge on the ball. The positive which flows to B raises it to the same potential as A (fig. 99).

If the conductor B is the earth, the potential of A falls to zero. A gets a greater negative charge, and more positive flows from the outer surface of the vessel to the inner surface (fig. 100).

Exactly similar results may be obtained, without actually charging the vessel, if it is brought under the influence of another electrified body, as in fig. 94. In this case, when

the ball is earthed, the charge which goes to the inner surface of the vessel is derived from the repelled induced charge on the outside of the vessel.

169. Solution of Electrostatic Problems.

No very definite rules can be given for solving descriptive electrostatic problems in general, but the following points should be kept in mind.

(1) The *independent* charges should be first considered, and the charges which these are likely to induce next determined.

(2) The general form of the lines of force should then be made out, and the variation of potential in the principal parts of the field shown by a graph. Each conclusion arrived at must be tested by seeing that it agrees with all the following laws:—

(i) Induced charges (+ and −) appear in equal quantities simultaneously, and also disappear in equal quantities simultaneously.

(ii) All parts of a conductor or set of connected conductors must be at the same potential.

(iii) Earth-connected conductors are at zero potential.

(iv) There is a fall of potential from each positive charge to the *corresponding* negative charge.

(v) The potential is uniform where there is no field of force.

(vi) In the case of hollow conductors, Rules I to VII above must be satisfied.

There are no exceptions to these laws in frictional electrostatics.

(3) Variations in the potential of conductors may be traced as explained in Art. 151.

(4) If two conductors are to be connected, their relative potentials must first be found. The direction in which electricity will flow is then at once decided.

170. Examples.

1. An insulated sphere having a diameter of 20 cm. is charged. It is then connected to an electrometer by a fine wire, the deflection being 50 divisions. An insulated and uncharged sphere of 16 cm. diameter is then joined to the first by a long wire, and the electrometer deflection falls to 32. Calculate the capacity of the electrometer. (1906.)

The charge Q on the whole system is unaltered by the connections. Let S_1 be the capacity of the first sphere, S the capacity of the electrometer, S_2 the capacity of the second sphere.

Common potential of electrometer and sphere after first connection

$$= \frac{\text{Charge}}{\text{Total capacity}} = \frac{Q}{S_1 + S} = V_1.$$

Common potential of electrometer and two spheres after second connection

$$= \frac{\text{Charge}}{\text{Total capacity}} = \frac{Q}{S_1 + S + S_2} = V_2.$$

$$\therefore \frac{V_1}{V_2} = \frac{S_1 + S + S_2}{S_1 + S}.$$

But the electrometer deflections represent the potentials.

$$\therefore \frac{\theta_1}{\theta_2} = \frac{S_1 + S + S_2}{S_1 + S}.$$

$$\frac{50}{32} = \frac{10 + S + 8}{10 + S}.$$

$$S = 4\tfrac{2}{9}.$$

2. Two equal insulated cans A and B, some distance apart, are joined by a wire. A + charged rod is held so as to act on A; the wire connection is then removed, and lastly the rod is taken away. Compare the charges on A and B, (i) when the rod is held outside A and near it, (ii) when the rod is held inside.

(i) No charge is induced inside A (Rule I). Part of the positive induced charge on A flows to B until the potentials are equal. After removal of the wire and rod, A and B are left with equal amounts of unlike charges, both less than the inducing charge on the rod.

(ii) Induced electrification, as in the ice-pail experiment. *Half* the induced charge on outside of A goes to B (Rule III). After

removal of wire and rod, A and B are left with equal unlike charges, each being equal to half the inducing charge on the rod.

3. In what position must an earthed conductor be held near a charged hollow conductor to obtain the greatest induced charge?

If there were no induced charge on the earthed conductor its potential would be that of the region in which it is placed. If this potential is positive, the induced charge must be sufficient by itself to produce an equal negative potential, so that the *resulting* potential of the earthed conductor is zero. Hence the greatest induced charge is obtained where there is initially the greatest potential to be neutralized, *i.e.* in the interior space of the vessel.

4. Three ice-pails A, B, C are placed one inside another, A being innermost. They are all insulated from one another and from the earth. A charged conductor X is suspended within A. What would be the effect of—

(i) earthing X, A, B, or C;
or (ii) joining A with C;
or (iii) joining X with B?

Initially in each case we start with a (say) negative charge on X and positive and negative charges on the inner and outer surfaces respectively of A, B, and C.

(i) If X is earthed, all the charges disappear.

If A is earthed, the charges disappear completely from B and C and from the outer surface of A.

If B is earthed, the charges disappear completely from C and from the outer surface of B. X and A are unaltered.

If C is earthed, the charge goes from its outer surface. X, A, and B are unaltered.

The inner surface of the earthed pail is in each case positively charged.

(ii) When A is joined to C the induced charges disappear completely from B, from the outer surface of A, and the inner surface of C. The remaining charges are unaltered.

(iii) When X is joined to B the charges disappear completely from X and A and from the inner surface of B. The charges on the remaining surfaces are unaltered.

The student should draw sectional diagrams showing the distribution of charge and lines of force for each of the above steps.

EXERCISES

Calculations of Capacity, Energy, etc.

1. Prove that the work required to charge an insulated sphere of radius a with e units of electricity is $\frac{e^2}{2a}$. (1904.)

Note.—This should be obtained from first principles, not by substitution in the formula $\frac{1}{2}QV = W$.

2. Two insulated spheres, having radii of 3 and 1 cm. respectively, are placed a long way apart, and a charge of 15 units is given to the larger sphere; what charge must be given to the smaller in order that the larger sphere may neither gain nor lose charge when the two spheres are connected by an insulated wire? (1903.)

3. Does the energy of an electric charge depend upon the magnitude of the charge only? If not, upon what other circumstances does it depend? Find an expression for its magnitude. (1900.)

4. Eight equal metal cubes are placed at a distance from each other and given equal charges. Without being discharged they are then placed on an insulator and built up so as to form a single cube. Compare the original density at a given point on one of the small cubes with the final density at a corresponding point on the large cube. What would the result have been if one of the small cubes had been accidentally discharged? (1898.)

[See Art. 162 and example.]

5. Equal quantities of positive electricity are communicated to two insulated metallic spheres whose radii are as five to one. What are the relative electrical potentials? The spheres are then put in conducting communication by means of a long thin wire, which is afterwards removed. What are now the relative electrical surface densities of the spheres? State in each case which sphere has the greater potential or surface density. (1895.)

6. An insulated uncharged metal sphere is placed at a certain distance from another precisely similar but charged sphere. If the two spheres be connected by a fine wire, will (1) the quantity of electricity which passes, (2) the energy of the discharge, be affected by the distance between the spheres at the moment when the discharge takes place? (1891.)

7. Two equal and equally-charged drops of water coalesce into a single drop. Find an expression for the ratio in which the potential is altered.

Descriptive Problems, Hollow Conductors, etc.

8. A gold-leaf electroscope is placed on an insulated metal stand. State and explain the indication of the leaves when the stand receives a positive charge. How would the indication be modified if the leaves were earth-connected? (1898.)

[*Note.*—In this and the following questions an electroscope may be assumed to have the form shown in fig. 63, the walls forming a wooden or metal conductor almost completely enclosing the leaves.]

9. An uncharged metal ball connected with an electroscope is hung inside a hollow conducting vessel, which is then charged positively. What are the signs of the charges on the ball and the electroscope (1) when the latter is outside the vessel, (2) when it is inside? (1902.)

[Note that in (2) the ball and the electroscope are both inside the vessel.]

10. A hollow metal vessel is insulated and charged to potential V, and the following operations are successively performed:—(1) An insulated metal ball is lowered into the jar without touching it, (2) the ball is momentarily earth-connected, (3) the jar is momentarily earth-connected, and (4) the ball is removed to a distance. State the changes of potential of the jar and the ball at each stage. (1901.)

11. An electroscope is joined to a metal can. How is the potential of the electroscope affected, if at all, by the size, shape, position, or charge of an electrified proof-plane held within the can?

12. Prove the truth of Rule VI, by considering the work done in taking a unit charge from the earth to the conductor and applying Rule III.

13. An electroscope is placed well inside a charged insulated vessel. It is then connected by a wire with a second electroscope placed outside at some distance. Describe the effects on each electroscope, giving the charge and potential diagrams.

14. In the same arrangement (Ques. 13), what will be the effect produced on each electroscope if that outside the can is joined to earth?

15. The stem of an electroscope projects some distance above the conducting case of the instrument. If the whole is insulated and a charged rod brought near the case, the leaves slightly diverge. Explain this.

16. An electroscope is insulated. What happens if the stem is earthed whilst an electrified rod is held near? Explain fully.

17. A gold-leaf electroscope stands inside a large metal jar and is insulated from it. The jar is insulated and charged positively. A conducting bell, large enough to cover the electroscope, is lowered into the jar by a wire held in the hand until it covers but does not touch the electroscope. Describe the effect on the leaves, and give the charge and potential diagrams supposing the bell does not touch the jar.

18. Starting with the arrangement described in Art. 170, Example 4, what is the effect of joining A and C with a wire, and then connecting B to earth?

19. How should a gold-leaf electroscope be constructed for measuring very small charges of electricity? Describe and explain the action of the condensing electroscope for measuring small differences of potential. (1905.)

[For condensing electroscope see Chap. XV.]

CHAPTER XIII

CONDENSERS—SPECIFIC INDUCTIVE CAPACITY

171. The arrangement known as a *condenser* plays an important part in electricity, and it will be necessary to consider its properties in detail. We shall first study the form of condenser most commonly used in frictional electricity—the Leyden jar.

Leyden Jar.—This generally consists of a jar of thin, colourless glass, coated on the inner and outer surfaces with tinfoil nearly to the shoulder. A bung of baked wood carries a vertical brass rod, ending in a knob above, and having a chain attached to the lower end. The chain makes connection with the inner coating. The conducting portions are shown, in section, in the figures.

The jar possesses virtually four conducting surfaces, and we shall find it convenient to refer to these by distinguishing letters. Thus we shall term the knob, stem, and inner surface

of the inner coating, A; the outer surface of the inner coating, B; the inner surface of the outer coating, C; and the outer surface, D. The inner surface of the inner coating carries no appreciable charge, since it forms a hollow conductor. "A" will therefore denote more particularly the knob and upper portion of the stem. The jar is said to be charged with + when the surface B is +. It is said to be insulated when both coatings are insulated, and earthed when *one* coating is earthed.

172. Properties of a Leyden Jar.

The most important points in connection with a condenser may be examined as follows:—

(1) Insulate the jar on a thick block of wax, and proceed to charge it as fully as possible from an electrophorus (or electrical machine). After three or four sparks the charging will cease, just as with a simple conductor of about the same size. Fig. 101 (*a*).

Explanation.—The + charge transferred to the inner coating collects almost entirely on B. It induces equal − and + charges on C and D respectively. The − charge on C tends to lower the pressure of the inner coating, but this effect is neutralized by the + charge on D. Hence the potential of the inner coating is about the same (for a given charge) as it would be if the outer coat were removed.

(2) Next remove the wax and place the jar on the table to earth-connect D. It will be found now that a torrent of sparks can be made to pass between the prime conductor of the machine and the knob of the jar. After some time the sparks cease and the charging is completed (fig. 102). If the jar is now discharged with tongs, a very bright spark and more or less violent report occur.

Explanation.—When the + escapes from D the − left on C lowers the pressure of the inner coating. The latter is thus brought to a lower pressure than the prime conductor, and more electricity passes to the jar. This induces a further − charge on C and + on D, the latter escaping to earth. The increased − charge on C still holds down the potential of the inner coat. This process continues until, in spite of the large − charge on C, the inner coat acquires the potential of the prime conductor. (More strictly the

potential of A is somewhat below that of the prime conductor, owing to the P.D. necessary to break across the spark gap.)

It is evident, therefore, both from theory and experiment,

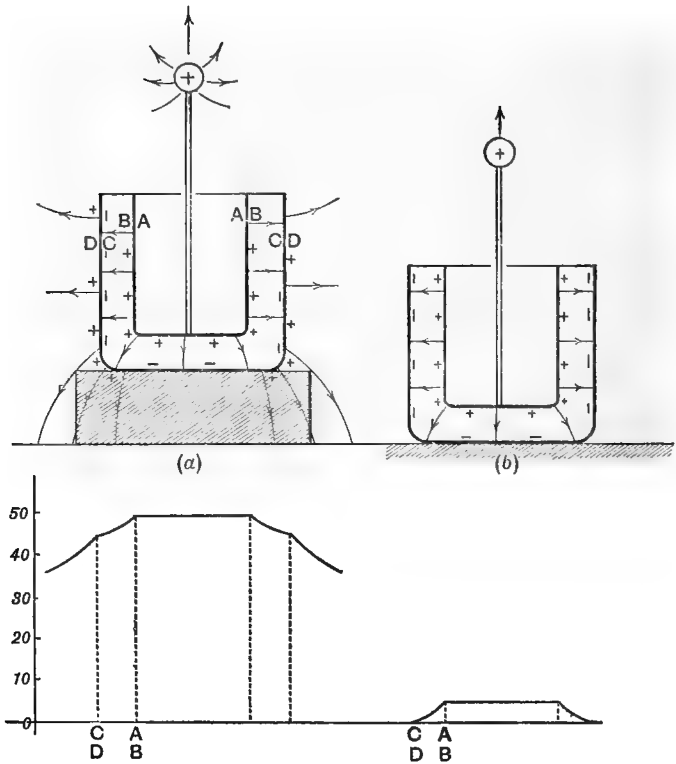
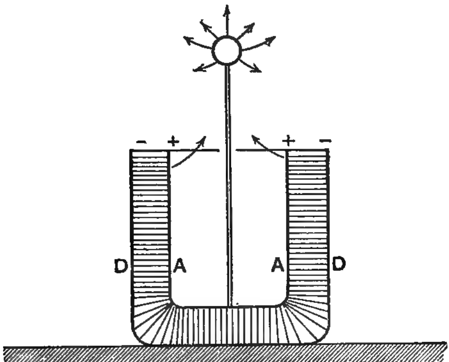


Fig. 101

that the capacity of the inner coating is much increased by earthing the outer coating. Similarly, if we earth the inner coat, the capacity of the insulated outer coat is much increased.

Fig. 101 (*b*) shows the condition of the Leyden jar after it

has been charged as in (1), and then placed on the table to earth the outer coat, but before it receives a further charge from the machine.



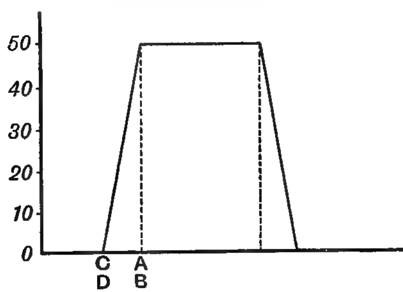


Fig. 102

173. Potential-Difference and Capacity.

An important property of the Leyden jar is that the field in the narrow space between the coatings (*i.e.* in the glass) is cut off from external influence. The shape of the tubes in this space is unaffected by exterior charges or earth-connections. The density may vary but the *distribution* of the tubes is unaltered. (Compare figs. 101, 102.) It follows from this, that—

(1) the density is proportional to the charge;
(2) the electric force is proportional to the charge, for $F = 4\pi\sigma$;
(3) the potential-difference of the coatings is proportional to the charge, for $V = F \times d$.

The charges on the surfaces B and C are always equal and opposite.

In consequence of (3) the capacity of the Leyden jar is defined as—

$$\frac{\text{Positive charge on B (or C)}}{\text{Potential-}\textit{difference}\text{ between the coatings}}.$$

This must be carefully distinguished from the capacities of the inner and the outer coats treated as *separate* conductors. For example, let us consider the experiment described above. (i) If the first charge Q brings the inner coat of the insulated jar to potential, say 50, then it is evident from the lines of force that the potential of the outer coat will be a little less, say 45, fig. 101 (*a*). (ii) When the jar is placed on the table the lines of force (in the glass) are unaltered, and therefore there is the same potential *difference* (fig. 101 (*b*)). (iii) But when the inner coat is again brought to the potential of the machine there is ten times the P.D., and therefore, by (3) above, the charge is = 10Q (fig. 102).

Thus in the three cases we have—

Capacities of inner coating,

$$\text{(i) } \frac{Q}{50}, \text{ (ii) } \frac{Q}{5}, \text{ (iii) } \frac{10Q}{50}.$$

Capacities of Leyden jar,

$$\text{(i) } \frac{Q}{50 - 45}, \text{ (ii) } \frac{Q}{5 - 0}, \text{ (iii) } \frac{10Q}{50 - 0}.$$

Observe carefully that whilst the capacity of one of the coatings varies with the earth-connection of the other, the capacity of the Leyden jar, *i.e.* of the *space between* the coatings, is unaffected.

174. Condensers.

A Leyden jar is spoken of as an *electric "condenser"*, because, when it is suitably connected to a source of electrification, the tubes of induction collect densely in the space between the coatings. The term, however, has now acquired a somewhat wider meaning, being applied to any pair of conductors enclosing a space which is protected from the disturbing influence of external bodies, especially the earth.

> **A condenser, or leyden, consists of two conductors, so arranged that the distribution of an electric field between (and terminating on) portions of their surfaces is uninfluenced by the earth-connection of either.**

Distribution here refers to the shape of the lines of force (not their direction).

If a Leyden jar, with its two coatings joined together, is insulated, a charge given to the coatings will distribute itself on the surfaces A and D only. These are conveniently termed the *free surfaces*, to distinguish them from what are more properly termed the *condenser surfaces*, namely, B and C.

The **capacity of a condenser** is the ratio of the charge on the condenser surfaces to their potential-difference.

Condensers used in practice may be divided into two classes.

(1) **True Condensers.**—This term may be applied when the distance between the conductors is everywhere small compared with all their other linear dimensions; *i.e.* when the free surfaces have capacities small compared with the condenser capacity.

(2) **Enclosed Systems.**—An enclosed system (or quasi-condenser) consists of a hollow conductor enclosing an insulated conductor of any form.

Two sheets of tinfoil pasted on opposite sides of a sheet of glass form a simple condenser of the first class known as a *Franklin's pane.* Fig. 103 shows an example of the second

class, known as a *spherical* condenser. A *cable* is another example; the outer surface of the core and the inner surface of the sheath, with the dielectric between them, form the condenser proper.

In applying the formula $S = \frac{Q}{V}$ to simple conductors, we must remember that V is the potential-difference between the *conductor and the earth* (or an infinitely distant point). In dealing with condensers of whatever form, the same formula applies, but this time V means the potential-difference between the *two coatings or terminals*, without any reference to the earth.

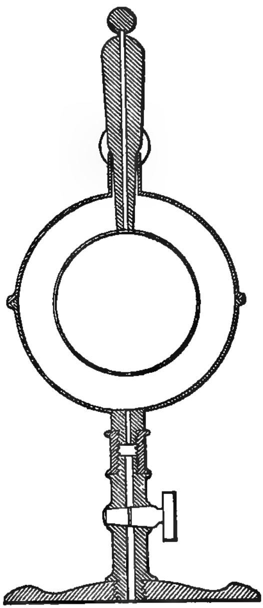


Fig. 103.—Faraday's Spherical Leyden

The capacity of a simple conductor was seen (Art. 159) to be subject to variation from a number of disturbing influences. The capacity of a condenser is fixed and depends on (1) its size and shape, and (2) the nature of the dielectric medium. We shall proceed to consider the effect of the dielectric on the capacity of a condenser.

175. Specific Inductive Capacity.

The first systematic quantitative experiments on dielectrics were made by Faraday. The principle of Faraday's method was as follows. Two spherical condensers of the form shown in fig. 103 and of the same size were used. One (A) contained air, and the other (B) the material under test. The air condenser was charged, and the surface density on its knob was tested. The two coatings of the air condenser were then placed in connection with the corresponding coatings of the other condenser (fig. 104). The

charge was thus shared between the two condensers. The surface density on the knob of A was again tested. It was found to have less than half the previous value, showing that the condenser containing the solid or liquid dielectric had received the greater share of the charge.

It was found that the ratio of surface densities on the knob (which is the same as the ratio of potentials) was always constant for a given medium. It follows that the substitution of

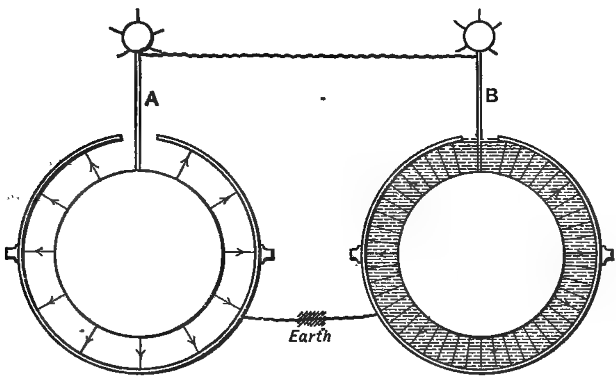


Fig. 104.—Faraday's Experiment on Specific Inductive Capacity

a given dielectric for air in a condenser increases the capacity in a definite ratio. This may be proved as below.

Let S_1 be the capacity of the air condenser, V its initial potential difference, and Q its charge. Thus—

$$Q = S_1V.$$

Let v be the common P.D. after the connection, q_1 the share of charge retained by the air condenser, q_2 the portion received by the other condenser (capacity S_2). Then—

$$q_1 = S_1v,\ q_2 = S_2v.$$

But $$Q = q_1 + q_2.$$

Therefore $$S_1V = S_1v + S_2v,$$

$$\text{or } \frac{S_2}{S_1} = \left(\frac{V}{v} - 1\right),$$

and by the experimental result stated above the right-hand member is constant.

This constant of a dielectric material is termed its "specific inductive capacity" (symbol K).

> **The specific inductive capacity of a dielectric is the ratio of the capacity of a condenser filled with the dielectric to the capacity of an air condenser of the same size and form.**

Faraday found the specific inductive capacity of several materials. He obtained the value 2·24 for sulphur and 1·5 for shellac.

The value of K for gases is practically the same as for air and a vacuum. Composite substances like ebonite and glass vary according to composition. K for ebonite varies between 2 and 3; for flint glass, between 6 and 10.

In Faraday's experiments only one-half of the condenser was filled with dielectric, and allowance had to be made for this. The method of comparing potentials by surface densities is not experimentally accurate, and is no longer used. An electrometer is now always employed for this purpose. The accurate determination of K is a matter of some difficulty, and will be dealt with in Chap. XVI.

It follows from the definition of K, that if S_2 is the capacity of a condenser *filled* with the dielectric, and S_1 the capacity of the corresponding air condenser,

$$K = \frac{S_2}{S_1}, \text{ or } S_2 = KS_1.$$

176. **Calculation of Condenser Capacities.**

(1) *True Condensers* (Franklin's pane, Leyden jar, etc.).

Let a = area of *one* coating,
t = thickness of dielectric,
σ = surface density of charge,
V = potential difference of coatings,
and F = electric force between coatings.

If the dielectric were air, we should have—

$$V = F \times t,$$
$$\text{also } F = 4\pi\sigma,$$
$$\text{and } Q = a\sigma.$$
$$\text{Thus } S_1 = \frac{Q}{V} = \frac{a\sigma}{4\pi\sigma \times t} = \frac{a}{4\pi t}.$$

Hence if glass or other dielectric is used—

$$S_2 = KS_1,$$
$$\therefore S_2 = \frac{aK}{4\pi t} \qquad \text{(1)}$$

(2) *Spherical Leyden* (fig. 103).—Let r and R be the radii of the outer surface B of the enclosed sphere and the inner surface C of the shell, respectively. Let Q and $-Q$ be the charges on these surfaces, and V the potential-difference.

$$\text{Potential of B due to } Q = \frac{Q}{r}.$$
$$\text{,, \quad C \quad ,,} \quad = \frac{Q}{R}.$$

The negative charge $-Q$ on the surface C produces potential $-Q/R$ at this surface, and therefore at all points included by it (Art. 166 V).

Therefore the potential of B,

$$V_B = \frac{Q}{r} - \frac{Q}{R}.$$

Also, the potential of C,

$$V_C = \frac{Q}{R} - \frac{Q}{R} = 0.$$

If there are exterior charges the potentials V_B and V_C will be altered, but their difference will be unaffected.

$$\text{Thus } V_B - V_C = \frac{Q}{r} - \frac{Q}{R} = Q\frac{R-r}{Rr}.$$
$$S_1 = \frac{Q}{V_B - V_C} = \frac{rR}{R-r}.$$

For dielectrics other than air we must write—

$$S_2 = \frac{rR}{R-r} \cdot K \qquad \text{(2)}$$

(3) *Cylindrical Leyden* (*e.g.* a cable). The capacity depends on the *ratio* of the diameters of core and sheath. It may be shown that—

$$S = \frac{lK}{2 \log_\epsilon \frac{R}{r}} \quad \text{.......................(3)}$$

EXAMPLES.—1. Each side of a Franklin's pane is coated with a sheet of tin-foil 25 cm. square. The glass is 5 mm. thick, and has S.I.C. = 8. Find the capacity of the condenser.

$$S = \frac{aK}{4\pi t} = \frac{625 \times 8}{12{\cdot}57 \times {\cdot}5}$$
$$= \frac{10{,}000}{12{\cdot}57} = 795 \text{ electrostatic units.}$$

2. Find the capacity of a spherical condenser formed of an inner sphere 10 cm. diameter and a spherical shell 1 cm. thick and 15 cm. external diameter, the intervening space being filled with paraffin (inductive capacity = 2).

$$R = 6{\cdot}5 \text{ cm.} \qquad r = 5 \text{ cm.}$$
$$S = \frac{6{\cdot}5 \times 5}{1{\cdot}5} \times 2 = 43\tfrac{1}{3} \text{ electrostatic units.}$$

3. Find the capacity of 1 kilometre (about $\frac{5}{8}$ mile) of cable, radii of core and sheath 2 mm. and 5 mm. S.I.C. of insulation = 5.

$$S = \frac{lK}{2 \log_\epsilon \frac{R}{r}} = \frac{100{,}000 \times 5}{2 \log_\epsilon \frac{5}{2}}.$$

$$\log_{10} \frac{5}{2} = {\cdot}3979.$$

$$\log_\epsilon \frac{5}{2} = {\cdot}3979 \times 2{\cdot}3026.$$

$$\therefore S = 2{\cdot}73 \times 10^5 \text{ electrostatic units.}$$

177. Further Study of the Leyden Jar.

The condenser proper in a Leyden jar or enclosed system consists of the surfaces which we have marked B and C, with the dielectric between them. The knob (A) and the outermost surface (D) are not considered in the calculation of the *condenser* capacity, and must be regarded as *simple conductors* connected with the condenser coatings. We have already

referred to A and D as the "free" surfaces of the system. The two following points must be clearly kept in mind:—

(1) The potential-*difference* of the coatings depends only on the charges on the condenser surfaces B and C and the capacity of the *condenser*.

(2) The *separate potentials* of the coatings (with respect to the earth) depend on the charges on the free surfaces as well, and external influences, if any exist.

These statements follow from Rules VI, VII, Art. 166. We shall proceed to some numerical illustrations, representing the Leyden jar by a spherical condenser.

1. Let the inner sphere have radius 14 cm. and outer sphere have internal radius 15 cm. Also let the thickness of the outer sphere be 1 cm. If the condenser is insulated and then charged with 320 units of positive electricity on its inner coat, what are the potentials of the two conductors?

A negative charge −320 is induced on the inner surface of shell, and positive charge 320 on its outer surface.

The capacity of the condenser

$$\frac{15 \times 14}{15 - 14} = 210.$$

The capacity of the outermost surface (considered as a simple conductor) = 16.

∴ the potential-difference of the conductors is—

$$\frac{320}{210} \text{ or } 1\tfrac{11}{21} \text{ units.}$$

The potential of the outer "free" surface with respect to the earth is—

$$\frac{320}{16} \text{ or } 20 \text{ units.}$$

The potential of the inner conductor is therefore—

$$20 + 1\tfrac{11}{21} \text{ or } 21\tfrac{11}{21} \text{ units.}$$

Notice that the same charge which produces a potential of 20 on the free surface only produces a P.D. of 1·52 between the condenser surfaces. This is due to the relatively large capacity of the condenser portion.

2. Let the same condenser be insulated, and let a charge of 100 units be given to the outer conductor. The circumstances are now like those considered in Art. 168. (The inner sphere corresponds to the ball and the outer sphere to the can.) There is no charge in the condenser, and the whole arrangement is raised to a potential—

$$\frac{100}{16} \text{ or } 6\tfrac{1}{4}.$$

Now let the inner conductor be connected to earth. It acquires a negative charge exactly as described in Art. 168, and some of the charge is drawn from the outer free surface to the inner surface. The condenser thus becomes charged. Let $-q$ and q be the charges on the condenser surfaces; then $100-q$ is the charge left on the free surface.

The potential-difference in the condenser is (charge $\div$ capacity) or $\frac{q}{210}$, and there is a fall of potential from the outer to the inner conductor.

But the potential of the inner conductor is zero. Hence the potential of the outer is $\frac{q}{210}$.

Again, the potential of the outer free surface is $\frac{100-q}{16}$.

$$\therefore \frac{q}{210} = \frac{100-q}{16}.$$

$$\therefore q = 93 \text{ nearly}.$$

Thus 93 per cent of the charge goes to the condenser surfaces and produces there a potential-difference ·44, whilst the remaining 7 per cent produces an equal potential at the free surface.

This example illustrates the process by which a jar becomes charged when the knob is held in the hand and the outer coating is brought to the prime conductor of a machine.

If the two coatings of the condenser charged as above are connected, the charges $+93$ and -93 on the condenser surfaces neutralize each other, whilst (the whole jar remaining insulated) the charge $+7$ on the free surface remains.

178. Alternate Discharge.

Suppose that a Leyden jar is charged and then insulated. A fraction k of the charge is free on the knob. If the inner

conductor is then earthed, the free charge from the knob, and a fraction k_1Q (from the inner coat) which becomes free at the moment of earthing, escape. At the same time a fraction k_1 of the charge (Q) on the condenser becomes free on the outer conductor. If the inner conductor is now disconnected from earth and the outer one earthed instead, the free charge k_1Q, and some which becomes free at the moment of earthing (a fraction k of the *remaining* charge), escape. At the same time a portion of the charge on the inner conductor (a fraction k of the charge which remains) becomes free on the knob. Hence if the coatings are touched *alternately*, a constant fraction of the charge in the *condenser* escapes at every earthing of the + side, and another constant fraction of the charge escapes at every earthing of the – side. Hence in time the charge left in the jar will be reduced to a negligible quantity.

The following point, illustrated by the examples above, should be kept in mind:—

With true condensers the charge on a free surface necessary to bring it to a given potential above the earth is very small compared with the charge required in the condenser to produce the same P.D. between the coatings.

179. Connected Condensers.

Leyden jars and other condensers are frequently joined together in groups, the object being to vary conveniently the total capacity. There are two main groupings—

(*a*) the "battery" or "parallel" arrangement;
(*b*) the "cascade" or "series" arrangement.

(1) **Battery Arrangement.**—In this all the inner coats or knobs are joined, and similarly all the outer coats. Thus whatever the capacities of the separate jars,

all the jars have the same P.D.

Thus for three jars, if Q_1, Q_2, Q_3 are the charges, and S_1, S_2, S_3 the capacities—

$$Q_1 = S_1V;\ Q_2 = S_2V;\ \text{and } Q_3 = S_3V.$$
$$\therefore \text{ total charge } = (S_1 + S_2 + S_3)V.$$

The total capacity is the ratio of the total *condenser* charge to the P.D. of the terminals; therefore—

$$S = \frac{Q}{V} = \frac{(S_1 + S_2 + S_3)V}{V}.$$

$$S = S_1 + S_2 + S_3 \qquad \ldots (4)$$

The proof is evidently applicable to any number of condensers, and to enclosed systems as well as true condensers.

The total capacity of a number of condensers arranged in parallel is the sum of the separate capacities.

Leyden jars are connected in parallel by placing them in a box lined with tinfoil and joining the knobs by thick metal rods.

(2) **Cascade Arrangement.**—Here the jars are insulated and joined in series, that is, the outer coating of each jar is joined to the inner coating of the next. The terminals are joined to the knob of the first jar and the outer coating of the last. Suppose that a charge $+Q$ is sent into the first jar through A. This induces $-Q$ on the inside of the outer coat and $+Q$ on the outside. If the condensers belong to class (1) (see Art. 174), the repelled charge $+Q$ practically all goes into the second jar. The process of induction is repeated, and so on through the series. Hence, for a moderate number of *true* condensers arranged in cascade, whatever the capacities of the separate condensers, we may say that—

all the jars acquire the same charge.

This is the same as the charge Q sent into the inside of the first jar and expelled from the outside of the last.

For three condensers we may write—

$$Q = S_1V_1; \; Q = S_2V_2; \text{ and } Q = S_3V_3.$$

The potential difference between the terminals is the sum of the P.D.'s in the separate jars.

Thus—
$$V_A - V_B = V_1 + V_2 + V_3$$
$$= \frac{Q}{S_1} + \frac{Q}{S_2} + \frac{Q}{S_3}.$$
$$\therefore \frac{V_A - V_B}{Q} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3}.$$

The total capacity must be defined as the ratio of the total charge sent in at one terminal to the P.D. produced between the terminals. Thus—

$$S = \frac{Q}{V_A - V_B}.$$
$$\therefore \frac{1}{S} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} \quad \text{.................. (5)}$$

The reciprocal of the total capacity of a number of true condensers in cascade is the sum of the reciprocals of the separate capacities.

EXAMPLE.—Three leydens have capacities 2, $\frac{1}{3}$, $1\frac{1}{4}$. Find their total capacity when arranged (*a*) in battery, (*b*) in cascade.

$$(a)\ S = 2 + \tfrac{1}{3} + 1\tfrac{1}{4}$$
$$= 3\tfrac{7}{12}.$$
$$(b)\ \frac{1}{S} = \tfrac{1}{2} + \tfrac{3}{1} + \tfrac{4}{5}$$
$$= 4\tfrac{3}{10}$$
$$S = \tfrac{10}{43}.$$

180. Energy Stored in a Charged Condenser.

The energy of a system of conductors is given by—

$$W = \tfrac{1}{2}Q_1V_1 + \tfrac{1}{2}Q_2V_2 + \text{etc.} \quad \text{.......... (Art. 161)}$$

If the inner and outer coatings of a condenser are at potentials V_1, V_2, with total charges Q_1, Q_2, the energy

$$= \tfrac{1}{2}(Q_1V_1 + Q_2V_2).$$

To find the energy in the *condenser proper* we must suppose the free surfaces practically uncharged. Then—

$$Q_1 = -Q_2 \; (= Q, \text{ say}),$$

and

$$\begin{aligned} W &= \tfrac{1}{2}QV_1 - \tfrac{1}{2}QV_2 \\ &= \tfrac{1}{2}Q(V_1 - V_2) \\ &= \tfrac{1}{2}QV \quad \ldots\ldots\ldots\ldots (6) \end{aligned}$$

where V is the difference of potentials.

Substituting from $Q = SV$, we obtain—

$$W = \tfrac{1}{2}\frac{Q^2}{S} = \tfrac{1}{2}SV^2.$$

If a battery or cascade of equal jars is charged to a potential-difference V at its *terminals*, we may express the energy in terms of that stored in a single jar charged to the same potential-difference. Suppose that a charge Q in a single jar produces a potential-difference V.

(1) Battery arrangement, n leydens.

$$\begin{aligned} \text{Potential-difference} &= V. \\ \therefore \text{ the charge in each jar} &= Q. \\ \therefore \text{ energy in each} &= \tfrac{1}{2}QV. \\ \text{Total energy} &= n \times \tfrac{1}{2}QV \\ &= n \text{ times the energy in a single jar.} \end{aligned}$$

(2) Cascade arrangement, n equal jars.

Potential-difference in each jar $= \dfrac{V}{n}$.

$\therefore$ charge in each $= \dfrac{Q}{n}$.

$\therefore$ energy in each $= \tfrac{1}{2}\dfrac{QV}{n^2}$.

$$\begin{aligned} \therefore \text{ total energy} &= n\,\tfrac{1}{2}\frac{QV}{n^2} \\ &= \frac{1}{n}\text{th of the energy stored in a single jar.} \end{aligned}$$

181. Experimental Tests.

The heat generated in the discharge circuit is only a portion of the energy of discharge. The remainder is dissipated in the form of light, sound, and electro-magnetic waves.

Experiments showing that the heat is roughly proportional to the total energy of discharge have been made with the apparatus shown in fig. 105, termed a "Riess calorimeter".

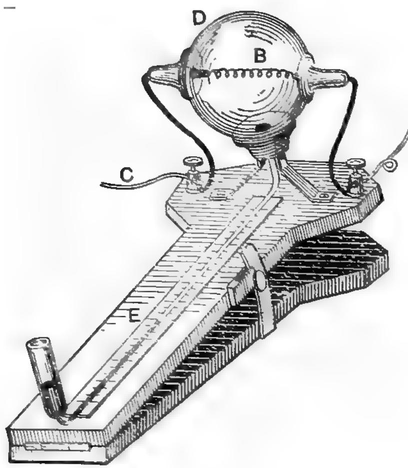


Fig. 105.—Riess Calorimeter

A wire B is fitted air-tight through the bulb D of a large air-thermoscope. The movement of the column of liquid E is proportional to the expansion of the air and (roughly) to the heat generated. By discharging one, two, three, or more equal jars (in parallel) through the wire, Riess was able to show the proportionality between W and H.

182. Enclosed Systems in Cascade.

The condition that the cascade formula may hold is that each jar shall receive the same charge. This condition is practically fulfilled when there is a moderate number of true condensers, but, in general, not when enclosed systems (quasi-

condensers, where there is considerable thickness of dielectric) are arranged in cascade.

EXAMPLE.—Two equal spherical air leydens have each radii 4 and 6 cm., the outer shells being of negligible thickness. They are joined in cascade with the outer coat of the second to earth. If the inner sphere of the first is charged to potential 30, find the

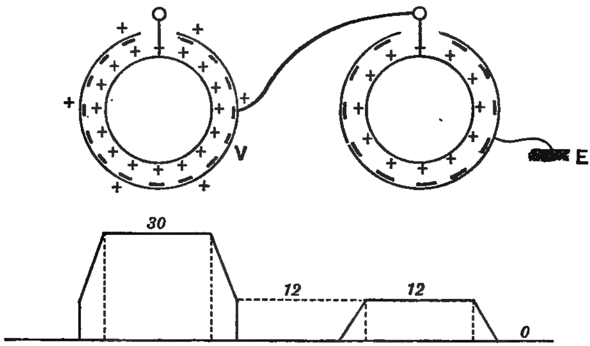


Fig. 106.—Quasi-condensers in Cascade

charges on the six surfaces and the potentials of the four conductors (fig. 106).

Let V be the potential of the outer coat of the first condenser; then the required potentials are 30, V, V, 0.

Capacity of each condenser $= \dfrac{6 \times 4}{6 - 4} = 12.$

Capacity of first outer surface $= 6$.

Charge in the first condenser $= Q_1 = 12 \times (30 - V)$.

Charge in the second condenser $= Q_2 = 12 \times (V - 0)$.

Let q be the charge retained on the free outer surface of the first condenser.

Then $$\frac{q}{6} = V.$$

$$\therefore q = 6V.$$

But $$q + Q_2 = Q_1.$$

$$\therefore 6V + 12V = 360 - 12V,$$

or, $$V = 12.$$

Hence $$Q_1 = 360 - 144 = 216.$$

Also $$Q_2 = 144,$$

and $$q = 72.$$

Thus the charges on the six surfaces are respectively

$$+216, \ -216, \ +72, \ +144, \ -144, \ 0;$$

and the potentials

$$30, \ 12, \ 12, \ 0.$$

The capacity of such a system cannot, *in general*, be defined or calculated in the same manner as the capacity of a system of "true" condensers. Exceptions occur, however, when the intermediate free surfaces are earthed and the terminal surfaces are insulated. Thus, if two parallel cables have their cores insulated at one end and connected to the terminals of an insulated battery at the other, the ordinary cascade formula applies. The free surfaces of the sheaths are earthed (or otherwise joined), and are therefore discharged. The Leyden jars attached to a Wimshurst machine are connected in cascade on the same principle.

183. Examples.

1. A Leyden jar standing on an insulated stool is electrified by a machine, while its outer coating is touched by the knob of an exactly similar Leyden jar held in the hand. The first jar being now disconnected from the machine, it is taken in hand, either (1) by its outer coating and presented with its knob to that of the second jar, or (2) by its knob and presented with its outer coating to the knob of the second jar. Does a spark pass in either case? Explain the action. (1896.)

We shall assume the glass so thin that the jars act as true condensers (Art. 174).

The jars become charged in cascade. If v is the potential of the machine, $\frac{v}{2}$ is the potential-difference of the coatings of the first jar; $\frac{v}{2}$ is also the potential-difference for the second jar. The four coatings in order, from the machine end, have potentials

$$v, \ \frac{v}{2}, \ \frac{v}{2}, \ 0.$$

(i) If the first jar is taken in the hand by its outer coating, *the small free charge on its outermost surface goes to earth.* Both

coatings drop in potential by the amount $\frac{v}{2}$. The four potentials are now

$$\frac{v}{2},\ 0,\ \frac{v}{2},\ 0.$$

The knobs therefore come to practically the same potential $\frac{1}{2}v$, and no appreciable spark occurs.

(ii) If the first jar is taken in the hand by its knob, the small free charge goes from the knob to earth, together with a little newly freed charge from the interior. The escape of the latter reduces the + charge on the surface touching the glass. Thus a little negative will be liberated on the outer coating and will go to the outside free surface of the jar. This lowers the potential of the outer coat to nearly $-\frac{v}{2}$, whilst the potential of the inner coat is made zero. On bringing the parts in contact as stated, it is evident that the charges on the coatings connected will nearly neutralize each other, and the system becomes practically discharged. A spark passes in this case.

2. An insulated sphere of 2 cm. radius is connected by a long thin wire with another insulated sphere, the radius of which is 6 cm. and which is surrounded by a third sphere of 8 cm. radius concentric with it. The wire which connects the first and second spheres passes through a small hole in the third so as not to touch it. All the spheres are conductors. Calculate the capacity of the two connected spheres. (1896.)

(i) Let the third sphere be insulated. If its thickness is negligible the charges induced on its inner and outer surfaces neutralize each other's effects on the potential of the enclosed sphere. The capacity of the enclosed sphere is the same as if the thin shell were removed, *i.e.* $S = 6$.

Thus the total capacity of the connected spheres is $(2 + 6)$, or 8 C.G.S. units (centimetres).

(ii) Let the third sphere be earthed. The capacity of the enclosed sphere is then the same as the capacity of the spherical condenser, namely,

$$\frac{6 \times 8}{8 - 6}.$$

Thus the total capacity is $24 + 2$, or 26 C.G.S. units (centimetres).

3. Within a spherical vessel of brass 1 cm. thick, the external

diameter of which is 14 cm., a brass ball 8 cm. in diameter is hung by a silk thread so that the centres of the two spheres coincide. If the ball is charged with 36 units of positive electricity, and if the potential of the vessel is 7, what is the potential of the ball? (1897.)

(i) We know the dimensions of the vessel and therefore its capacity. We also know the charge on the *condenser* surfaces. Hence we can find the potential-*difference* of the coatings.

$$S = \frac{Rr}{R - r} = \frac{6 \times 4}{6 - 4} = 12.$$

$$V = \frac{Q}{S} = \frac{36}{12} = 3 \text{ (potential-difference)}.$$

(ii) We do not know the charge on the *free* (or outermost) surface. The potential of the walls of a hollow conductor depends on exterior charges (Art. 166), but in the present instance we are not told what the charges are—only that they produce potential 7, which is all we require to know to complete the calculation. Thus—

Potential-difference	= rise of 3 (outer to inner).
Potential of outer conductor	= 7.
∴ potential of inner conductor	= 7 + 3
	= 10.

4. A Franklin's pane is formed of an ebonite sheet $\frac{1}{2}$ cm. thick, coated on each side with a tin-foil sheet 20 cm. square. What must be the thickness of a similar pane formed of glass coated with tin-foil 30 cm. square in order that the two condensers may store equal energies for equal charges? The specific inductive capacities of glass and ebonite are as 5 : 2.

Since $W = \frac{1}{2}\frac{Q^2}{S}$, the condensers will only store equal energies for equal charges when the capacities are equal.

Thus—

$$\frac{a_1K_1}{4\pi t_1} = \frac{a_2K_2}{4\pi t_2}.$$

$$t_2 = \frac{a_2K_2t_1}{a_1K_1}.$$

$$= \frac{900 \times 5 \times 1}{400 \times 2 \times 2} = 2\tfrac{13}{16} \text{ cm.}$$

5. A Leyden jar consists of two concentric spherical surfaces of 5 and 6 cm. diameter respectively, the intervening space being

filled with air. The outer sphere is uninsulated and the inner is charged with 20 units of electricity. How much work is done when the inner sphere is put to earth? (1895.)

$$\text{Capacity of the condenser} = \frac{3 \times 2\frac{1}{2}}{3 - 2\frac{1}{2}} = 15.$$

$$W = \frac{1}{2}\frac{Q^2}{S} = \frac{1}{2} \cdot \frac{400}{15} = 13\tfrac{1}{3} \text{ ergs.}$$

This is the work done by the electric forces during the discharge.

6. A Leyden jar A, of capacity 3, is insulated, and the outer coating is connected by a wire with the inner coating of another Leyden jar B, of capacity 2, the outer coating of which is uninsulated. If the inner coating of A be charged so that the potential is V, what is the potential of the inner coating of B? (1899.)

Let V_1 be the required potential.

Then in first jar $Q = S \times (\text{potential difference})$
$= 3 \times (V - V_1)$.

Also in the second jar $Q = 2 \times (V_1 - 0)$.

$\therefore\ 2V_1 = 3V - 3V_1$

$V_1 = \frac{3}{5}V$.

EXERCISES

1. A sphere of radius 40 mm. is surrounded by a concentric sphere of radius 42 mm., the space between the two being filled with air. What is the relation between the capacity of this system and that of another similar system in which the radii of the spheres are 50 and 52 mm. respectively, and the space between them is filled with paraffin of specific inductive capacity 2·5? (1892.)

2. Two Leyden jars are exactly alike, except that in one the tin-foil coatings are separated by glass and in the other by ebonite. A charge of electricity is given to the glass jar, and the potential of its inner coating is measured. The charge is then shared between the two jars, and the potential falls to 0·6 of its former value. If the S.I.C. of ebonite be 2, what is that of glass? (1893.)

3. The inner coating of one spherical Leyden jar, whose surfaces have radii 12 and 14 respectively, is charged with 25 units of positive electricity, and the inner coating of another, with surfaces of radii 8 and 12, is charged with 5 positive units, the outer coatings of both being earth-connected. The inner coatings are then momentarily joined by a fine wire; in which direction will electricity

pass, the dielectric in both jars being air and the distance between the jars considerable? Give full reasons for your answer. (1894.)

4. Two condensers are exactly alike, except that one has air and the other glass for the dielectric. Equal charges are given to the two condensers. In which is the energy of the charge the greater? (1897.)

5. A conducting sphere, of diameter 6, is electrified with 105 units; it is then enclosed concentrically within an insulated and unelectrified hollow conducting sphere formed of two hemispheres, of thickness $\frac{1}{4}$ and internal diameter 7. The outer sphere is then put to earth. Determine the potential of the inner sphere before and after the outer sphere is earth-connected. (1899.)

6. What is meant by charging Leyden jars in cascade? Three Leyden jars whose capacities are $\frac{1}{2}$, 1, $1\frac{1}{2}$ are arranged in cascade. What is the capacity of the combination? (1900.)

7. If the charge sent into the first jar of the cascade arrangement mentioned in Question (6) is 3 units, what are the potentials of the coatings taken in order, when the outer coating of the last jar is connected to earth? Draw the potential graph.

8. Two Leyden jars are charged with quantities of electricity in the ratio 2 : 3. If in the jar which receives the larger charge, the tin-foil surface is twice as great and the glass twice as thick as in the other, compare the quantities of heat produced by discharging them. (1903.)

9. A brass ball 7 centimetres in radius is suspended concentrically inside a spherical brass vessel of internal radius 9 centimetres and external radius 10 centimetres. If the charge on the ball is 56 units and the potential of the outer vessel 5, what is the potential of the ball? (1904.)

10. Show that the energy in a charged condenser is equal to $\frac{1}{2}QV$, when Q is the charge on one plate and V the potential difference between the plates. (1905.)

11. What is meant by the *capacity of a condenser*? Calculate the capacity of a parallel plate air condenser of which each plate has an area 400 square centimetres, the distance between the plates being half a millimetre. Be careful to state the unit in which you express your answer. (1906.)

12. A sphere, radius 15, is charged with 300 units of positive electricity. It is then joined with the inner coating of a spherical condenser, the outer coating of which is to earth. The radius of the inner sphere is 5 cm. The external radius of the outer sphere

is 7 cm., and it is 1 cm. thick. Find the charge acquired by the condenser.

13. Compare the capacity of a spherical condenser, radii 5 and 15, with that of the outer shell. (The shell is of negligible thickness.) What is the condition that the capacity of a spherical condenser may be less than that of its outer conductor, supposing the latter thin?

14. Three equal spherical air leydens have radii 4 and 6 cm., the outer shells being of negligible thickness. They are joined in cascade, and the last shell is put to earth. If a charge of 60 units is given to the first inner conductor, find the charges on the nine surfaces, and the potentials of the six conductors.

15. Two Leyden jars, each of capacity F, are separately insulated, and the outer coating of the first is connected to the inner of the second, the outer of the second being earthed. If the terminals of a battery of e.m.f. E are connected to the inner and outer coatings of the first jar, with what charges and to what potentials will the jars be charged? (1904.)

16. A and B are two similar condensers, but the dielectric between the plates of A is air, and that between the plates of B is paraffin of inductive capacity 2. Both are insulated and are connected in cascade or series, the inner coating of A and the outer of B being connected to the terminals of an electric machine. Calculate the ratio of the energies of the charges possessed by the two condensers. (1903.)

17. Three similar Leyden jars, A, B, C, are connected in series, the inner coating of A to the outer of B, the inner of B to the outer of C, the inner of C to the outer of A. The terminals of an electrical machine are connected to the inner coatings of A and C. The jars are charged and disconnected. Compare the energies of the charges in each. (1902.)

CHAPTER XIV

MECHANICAL FORCE—ELECTROSTATIC MEASUREMENTS

184. Mechanical Force Exerted on Conductors.

The attraction and repulsion observed between charged bodies can be explained by supposing that the tubes of induction tend to contract lengthwise and expand sideways. In

many cases this method is more useful than that of considering the component forces produced by the point charges. We may, for example, suppose that the leaves of a gold-leaf electroscope are pulled apart by the action of the tubes, instead of supposing that they are repelled apart by the action of the like charges on the leaves. We proceed to calculate the force exerted on unit area of the surface of a conductor, and we shall give a proof according to each of the methods just mentioned.

Tension in Resultant Tubes of Induction.

I. *Calculation by Properties of Resultant Tubes.*

Let A, B be two parallel discs placed a short distance apart in air. Let A be electrified with + and B earthed. The field between the plates is uniform (except just near the edges). Very little charge collects on the outer surfaces of the plates.

The total energy of the plate condenser so formed is given by—

$$W = \tfrac{1}{2}QV.$$

If d is the distance apart of the plates, and F the electric force between them—

$$V = F \times d.$$

Also, if σ is the surface density of the charge, and a the area of each plate—

$$F = 4\pi\sigma, \quad Q = \sigma a.$$

$$\therefore W = \tfrac{1}{2}(\sigma a)(4\pi\sigma d)$$
$$= 2\pi\sigma^2 ad.$$

Let the plate A be moved towards B through a short distance x. The tension in the tubes helps the motion, and the work done by the tubes—

$$= f \times x,$$

where f is the mechanical pull which the tubes exert on the plate. The energy remaining is—

$$2\pi\sigma^2 a \times (d - x).$$

By the principle of the conservation of energy, the work done by the tubes is equal to the energy which they lose.

$$\therefore f \times x = 2\pi\sigma^2 ad - 2\pi\sigma^2 a(d - x),$$
$$f = 2\pi\sigma^2 a.$$

Since the pull per unit area is required, we have—

$$\frac{f}{a} = 2\pi\sigma^2 \quad \text{.................................(1)}$$

(This applies to any conductor, since the field is always of the same nature and properties.)

II. *By the Method of Component Forces.* (See also Art. 126.)

Let AB be a small thin metal disc laid on a charged conductor. The disc forms virtually a part of the surface, and the force exerted on it is equal to that which would be exerted on the surface which it covers, if it were removed. The charge on the disc ($= q$ say) cannot exert any force on itself. Thus the force exerted on the disc is due to the attractions and repulsions exerted on q by all the *other* charges in the field. Adopting the notation of Art. 126, we have—

$$f = F_2 q.$$

$$\text{But } F_2 = 2\pi\sigma, \text{ and } q = a\sigma.$$

$$\therefore f = 2\pi\sigma^2 a.$$

$$\therefore \frac{f}{a} = 2\pi\sigma^2, \text{ as before.}$$

In these formulæ distinguish carefully between "electric" and "mechanical" force at the surface of the conductor. The former (F) is the force exerted on a unit charge placed *just outside* the surface, and $= 4\pi\sigma$. The mechanical force (f) is the pull exerted on an element of the conductor, and its value per unit area is $2\pi\sigma^2$. The charge σ helps to produce the electric force outside the conductor (Art. 126), but exerts no mechanical force *on itself*; hence the factor 4 in one case and 2 in the other.

The first proof of Equation (1) given above shows that—

The energy per unit volume $= 2\pi\sigma^2$.

Note that the force per unit area or *electrostatic tension* is proportional to the *square* of σ. If the density is doubled the action in each tube is twice as intense, and there are twice as many unit tubes on unit area; thus the pull is made 4 times as great.

The above expressions must be modified if the dielectric is other than air.

EXAMPLES.—1. A metal plate 400 sq. cm. area is + charged and placed ·5 cm. from a parallel earth-connected plate. It is charged to potential 30. Find the total pull.

We have
$$\begin{aligned} V &= F \times d. \\ \therefore\ 30 &= F \times \cdot 5 \text{ and } F = 60. \\ \therefore\ 60 &= 4\pi\sigma \text{ or } \sigma = \frac{60}{4\pi}, \\ f &= 2\pi\sigma^2 a \\ &= 2\pi \times \frac{60^2}{16\pi^2} \times 400 = 56{,}000 \text{ dynes.} \end{aligned}$$

2. An earthed circular disc 5 cm. radius is attracted with a force of 200 dynes when placed at a distance of 7·5 mm. from a similar parallel charged disc. Find the P.D. between the two.

$$\text{Mechanical force per unit area} = \frac{f}{\pi r^2} = \frac{200}{\pi \times 25} = \frac{8}{\pi}.$$

$$\begin{aligned} \therefore\ 2\pi\sigma^2 &= \frac{8}{\pi}. \\ \therefore\ \pi\sigma &= 2 \text{ and } F = 4\pi\sigma = 8. \\ \text{P.D. required} &= F \times d \\ &= 8 \times \cdot 75 \\ &= 6 \text{ ergs per unit charge.} \end{aligned}$$

185. **Measurement of Potential-Difference.**

The last example indicates how we may measure a potential in e.s. units, by determining the force of attraction between two parallel metal plates. An instrument which enables us to measure potentials by means of the mechanical pull of the tubes of induction is called an **electrometer**. The following experiment illustrates the principle of the **attracted-disc** electrometer.

A disc AB is suspended from one beam of a balance and is earth-connected through the balance (shown diagrammatically in fig. 107). CD is an insulated disc connected to the conductor to be tested. CD thus acquires the same potential as the conductor. E is a device due to Lord Kelvin, and is known as the *guard-ring*. It consists of a large disc in which a hole is cut just large enough to permit of its being placed round AB. When

in position it lies in the same plane as AB, and is connected with this disc through the pillar of the balance. The object of the guard-ring is to keep the field between the plates uniform. The formulæ are worked out on the assumption that the field between the discs is uniform, but this condition is not satisfied near the edges of the discs; the tubes here tend to bulge outwards. The use of a guard-ring introduces a set of tubes tending to bulge in the opposite direction, and the field is thus made very uniform.

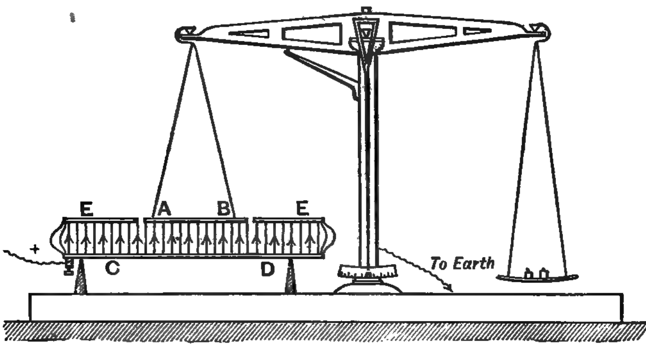


Fig. 107.—Electrostatic Balance

The guard-ring is separately supported and fixed in position so that when the attracted disc (AB) is in its plane, the pointer of the balance reads zero. The play of the balance-arm should be limited by stops.

In using the instrument, CD is first earthed and AB carefully counterpoised when neutral. CD is then charged to the potential to be tested, and the weight w necessary to bring AB to the normal or "sighted" position determined. Then we have—

$$\text{Total pull on AB} = w \text{ grams wt.}$$
$$= w \times 981 \text{ dynes.}$$

Also $$\text{total pull} = 2\pi\sigma^2 a = f.$$

Again, $$F = 4\pi\sigma,$$

and $$V = F \times d.$$

$$\therefore V = d.\sqrt{\frac{f \times 8\pi}{a}}.$$

Thus $$V = d\sqrt{\frac{8\pi \times w \times 981}{a}} \qquad \text{(2)}$$

The quantities to be measured are the distance between the discs (in centimetres), the area of AB (in square cm.), and w in grams.

In some instruments to be described presently, the force f is kept constant, and different potentials are measured by varying the position of the attracting disc, *i.e.* by varying d.

186. Calibration of a Gold-leaf Electroscope.

The electrostatic balance furnishes an instructive method of calibrating an ordinary gold-leaf electroscope or other simple form of electrometer. Fig. 108 shows the arrangement for

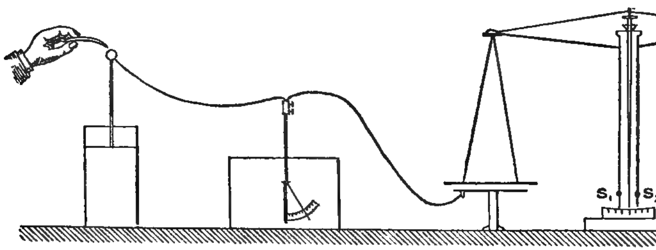


Fig. 108.—Calibration of Electroscope

this purpose. The upper disc is a light aluminium plate about 10 cm. diameter, and is earth-connected through its suspension. The lower disc (about 7 cm. diameter) is well insulated on an ebonite rod, or block of freshly pared paraffin wax. It is joined to the electroscope and a Leyden jar.

First counterpoise the suspended disc so that the pointer rests half-way between the stops. The distance between the discs should then be about 3 mm. Now charge the jar from a small electrophorus so that the leaves diverge widely and the pointer is pulled over to the stop S_2. Place a small weight in the pan, say ·5 gram. Next, by means of a dry thread, strip of cardboard, or other *partial* conductor, let down the charge in the jar until the pointer leaves S_2 and comes to the zero mark. With a little practice this can be readily done. Note the divergence of the leaves.

Now recharge the jar and repeat the experiment, with say ·4 gram; and similarly with other weights.

The relative potentials can then be obtained from Equation (2).

Care must be taken to obtain exactly the same balancing position each time. An alternative method is to *note the divergence of the leaves at the moment the pointer leaves the stop S_2*, the initial balancing of the disc being made for this position also. (This method is preferable if the potentials are fairly high.) If the stops are at s_1, s_2, the pointer of the balance must be rigid; a better position is under the beam of the balance, or under the edge of the attracted disc. For absolute measurement a guard-ring must be used.

The following are the results of an experiment with an aluminium-foil electroscope:—

Weight (grams).	Potential (relative).	Divergence.
·1	1	1·0
·4	2	2·1
·9	3	3·5
1·6	4	5·0
2·5	5	6·3
3·6	6	7·0

It will be observed that for small deflections the potential is nearly proportional to the deflection.

187. **The Electrostatic Gauge.**

When we require to maintain a conductor at a certain potential, it is necessary to have some means of detecting any variation which may occur, through leakage or other cause. The electrostatic gauge devised by Lord Kelvin for this purpose is a small attracted disc electrometer controlled by torsion. The movable portion (fig. 109) consists of a strip of aluminium cut to form a circular disc, about 1 cm. in diameter, with a long arm. The arm is forked at the end, and a small hair is attached across the prongs. This movable part is supported on a wire W stretched between springs, which are attached to the guard-ring, near its edge. The disc and its arm form a lever, and the wire serves as the fulcrum. When the disc is in the plane of the guard-ring, the hair viewed

through the lens appears exactly half-way between two black dots on a small upright. This is called the "sighted" position. (The upright is between the prongs of the fork, but clear of them.)

The attracting disc is placed below the aluminium disc, and it can be clamped in different positions so that the instrument may be adjusted to indicate different potentials. The upper disc and guard-ring are earthed, and the lower disc is joined

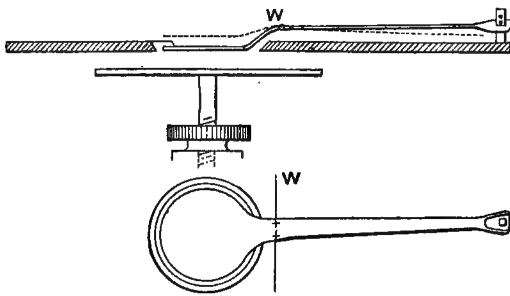


Fig. 109.—Kelvin Electrostatic Gauge

to the conductor to be kept at constant potential. The upper disc is normally raised just above the plane of the ring by the twist on the wire, but when the conductor is charged, the disc is pulled down. The charging is continued until the arm rises into the sighted position.

The instrument is used in connection with the replenisher.

188. **Kelvin Absolute Electrometer.**

This differs from the electrostatic balance in that means are provided for measuring the attraction between the discs with the aid of a spring. It is the standard instrument used for measuring potentials in electrostatic units. The essential parts are shown diagrammatically in fig. 110. They are nearly all enclosed within a large glass cylinder mounted on levelling screws.

1. The attracted disc AB is suspended by means of a spring

resembling in form an ordinary coach-spring. This is attached to the lower end of a rod P, which passes through the cover and is adjustable by means of a micrometer screw.

The sighting arrangement.—Two small uprights, to which a short hair is attached horizontally, are fixed to the disc AB. An image of the hair is thrown between the two fixed points F by means

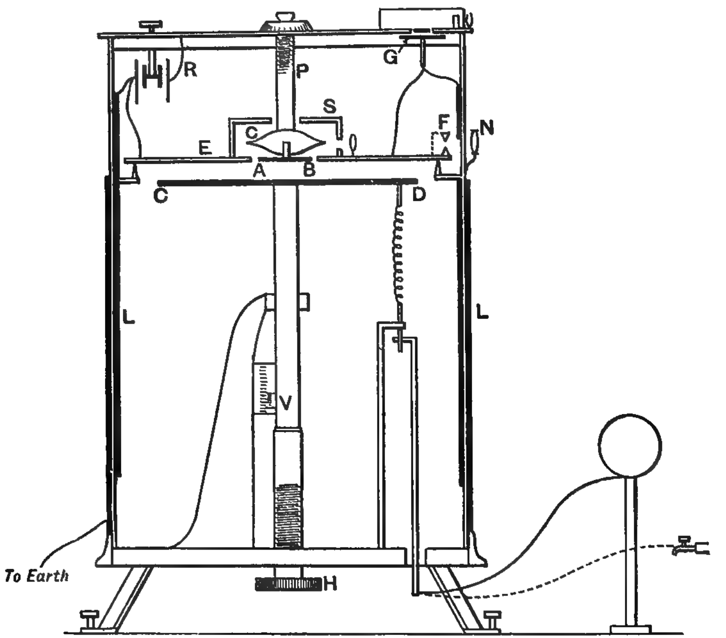


Fig. 110.—Absolute Electrometer

of a lens. On looking through a second lens N, the image and the points can be seen, and the latter are adjusted so that when AB is in the plane of the guard-ring the hair appears exactly between the points.

2. The guard-ring E is supported in the upper part of the instrument on insulated points. It is connected with AB, and can be reached by a metal rod, through a hole in the cover.

3. The attracting disc CD is supported on an insulating rod

provided with a fine screw adjustment, worked from a milled head H. The position of the attracting disc can be accurately observed with the vernier V and a micrometer (not shown in the figure). The disc is provided with an insulated terminal.

Theory of the Instrument.

Preliminary Experiment.—The plates are first joined to earth and so discharged, and the spring holds the disc AB a little above the plane of the guard-ring. A constant force, f dynes, is then necessary to bring it into this plane. To determine this force, weights are placed on the disc AB until it comes into the sighted position. If the weight necessary is m grams,

$$f = m \times 981 \text{ dynes.}$$

If the weight is removed, the disc springs back to its former position above the plane of the guard-ring.

Measurement of Potential.—The conductor to be tested is then joined to CD, and the plates become oppositely charged. The mechanical force due to the potential-difference of the plates will, if we make a suitable adjustment of the distance d between the lower plate and the guard-ring, attract the disc AB into the plane of the guard-ring. The force is given by the equation—

$$\mathrm{V} = d\sqrt{\frac{8\pi f}{a}},$$

$$\text{or } f = \frac{\mathrm{V}^2 a}{8\pi d^2}.$$

But this force f has been determined in the preliminary experiment and equals $m \times 981$ dynes.

$$\therefore \frac{\mathrm{V}^2 a}{8\pi d^2} = 981\, m,$$

$$\text{or } \mathrm{V} = d\sqrt{\frac{8\pi m \times 981}{a}}.$$

Since all the quantities are known except V, the latter can be calculated.

The quantity under the root sign is constant and is called the constant of the instrument. It can be calculated from the preliminary experiment. Denoting it by k—

$$\mathrm{V} = kd.$$

Theoretically, the simplest way to find the potential of a

conductor would be to connect the lower plate to the conductor and the upper plate to earth, observing the distance d necessary to bring the disc into the sighted position. Practically, however, it is difficult to determine the absolute distance between the discs with great accuracy. Hence a *method of differences* is adopted.

The attracted disc and guard-ring are insulated and charged to a potential V_1. The body to be tested is joined through the terminal to the lower disc, and the instrument adjusted. The reading d_1 corresponds to a potential-difference $(V_1 - V)$ where V is the potential of the body under test; also

$$(V_1 - V) = kd_1.$$

Next, the terminal is connected to earth (*e.g.* to the water pipes). A second adjustment gives a reading d_2 corresponding to a potential-difference $(V_1 - 0)$, and therefore

$$(V_1 - 0) = kd_2.$$

Thus by subtraction—

$$V = k(d_2 - d_1).$$

The result thus depends on a *difference* of readings, and this can be accurately determined. The method of using an *independent* charge on the attracted disc is called the *heterostatic* method. That described in connection with the electrostatic balance, where the charge on one plate was induced by the charge derived from the body under test, is called the *idiostatic* method.

The heterostatic method requires the following accessory parts in the instrument:—

(1) The screen S. This is necessary to prevent the independent charge from spreading over the upper surface of AB. The screen is joined to the guard-ring.

(2) The gauge G (Art. 187).

(3) The replenisher R to bring back the potential of AB when it varies through induction or leakage.

(4) The condenser. This consists of tin-foil strips pasted to the inner and outer surfaces of the glass cylinder,

the inner set being joined to AB through the guard-ring. Its object is to diminish the variations of the potential (V_1) of the attracted disc which take place when the distance between the discs varies. It also diminishes the effect of leakage.

If the instrument is used to measure the potential of an isolated conductor, it must be observed that the potential indicated is less than that which existed before the test.

189. Absolute Measurement of Charge.

Accurate measurements in electrostatic units always depend on the determination of potential by means of an attracted-disc electrometer. The capacity of a conductor or air condenser of regular form must be calculated from its dimensions. Charge is then determined from the relation—

$$Q = SV.$$

Surface density (when of sufficient magnitude) may be determined in a similar manner. The electric force in a uniform field between two plates is found by dividing the distance between the plates into the potential-difference between them. ($F = V/d$.)

Such *absolute* measurements in electrostatic units are very seldom made. Certain *comparative* electrostatic measurements are, however, of practical importance, *e.g.* spec. ind. capacity. To avoid the difficulties connected with leakage, it is usual to employ batteries to produce the necessary charges. Comparative measurements will be dealt with in the next two chapters.

190. Example.

Obtain, in terms of electric force, expressions for the tension in the electrostatic induction tubes—

(*a*) per unit area;
(*b*) per unit tube.

Assuming the equations already obtained, we have—

$$\frac{f}{a} = 2\pi\sigma^2 \text{ and } \sigma = \frac{F}{4\pi}.$$

Thus, by substitution, the pull per unit area—

$$\frac{f}{a} = \frac{F^2}{8\pi} \quad \text{.................................}(3)$$

Again, since there are σ tubes per unit area the pull exerted by each unit tube—

$$= 2\pi\sigma$$
$$= \tfrac{1}{2}F \quad \text{.................................}(4)$$

These expressions also represent the energy. For supposing the tubes between two plates to shorten, pulling the plates together, we may easily prove that—

Mechanical force per unit area = energy per unit volume.
Mechanical force per unit tube = energy per unit length of unit tube
$= \tfrac{1}{2}F$.

QUESTIONS

1. Explain what is meant by difference of potential, and describe some method by which it can be measured in absolute units. (1902.)

2. An insulated metal plate 10 cm. in diameter is charged with electricity and supported horizontally at a distance of 1 millimetre below a similar plate suspended from a balance and connected to earth. If the attraction is balanced by the weight of one decigram, find the charge on the plate. ($g = 980$ C.G.S.) (1902.)

3. If the force of attraction F between two large parallel plates charged with equal and opposite charges $\pm$ Q is independent of the distance between them, find the energy of the charge, and the difference of potential between them in terms of F and Q, when they are at a distance of 1 cm. apart. (1903.)

CHAPTER XV

MAINTAINED POTENTIAL DIFFERENCE

191. We have hitherto considered only *frictional* sources of electrification. But voltaic cells, dynamos, and other generators, which we regard primarily as sources of electric *currents*,

also produce *static charges* on conductors joined to their poles. In the present chapter we shall consider the production of static charges by voltaic cells; the conclusions arrived at will apply to other types of generators.

192. Voltaic Cells.

If plates of copper and amalgamated zinc are immersed in a vessel containing dilute sulphuric acid, no chemical action occurs so long as the plates remain separate. But if the copper is allowed to touch the zinc either inside the liquid or outside a rapid chemical action is at once set up; the zinc dissolves and hydrogen is liberated. A similar result occurs if the plates are connected by a wire or other good conductor.

The production of chemical action by the contact of dissimilar metals was discovered by Volta about the year 1800. The arrangement just described is termed a *Volta's cell.*

Similar results are obtained when the copper is replaced by other metals, *e.g.* silver, platinum, mercury. Again, for the zinc we may substitute iron, cadmium, and some other metals. For the acid we may substitute various solutions which under ordinary circumstances have no action on the metals used. We thus obtain different forms of simple voltaic cells, in all of which chemical action is at once set up when the plates are connected. Useful simple cells are—

(1) *Iron and carbon plates in a solution of ferric chloride.*
(2) *Zinc and copper plates in a solution of zinc chloride.*

Types of compound voltaic cells specially used as sources of *current* will be described in Chap. XVIII.

193. Condensing Electroscope.

If the plates of a cell are joined to an ordinary electroscope, no divergence of the leaves is obtained. But by combining a condenser with the electroscope Volta was able to show that the plates were charged. The combination is termed a "condensing electroscope". It consists of two brass discs ground perfectly plane, and of sufficient thickness not to get out of

truth through use. An electroscope is joined to the lower disc (fig. 111). The ground surfaces are kept from actual contact by a thin coating of lacquer, thus forming a condenser in which the surfaces are very close together, and which has therefore a relatively large capacity.

Join the copper (or carbon) plate of a cell to the lower disc, making good contact at a place scraped free from lacquer. Make similar connection between the zinc plate and top disc. Remove the wires and immediately lift the top disc. The leaves show a small divergence, and if tested will be found + charged. Repeat

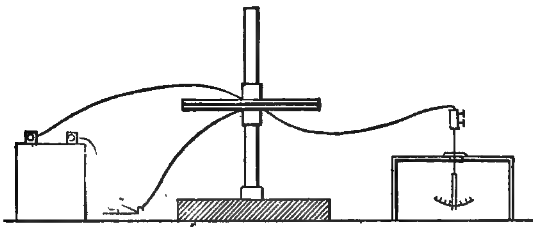


Fig. 111.—Condensing Electroscope

the experiment with the zinc joined to the lower disc. The leaves acquire a − charge.

It is evident, therefore, that the zinc and copper plates become electrified when dipped in the acid. But since there is no effect if the plates are joined direct to an ordinary electroscope, we conclude that the potentials must be very weak.

The condensing electroscope is not suitable for the exact comparison of potentials. The small potential differences between the + and − plates or "poles" of voltaic cells can, however, be accurately compared by a highly sensitive form of electrometer invented and developed by Lord Kelvin. This instrument, termed the "quadrant electrometer", we shall now describe.

194. The Quadrant Electrometer.

There are two parts essential to the instrument—

(1) A light flat "needle" of the form shown in fig. 112,

suspended by two threads (or other form of torsional control) and insulated.

(2) Four quadrant-shaped cells, A, A_1, B, B_1, which enclose the needle, but do not touch it. The opposite quadrants, A and A_1, are connected, also B and B_1. Each pair of quadrants is connected with a separate terminal, the A pair to T_A, and the B pair to T_B (fig. 112).

Theory.—The needle is charged to a large potential, usually with positive electricity. The positive charge induces an equal negative charge on the inner surfaces of the quadrants. The charge induced on the outer surfaces is removed by joining the quadrant terminals T_A and T_B to earth.

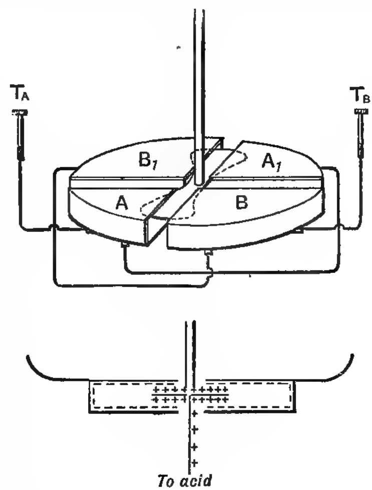


Fig. 112.—Electrometer—Theory

We may confine our attention to the front pair of quadrants, A and B. If the needle lies symmetrically between them, the charges induced on A and B will be equal, and the attractions which these exert on the needle will balance. Suppose now that the needle is kept at *constant potential*, say 3 e.s. units. Let B remain earthed whilst the potential of A is very slightly raised, say, by ·001 unit. The potential difference between the needle and A is now 2·999, whilst that between the needle and B is still 3·0. Thus more than half the charge on the needle collects on the B side, and the attractions no longer balance. The needle is deflected into the quadrant B at one end, and into B_1 at the other.

The deflection of the needle increases until the electric forces are balanced by the forces called into play in the suspending wire or threads. It can be shown[1] that the deflection produced, if small, is proportional to—

$$(V_A - V_B)\left\{V_n - \frac{V_A + V_B}{2}\right\}\dots\dots\dots\dots\dots\dots(1)$$

where V_A, V_B, and V_n are the potentials of the quadrants A and B and the needle respectively. If V_n is high compared with V_A and V_B, the second term in the second bracket may be neglected. Thus—

$$\delta \text{ is proportional to } V_n(V_A - V_B).$$

Since V_n is constant—

$$\delta \text{ is proportional to } (V_A - V_B).$$

When the needle is kept at a constant high potential, the deflection is proportional to the difference of the potentials of the neighbouring quadrants.

Construction.—The main points in the construction of the Kelvin instrument may be perceived from fig. 113.

The needle, N, is attached to a straight stiff wire P, which carries at its upper end a small concave mirror and T-piece. This system is suspended by means of two silk threads, which are attached to the T-piece, as shown in the figure[2], their upper ends being attached to two small screws on a brass plate carried by the insulating stem I. This is supported on the brass cover of a large inverted bell-jar of glass. The bell-jar is held in a frame provided with levelling screws. The vessel contains strong sulphuric acid, and is coated on the outside with strips of tin-foil, which are connected with the frame. A very fine platinum wire hangs from the rod P, and makes

[1] See Everett's "Electricity" (*Deschanel*, Part III, New Edition), p. 78.

[2] In some forms of electrometer the bifilar suspension described above is replaced by a fine torsion fibre or wire; or the controlling force may be supplied by the earth's action on a small magnet attached to the needle, the suspension in this case being a single *silk* fibre.

connection between the needle and the acid. It carries a small platinum weight, W, at its lower end, which, when rotating in the acid, damps the vibrations of the needle. The

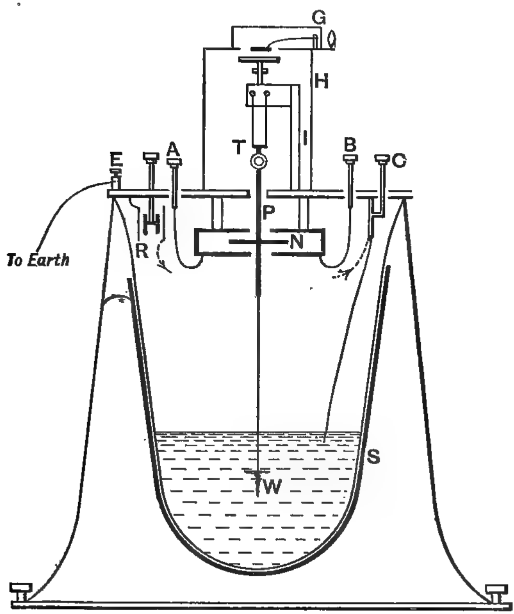


Fig. 113.—Kelvin Quadrant Electrometer

cover supports an insulated terminal, the "charging electrode" C, which is connected with—

(i) the acid (and therefore also the needle);
(ii) a replenisher R;
(iii) the attracting disc of an electrostatic gauge G.

One inductor of the replenisher is thus joined to the needle. The replenisher spindle can be rotated by means of the knob which projects above the cover. The gauge is mounted on a brass hood or shield H, which has a window in front of the mirror.

The above arrangements all have reference to charging the

needle and keeping it at constant potential. A few sparks from an electrophorus are given to C, until the gauge comes into the sighted position. The charge flows to the acid, replenisher, needle, and gauge-disc, but most of it collects on the surface of the acid in contact with the glass. This surface with the tin-foil strips forms a condenser of large capacity compared with the needle, etc. The condenser serves two purposes:—

(i) When the needle is deflected into either quadrant its capacity increases. It therefore requires an extra charge to keep its potential constant. This is supplied by the condenser, which, owing to its large capacity, suffers no appreciable change in potential by the loss of a small charge.

(ii) The rate of fall of potential due to unavoidable leakage is less with a condenser of large capacity. Also, the sulphuric acid keeps the interior of the instrument dry, and so diminishes leakage. If leakage occurs it is at once shown by the gauge, and may be corrected by the replenisher.

The quadrants are suspended by glass rods from the lower side of the cover. They are joined to two terminals, A and B, on the cover, termed the "quadrant electrodes". One of the quadrants can be moved inwards or outwards by means of a micrometer screw (not shown in the figure). An earth terminal E is joined to the water-pipes to bring the frame into good earth-connection.

Method of Experiment.—Join all the terminals to earth, and adjust the instrument so that when the needle lies symmetrically between the quadrants, the spot of light is reflected to the centre of the scale. Disconnect the charging electrode from earth, and pass sparks to it from an electrophorus until the gauge is in the sighted position. Owing to a slight want of symmetry in the positions of quadrants and needle, the spot of light may wander from the zero position during the charging process. Correct this by means of the micrometer screw.

Now disconnect the quadrant electrodes from earth and join them to the positive and negative poles respectively of the cell to be tested. The needle is deflected away from the quadrant joined to the copper or + terminal. Reverse the connections and take

the deflection on the other side of zero. If this is not quite equal to the former deflection, the mean may be taken.

NOTE.—Observe carefully that there is no connection between the neighbouring quadrants joined to A and B. Hence *no current* flows through the instrument. The connections to an electrometer are always simply *pressure* (*or potential*) *connections.*

195. Property of a Voltaic Cell.

By means of the electrometer we may show that the potential-difference between the poles of a cell depends on the materials used in its construction, and that this potential-difference is always the same for a given cell, when the cell is on **open circuit**; *i.e.* when it is not traversed by a current.

Join the poles of a cell to the terminals of a quadrant electrometer and observe the deflection. Now join the poles of the cell to insulated conductors of any size. The deflection is unaltered.

Join the poles to the electrometer as before. Earth-connect the negative pole. The deflection is unaltered. Next disconnect the negative pole from earth, and earth-connect the positive pole instead. The spot of light still rests in the same position.

These experiments prove the following important property of a voltaic cell:—

> **A voltaic cell of given construction always maintains the same potential-difference between its poles, provided that it is kept on open circuit.**

It should also be remembered that in accordance with the general law of production of charges the cell will always generate equal quantities of + and − electrification at the same time, although one of these may be removed by earth-connection.

196. Electromotive Force.

The small potential-difference of the cell is produced by the positive and negative charges on the poles. The experiments mentioned in the preceding article show that if we

remove the charges by connecting the poles to insulated conductors the cell immediately replenishes the charges so that the P.D. is maintained. There is therefore some action in the cell which tends to send electricity into the positive pole, withdrawing it at the same time from the negative pole. This action is called the "electromotive force" of the cell. In general electromotive force may be defined as "any cause which tends to set electricity in motion".

When the cell is on open circuit the poles are in conducting connection through the acid of the cell. Why is it, therefore, that the unlike charges on the poles do not neutralize each other through this conducting liquid? The explanation is that *the E.M.F. of the cell is always acting.* The tendency of the positive charge to run back through the liquid from the copper to the zinc is balanced by the electromotive force, which tends to drive it from zinc to copper. Thus we may say that on open circuit the E.M.F. and P.D. balance each other. Hence—

> **The potential-difference of the terminals when the cell is on open circuit is taken as a measure of the electromotive force.**

The unit of electromotive force is the same as the unit of potential-difference.

197. Steady Electric Current.

It will be convenient to speak of the direction in which the E.M.F. tends to send the electricity as the *forward* direction through the cell. The P.D. then tends to send the electricity *backwards* through the cell; and on open circuit the two effects balance.

Let us now suppose the poles joined by a wire. The electricity can now flow from the + to the − pole without going against the E.M.F. The flow thus set up along the wire relieves the difference of pressure (potential) at the poles. The E.M.F. is no longer balanced; more electricity is drawn from the negative pole and sent through the cell into the positive pole, flowing back to the negative pole through the

wire. Thus a continuous and steady current is set up. It is this current through the cell which causes the chemical action we have noticed. The cell is said to be on *closed* circuit.

The P.D. on closed circuit is less than the P.D. on open circuit owing to the escape of electricity from the pole along the wire. But the E.M.F. always maintains some P.D., for otherwise there would be nothing to cause the flow along the wire.

Join the poles of a cell to the terminals of an electrometer. Observe the deflection. Now, without altering the connections to the electrometer, join the poles to each other through a thin wire. The deflection shown is immediately decreased and becomes steady at a lower value.

The properties of electric current will be dealt with in Part III. In the remainder of the present chapter we shall be concerned only with cells on *open* circuit.

198. Voltaic Batteries.

Cells may be grouped in various ways to form *batteries.* If the cells are all of the same kind, the three chief methods of connecting them are—

(1) In *series*, where the + pole of one cell is joined to the − pole of the next.

(2) In *parallel*, where the positive poles are all joined to form one terminal, and the negative poles are joined to form the other terminal.

(3) In *compound circuit*, where the cells are formed into a number of equal series batteries, which are then joined in parallel.

In a series battery the P.D. of the terminals is the sum of the P.D.'s of the separate cells.

Hence if the cells are all alike, the total P.D. is equal to that of one cell multiplied by the number of cells.

If all the cells are in parallel the terminal P.D. is the same as for one cell.

In this arrangement there is a gain of conductivity, but this is of no importance until we deal with currents.

If the cells are in compound circuit, the P.D. of the battery terminals depends on the number of cells in each component series battery.

These statements may be directly proved with the electrometer.

Example.—A series battery of 6 cells gives a deflection of 54 divisions when joined to an electrometer. If the cells are rearranged to be (1) all in parallel, or (2) as two series batteries in parallel, what deflections will the battery give when joined to electrometer?

Each cell gives deflection $= \frac{54}{6} = 9$ divisions. $\therefore$ the all-parallel battery gives 9 divisions; and the compound battery gives $3 \times 9 = 27$ divisions.

If any point of a series battery on open circuit is joined to earth, that point is brought to zero potential, but the potential-*difference* of the terminals is unaltered. (If *two* points of the battery are joined to earth at the same time a circuit is completed through the earth. The consideration of this case falls under Current Electricity.)

Although the P.D. of the poles of a single cell is far too small to be shown by an ordinary electroscope, by taking a sufficient number of cells (100 or more) in series we may prove its existence by means of a simple gold-leaf electroscope.

199. Use of Cells in Electrostatics.

In electroscopes, electrometers, and other instruments which depend on the mechanical forces between charges, it is generally necessary that one of the conductors be maintained at a high potential. If the charge is given to the conductors from a frictional source, the inevitable leakage causes a fall of potential. This is a source of trouble and inaccuracy. In quantitative experiments, therefore, it is usual to maintain the potentials or potential-differences with voltaic batteries. The cell constantly replaces any charge which leaks away, and

steady potentials may thus be maintained even with indifferent insulation.

In batteries used for electrostatic work the resistance is of little account. The cells may therefore be very small and the liquid used need not be a particularly good conductor. A convenient form is the water battery consisting of a hundred or more copper-zinc couples dipping into small vessels of water. In the Kelvin form, all the couples can be lifted out of the liquid simultaneously. A useful installation consists of a hundred small accumulators arranged in series. These may be of simple construction (fig. 114), and can be readily charged from the lighting mains.

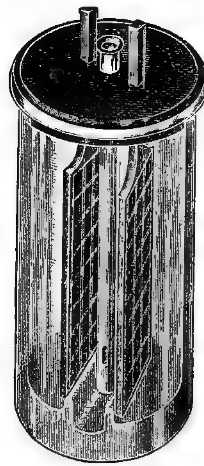


Fig. 114.—Cell for Potential Battery

A *Volta's pile* may be used for the same purpose. It consists of 50 to 100 compound discs arranged in the form of a pile with layers of flannel or felt between the discs. Each disc consists of a copper plate soldered to a zinc plate. The flannel is moistened with brine or dilute sulphuric acid. Suitable terminals are provided for the top and bottom discs. (The effects obtained with this arrangement gave great impetus to the study of Current Electricity in the opening years of the nineteenth century.)

In *dry piles* the liquid is dispensed with, moisture absorbed from the atmosphere probably taking its place. Zamboni's pile consists of discs of paper coated on one side with zinc- or tin-foil and on the other with manganese peroxide. These are built up so that the zinc of each disc is in contact with the peroxide of the next disc. With a few thousands of these discs, sparks can be obtained.

Note that the above arrangements are only suitable for producing high pressures. Batteries for producing strong currents will be considered later.

200. Practical Units of Electrostatic Quantities.

The units of charge, potential-difference, etc., which we have hitherto used belong to the C.G.S. electrostatic system.

The P.D. at the poles of a cell may be found in terms of electrostatic units as follows:—If we take a series battery of, say, 500 Daniell cells, we may measure the P.D. for the whole battery by means of an attracted-disc electrometer. If we divide the whole P.D. by the number of cells, we obtain the value for a single cell. This, for a Daniell cell, is about ·0036 of an electrostatic unit.

The electrostatic units are, however, seldom used except in theoretical calculations. In practice, the charges are derived from voltaic cells (or other current generators), and it has been found necessary to adopt a system of units based on the properties of the electric current. These are termed *electromagnetic* units. The definitions of electromagnetic units will be dealt with in Part III. We shall here, for immediate use, quote the relations of the electrostatic to the practical electromagnetic units.

The practical unit of potential is termed the volt.

1 e.s. unit of potential is equivalent to 300 volts.

The practical unit of capacity is termed the microfarad.

1 e.s. unit of capacity is equivalent to $\frac{1}{900{,}000}$ microfarad.

The practical unit of electrostatic charge is the microcoulomb.

1 e.s. unit of charge is equivalent to $\frac{1}{3000}$ microcoulomb.

Adopting these units, the equation $Q = SV$ may be read—

$$(\text{microcoulombs}) = (\text{microfarads}) \times (\text{volts}).$$

These practical units are chosen of such magnitudes that, whilst they bear a simple relation to the theoretical units, they are suitable for the expression of the quantities met with in practice, so that the numerical values are not inconveniently great nor inconveniently small.

Thus the E.M.F. of a Daniell cell is 1·08 volt, and the capacity of a mile of submarine cable is about $\frac{1}{3}$ of a microfarad.

It is convenient to remember that a spark 1 cm. long between two large surfaces corresponds to a P.D. of nearly 30,000 volts; and for less distances the sparking P.D. is nearly proportional to the spark length. The leaves of an electroscope (as shown in fig. 63) diverge rather more than a centimetre for a potential of 1000 volts.

Taking the volt as the standard of reference, we see that the potentials dealt with in frictional experiments must be considered to be very high, being usually reckoned in thousands of volts or kilovolts.

On the other hand, taking the microfarad as a standard of reference for capacities, the Leyden jars, insulated conductors, etc., of frictional experiments have very small capacities.

Thus the capacity of a sphere 1 inch in radius is ·0000028 microfarad, and the capacity of an ordinary Leyden jar is about ·0025 microfarad.

The practical unit of electric force is defined from that of potential. We have seen (Art. 147) that the electric force is equivalent to the rate of fall of potential along the lines of force, or to the potential-gradient. The practical unit is a gradient of *1 volt per centimetre.* Similarly electric surface density may be expressed in microcoulombs per square centimetre. In calculations it is advisable to use the equations for electrostatic units, and make the change to or from the practical units independently.

EXAMPLES.—1. Two pith-balls equally charged repel each other at a distance of 3 cm. with a force equal to the weight of ·5 milligram. Find the charge on each ball in microcoulombs.

$$\text{Weight of 1 gram} = 980 \text{ dynes,}$$

$$\text{Weight of } \frac{1}{2000} \text{ gram} = \cdot 49 \text{ dyne.}$$

In electrostatic units—

$$f = \frac{ee'}{d^2},$$

$$\cdot 49 = \frac{e^2}{9}. \quad \therefore e = 2\cdot 1.$$

Thus the charge expressed in microcoulombs is—

$$\frac{2\cdot 1}{3000} \text{ or } \cdot 0007 \text{ m.c.}$$

2. A battery of 200 secondary cells, each E.M.F. 2, has its negative pole earthed. Its positive pole is joined to a metal ball 5 cm. in diameter. Find the charge acquired by the ball.

$$\text{Capacity of ball in e.s. units} = 2{\cdot}5.$$

$$\therefore \text{ capacity in m.f.} = \frac{2{\cdot}5}{900{,}000}. \quad \text{The P.D.} = 200 \times 2 \text{ volts.}$$

The charge acquired—

$$= \frac{2{\cdot}5}{900{,}000} \times 400$$
$$= {\cdot}0011 \text{ microcoulomb.}$$

201. Condensers for Use with Batteries, etc.

We have seen that a Leyden jar has only a small capacity (if this is expressed in microfarads). If a condenser is to be charged from a cell, battery, or other generator of moderate voltage, then in order that it may acquire a measurable charge, it must have a capacity many times greater than a Leyden jar. On the other hand, the insulation need not be high, and a much thinner dielectric may be used. The form of condenser used with cells, etc., consists of a large number of sheets of tin-foil and paraffined paper arranged in alternate layers. The alternate sheets of tin-foil are joined together on one side, and the remaining sheets on the other. The arrangement thus corresponds to a number of Leyden jars arranged in parallel. The number of component condensers is equal to the number of sheets of paraffined paper.

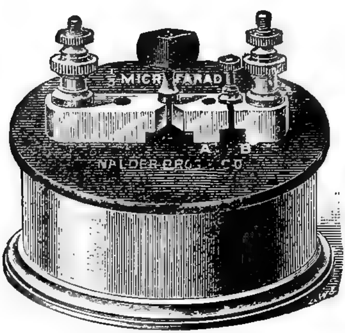


Fig. 115.—Standard Condenser

In the best standard condensers the insulation consists of sheets of mica, and the terminals are carefully insulated with ebonite. Fig. 115 shows the usual form.

Comparison of Condenser Capacities.—The capacities of two condensers may be compared by the method of *sharing the charge*, using an electrometer to indicate the P.D.'s.

The connections are shown in fig. 116. When the key M is depressed the condenser A becomes charged from the battery, and the P.D. is indicated on the electrometer. M is then opened, and the key N closed. The charge is shared between the condensers, and the P.D. at once drops. If θ_1 and θ_2 are the first and second electrometer deflections, then—

$$\frac{\text{Capacity of B}}{\text{Capacity of A}} = \frac{\theta_1 - \theta_2}{\theta_2}.$$

All connections on one side must be highly insulated. The other side may be joined to earth. The key N should be worked from an ebonite rod. A preliminary test must be made to ensure that the quadrants and condensers are not leaky.

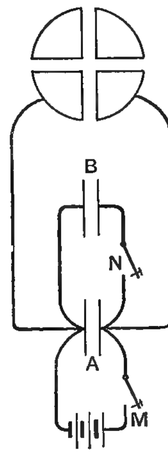


Fig. 116.—Ratio of Capacities

202. Use of Pressure Battery with Electrometer.

Instead of keeping the needle of an electrometer at a high potential by connecting it with a charged condenser, we may connect the needle to one terminal of a large series battery (or to a voltaic pile), the other terminal of the battery being to earth. The condenser, gauge, and replenisher may then be dispensed with, and the instrument becomes much simplified. The sensitiveness may also be readily varied by altering the number of cells in the battery. This is the most satisfactory method of using the electrometer.

Dolezalek Electrometer (fig. 117). In the form of electrometer recently invented by Dr. F. Dolezalek the suspension consists of a fine quartz fibre about ·01 mm. in diameter. The needle is of silvered paper, and its extreme

lightness renders the movements nearly dead-beat.[1] The quadrants are supported on amber pillars giving a high insulation. The needle is charged by connecting it to a battery giving 100 or 200 volts. Since the quartz is an insulator it is first rendered conducting by dipping it into a solution of calcium chloride. This solution never dries on the fibre, since the calcium chloride is hygroscopic. The upper end of the fibre is attached to an insulated head, which is joined to the charging battery.

The extreme fineness of the quartz fibre renders the instrument very sensitive. (One-tenth of a volt P.D. will give about 5 cm. deflection on the scale at a distance of 1 metre, when the needle is maintained at 100 volts.)

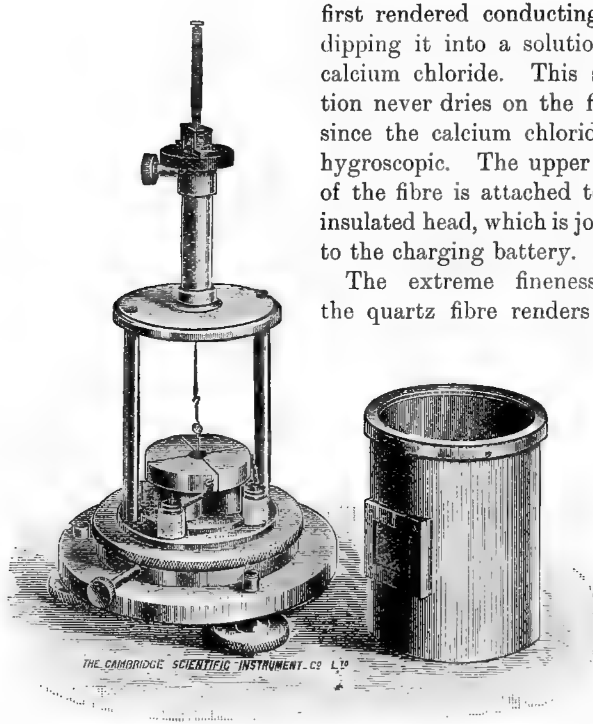


Fig. 117.—Dolezalek Electrometer

[1] With bifilar suspensions the weight of the needle affects the sensitiveness *directly*; with a single torsion fibre (or wire) suspension the weight of the needle indirectly influences the sensitiveness, since the small moment of inertia of such a needle allows the use of a very fine suspending fibre, without rendering the movement inconveniently slow.

203. Idiostatic Use of Electrometer.

When very great sensitiveness is required the electrometer is used as above, the needle being charged heterostatically, *i.e.* from an independent source (see p. 260). Potential-differences ranging between ·01 volt and 5 volts may then be measured. But for higher P.D.'s the idiostatic method is more convenient. The needle is joined to one pair of quadrants, so that there are then only two terminals to the instrument. These are joined to the poles of the battery or other arrangement to be tested.

Putting $V_n = V_A$ in the formula we have—

$$\delta \propto (V_A - V_B)\left\{V_A - \frac{V_A + V_B}{2}\right\}.$$

$$i.e.\ \delta \propto \frac{(V_A - V_B)^2}{2}.$$

When the electrometer is used idiostatically, the deflection is proportional to the square of the potential-difference.

A little thought will show that one pair of quadrants, namely, the pair joined to the needle, is unnecessary. If these quadrants are omitted the attraction between the needle and the remaining quadrants will produce the deflection. The needle and quadrants then correspond to the leaves and case respectively of an ordinary gold-leaf electroscope.

A special advantage of the idiostatic principle is that the instrument can be used to measure *alternating* P.D.'s.

Instruments constructed on the principle just described are widely used, and are termed in engineering "electrostatic voltmeters", in contradistinction to "electromagnetic voltmeters", although the former term might be applied with equal propriety to any form of electrometer.

204. Electrostatic Voltmeters.

Typical forms of these instruments, as devised by Lord Kelvin, are—

(1) The "multicellular", suitable for ranges of P.D. which lie between 40 and 600 volts, the actual range in each instrument being about two-thirds the maximum voltage for that instrument.

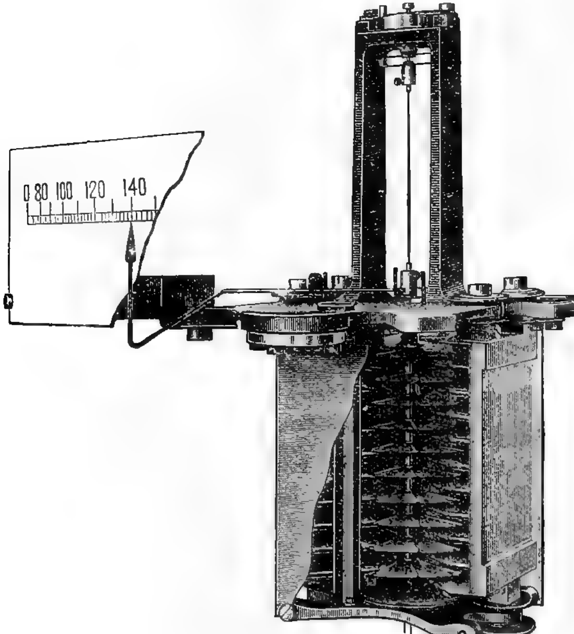


Fig. 118.—Kelvin Multicellular E.S. Voltmeter

(2) The "vertical" or gravity-control form, suitable for ranges of P.D. which lie between 500 and 20,000 volts, the range in each instrument being about three-quarters of its maximum voltage.

In the most recently introduced type of Kelvin multicellular (fig. 118) a number of vanes of aluminium are attached to a vertical rod suspended by a fine phosphor-bronze strip. An equal number of cells cast in type-metal are insulated

within the case. The vanes are suspended so that they can be attracted into the cells. Each pair of cells and its corresponding vane thus resemble one pair of opposite quadrants and the needle of an electrometer. The cells are joined to one terminal of the instrument, and the vanes (through the medium

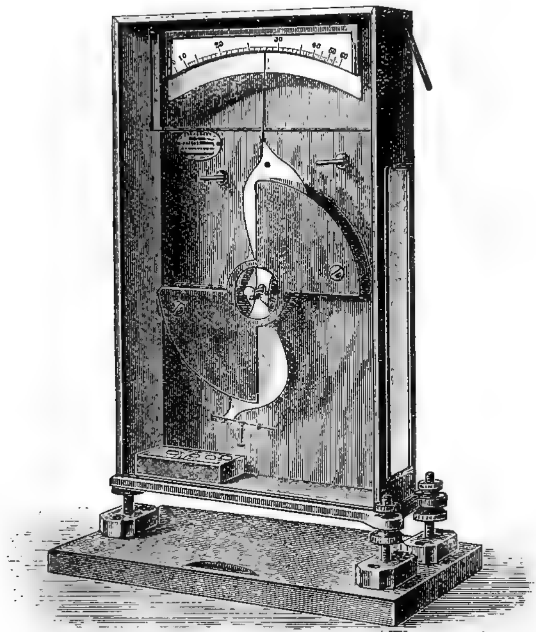


Fig. 119.—Vertical E.S. Voltmeter

of the suspending wire) to the other. An extension of the rod dips into a dash-pot and serves to damp the vibrations. A "coach spring" is placed between the suspending wire and its upper attachment to prevent sudden jerks on the wire when the instrument is moved. A pointer attached to the rod indicates the voltage on a suitably placed scale. The working parts are completely insulated from the case, and are protected by an insulated screen of tin-foil connected with the vanes.

The gravity-control instrument (fig. 119) resembles the electrometer with one pair of quadrants removed. But the "needle" is balanced on a horizontal axle which is supported on knife-edges. At the lower end of the needle or vane a light rod is attached, and carries a small weight. The moment of this weight about the axis supplies the controlling influence. A pointer attached to the upper end of the vane moves over a vertical scale. Different degrees of sensitiveness are obtained by attaching different weights to the lower end of the needle.

205. Volt Balance.

For the measurement of very high potentials ranging from 5000 to 30,000 volts, or 20,000 to 100,000, a form of electro-

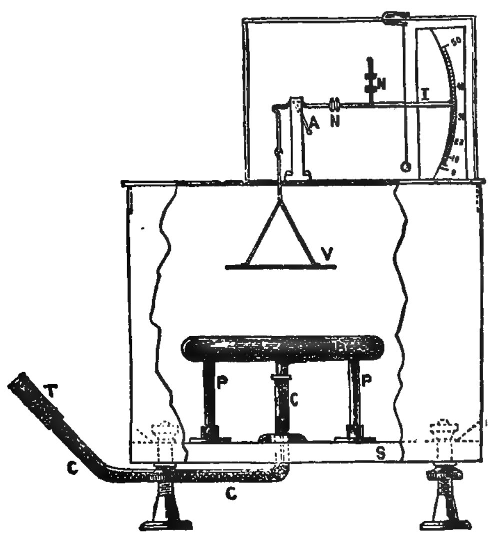


Fig. 120—Volt Balance

static balance is used. The Kelvin form is shown in fig. 120. The attracted disc is suspended some distance above the attracting plate, from one end of a beam with unequal arms. The longer arm of the beam serves as the pointer. The con-

trolling influence is supplied by gravity; a weight is attached to the beam so that when the disc is attracted the weight is carried farther from the vertical line through the axis. A position of balance is thus acquired for each voltage. The attracting plate or bun B is highly insulated. A magnetic damper is also provided which renders the movements practically dead-beat.

The scales of these instruments must always be calibrated, since the deflections do not obey any simple law.

206. Examples.

1. The plates of a condensing electroscope are 10 cm. radius and the lacquer (K = 2) is ·25 mm. thick. After joining a cell and removing the wires and top plate, the leaves diverge 5 mm. at their ends. Find how many cells in series (connected directly to the electroscope) would make the leaves diverge 1 mm.

$$\text{Capacity of the condenser} = \frac{aK}{4\pi t} = \frac{100 \times 2}{4 \times \cdot 025} = 2000 \text{ e.s. units.}$$

Capacity of the lower plate when the upper one is removed

$$= \frac{2r}{\pi} = \frac{20}{\pi}.$$

Since one of the condenser charges remains on the lower plate, the rise of potential when the upper plate is removed (neglecting capacity of electroscope) is in the ratio—

$$20/\pi \; : \; 2000,$$
$$\text{or} \quad 1 \; : \; 314.$$

Thus the P.D. of the poles of the cell is 1/314 of that required for 5 mm. divergence. 314 cells in series would produce 5 mm. divergence, and ∴ 63 cells would produce a divergence of 1 mm.

2. A is a gold-leaf electroscope, B a quadrant electrometer. For measuring small potential differences B is found to be more sensitive than A; does it necessarily follow that it will be more sensitive for the measurement of small charges? Give reasons for your answer. (1905.)

This is a question of capacity. The quadrants of an electrometer have a larger capacity than the rod and leaves of a gold-leaf electroscope.

Let V be the potential required to produce a measurable deflec-

tion in the gold-leaf electroscope; and v that required to produce a deflection measurable with equal accuracy on the electrometer.

Let S_1 and S_2 be capacities of electroscope and quadrants respectively.

Then if Q_1 and Q_2 are the charges for which the instruments are equally sensitive,

$$Q_1 = S_1V \text{ and } Q_2 = S_2v.$$

$$\therefore \frac{Q_1}{Q_2} = \frac{S_1}{S_2} \times \frac{V}{v}.$$

Thus if S_1/S_2 is less than v/V, we have Q_1 less than Q_2, and for *equal* charges, the electroscope is the more sensitive.

By making the leaves and rod of the electroscope very small (in this case the divergence must be observed through a microscope) S_1/S_2 can be made smaller than v/V; the electroscope is therefore used for the measurement of small charges.

3. The two plates of an electrostatic balance are joined to the poles of a battery of cells in series. The plates are 6 cm. in rad. and 5 mm. apart, and the weight ·05 gram is placed in the opposite pan. Balance is obtained when the number of cells is increased to 255. Find the P.D. at the poles of each cell in electrostatic units.

$$V = d\sqrt{\frac{8\pi f}{a}}. \quad \text{(Art. 185.)}$$

$$= \tfrac{1}{2} \sqrt{\left(\frac{8 \times \pi \times \cdot 05 \times 980}{\pi \times 36}\right)}$$

$$= 1{\cdot}65.$$

$$\therefore \text{P.D. at poles of each cell} = \frac{1{\cdot}65}{255} = \cdot 0064.$$

QUESTIONS

1. If you touch an isolated metal sphere 3 in. diameter charged to a potential of 200 volts, you experience no special sensation. But if you momentarily touch the lighting mains charged to the same potential, you feel a smart shock. How do you account for the difference?

2. Describe some good form of electrometer: explain its action and the mode of using it. (1900.)

3. Describe the construction of a condenser of moderately large capacity, and state how you would compare it with another condenser of which the capacity was known. (1897.)

CHAPTER XVI

DIELECTRICS

207. The term "dielectric" is used in preference to "insulator" when reference is made to the property of transmitting electric induction. We must now consider the dielectric properties of insulating materials in some detail.

208. Electric Force in a Dielectric.

Consider two condensers, the first consisting of two parallel plates in air, and the second of two plates separated to the same extent by some other dielectric. If S_1, S_2 are the respective capacities, V_1, V_2 the potential-differences, then by definition of specific inductive capacity—

$$K = \frac{S_2}{S_1} \qquad (1)$$

If equal charges are given to the condensers,

$$\frac{S_2}{S_1} = \frac{V_1}{V_2}.$$

Now the potential-difference V_1 is by definition $F_1 \times d$, where F_1 is the electric force and d the distance of the plates. We have therefore—

$$\left.\begin{aligned} V_1 &= F_1 d \\ V_2 &= F_2 d \end{aligned}\right\} \therefore \frac{V_1}{V_2} = \frac{F_1}{F_2}.$$

Therefore also,

$$K = \frac{F_1}{F_2},$$

$$\text{and } F_2 = \frac{F_1}{K} \qquad (2)$$

This relation may be expressed thus—

The electric force in a solid or liquid dielectric is 1/Kth of the electric force in air for the same density of the tubes of induction.

The electric force in air is given by $F_1 = 4\pi\sigma$,

$$\therefore F_2 = \frac{4\pi\sigma}{K}, \qquad \text{(3)}$$

where σ is the density of the induction tubes.

209. Energy in Unit Volume.

The energy stored in a condenser is given by—

$$W = \tfrac{1}{2}QV.$$

If a is the area of each plate, d the distance between the plates, F the electric force, and σ the surface density or density of induction tubes, then—

$$V = F \times d; \text{ also } F = \frac{4\pi\sigma}{K},$$

and $$Q = a \times \sigma.$$

Therefore by substitution,

$$W = \tfrac{1}{2} \cdot \frac{a\sigma \cdot 4\pi\sigma \cdot d}{K}$$

$$= \frac{2\pi\sigma^2}{K} \cdot ad.$$

But ad is the volume of the dielectric between the plates, and the field is uniform. Hence the energy per unit volume

$$= \frac{2\pi\sigma^2}{K} \qquad \text{(4)}$$

Thus the energy in unit volume is $1/K$th of the energy per unit volume in air, for the same density of induction tubes.

The student will find it helpful to imagine that the presence of the dielectric weakens the tubes of induction so that their pulling power is decreased. But it must be observed that the unit tube represents the same amount of *charge* whether it exists in air or in some other dielectric.

210. Influence of Dielectric on Distribution.

Since the energy in a solid or liquid dielectric is less than in air, by equation (4), the tubes will tend to collect in the dielectric; for it is a general mechanical principle that the

potential energy of a conservative system tends to decrease. The tubes of induction therefore crowd into the dielectric until any further increase of density would counteract the decrease of energy due to greater inductive capacity. An uncharged ball of insulator distorts the field very slightly,

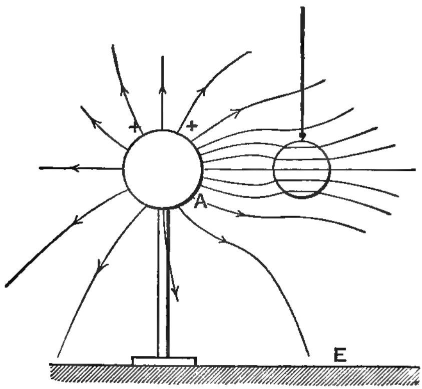


Fig. 121.—Influence of a Dielectric on the Field

as shown in fig. 121. The effect of soft iron on a magnetic field is similar in nature, but much more marked.

SOLID DIELECTRICS

211. Determination of Specific Inductive Capacity.

Faraday's method of determining this ratio has been replaced by other methods of greater accuracy, but the experiment is always one of some difficulty. The chief errors arise from dielectric absorption (see below), accidental charges on the dielectric, and imperfect insulating power of the material. The following method, due to Hopkinson, is one of the best yet devised.

It should be noticed that batteries are generally used to produce the charges (in place of frictional methods). For since the potential differences are thus maintained, difficulties due to leakage are to a large extent overcome.

The arrangements are shown in figs. 122, 123, the connections being somewhat modified for clearness. The condenser containing the dielectric is provided with a guard-ring round one

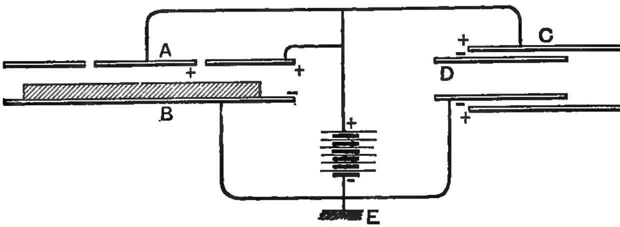


Fig. 122.—Measurement of Specific Inductive Capacity

plate A, the other plate B being of the same diameter as the ring. The dielectric is in the form of a slab.

An adjustable condenser is used, this being of the cylindrical form designed by Lord Kelvin. It consists in principle of two concentric tubes, one of which can be slid into the

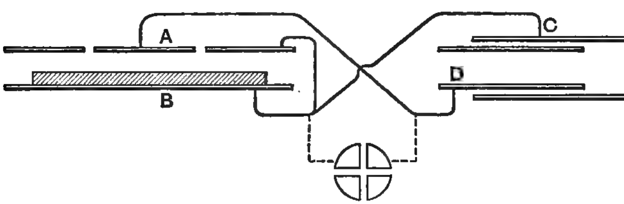


Fig. 123.—Measurement of Specific Inductive Capacity

other to any desired extent, so that the capacity may be varied. The capacity of this condenser can be calculated in terms of the length of that portion of the tube D which is within the tube C.

A *method of mixtures* is employed.

A battery of constant cells (50 to 100 volts) is joined up as shown in fig. 122, one pole of the battery being to earth. With a 100-volt battery, if B and D are at zero potential, then A, C, and guard-ring are at 100 volts. The charges on B and D are negative, and the charges on A and C are positive.

The connections are next altered to those shown in fig. 123. The positive charge on A mixes with the negative on D. The charge on the *portion* of B immediately opposite A may be considered to mix with that on C; for the charge on the guard-ring neutralizes the charge on the part of B immediately underneath it when the connections are made as shown.

The condensers are charged to the same potential difference. Hence, if their capacities are equal (not counting the field just under the guard-ring), their charges are equal also. In this case, when the charges mix, the neutralization will be exact. To test if this is so, immediately after the mixing, an electrometer is connected up as shown by the dotted lines. If the electrometer deflects, then the neutralization is not exact. The sliding condenser is then readjusted and the experiment repeated, until a position of D is found which makes the capacities equal.

The dielectric slab is now removed and the experiment carried out as before, until a position of D is again found which makes the capacities equal.

The ratio of capacities of the cylindrical condenser can now be found from the two positions of D. Then if S_2 and S_1 are the capacities with and without the dielectric respectively, we should have, if the dielectric *filled* the condenser—

$$K = \frac{S_2}{S_1}.$$

But since the dielectric does not fill the condenser in the actual experiment, we have (Example 1, Art. 224)—

$$\frac{Kd}{K(d-t)+t} = \frac{S_2}{S_1},$$

from which K can be calculated.

The experiment may also be carried out by adjusting the plate B, instead of the sliding condenser, until the capacities are equal.

The change of connections indicated in the figures must be made as quickly as possible by means of a specially arranged key. The slab of dielectric must be carefully freed from charge before testing, and care must be taken to avoid friction.

The values of K for the more important solid dielectrics are given in the following table:—

Substance.	Specific Inductive Capacity.
Dense flint glass...	9·89 (Hopkinson).[1]
Hard crown glass.	6·96 (Hopkinson).
Paraffin..............	2·29 (Hopkinson).
Ebonite..............	3·15 (Boltzmann).[2]
Mica	6·64 (Klemencic).[3]
Sulphur.............	3·84 (Boltzmann).
Resin	2·55 (Boltzmann).

The specific inductive capacity increases very slightly with rise of temperature.

212. **Dielectric Absorption.**

Charge a Leyden jar strongly from an electrical machine. Now discharge it with the tongs and make these *touch* both the knob and the outer coating. When the jar is thus apparently discharged, allow it to stand for a few minutes. Again apply the discharge tongs — a small spark is obtained. Allow the jar to rest again and repeat the test—a third spark may occur.

The charge which remains after the coatings of the jar were brought to the same potential by the tongs is called the *residual charge.* Since it produces at first no potential difference between the coatings, it is, for the time being, latent, but becomes active when the jar is allowed to rest.

The phenomenon of elastic recovery exhibits similar features. If you clamp a long strip of glass at one end, and bend it by applying a force at the other end, then if the force is released the glass flies back in consequence of its elasticity. But it does not return quite to its original position; it remains very slightly bent without exerting an elastic force. This "set", however, is only *subpermanent.* After a time the glass regains its original condition, or, if prevented from doing so, exerts elastic force.

The process by which a portion of the charge on the Leyden becomes residual or latent is termed *dielectric absorption.* Its nature is not fully understood, but it is no doubt due to the

[1] *Phil. Trans.*, 1881. [2] *Wien. Ber.*, 1872. [3] *Beiblätter*, vol. xii, p. 57.

same molecular properties as the imperfect mechanical elasticity of the material. This conclusion is supported by the fact that if the dielectric is a substance which shows perfect elastic recovery (*e.g.* quartz) no absorption occurs. The effect is best shown with substances of a complex composition, *e.g.* glass, shellac, paraffin-wax, ebonite, gutta percha. No absorption occurs in gaseous nor in most liquid dielectrics.

The phenomenon is rendered evident during the charging process as well as during discharge. If a Leyden is charged to a certain potential and then allowed to stand, its potential gradually falls off, but tends to a definite limit, provided the insulation is good. The effect is not due to leakage, for if the jar is now recharged to the original potential and allowed to stand insulated, its potential falls off, but not so much as before. If the charging potential is maintained by a battery we may distinguish, when the connections are first made—

(i) a sudden rush of electricity, which goes to charge the condenser;

(ii) a continued, but decreasing flow, which makes up for the loss of potential due to absorption.

(iii) a steady, but exceedingly minute flow due to leakage.

Instantaneous Capacity.—Dielectric absorption is a source of considerable trouble in the measurement of capacity. Its effect is equivalent to a slow increase of capacity whilst the condenser remains charged. In order to arrive at definite results, measurements have been made in which the charge was only maintained for a small fraction of a second, being then immediately reversed. The value so obtained is called the "instantaneous capacity". In some of Gordon's experiments the charge was reversed many thousands of times per second.

Absorption must be allowed for in cable-testing. In the measurement of cable capacity it is usual to allow a "one-minute" electrification. The paper- (really air-) insulated cables recently introduced greatly reduce the absorption.

It is important to observe that the effect takes place *all through* the dielectric—not at the surface; it takes place equally in all parts of the tubes of induction in the dielectric.

213. Dielectric Strain.

The material of a dielectric transmitting electric induction is subjected to a mechanical strain. In some cases this is a compression along the lines of force combined with an extension at right angles to them, whilst in other cases the reverse conditions hold. Kerr discovered and investigated these strains optically with the aid of polarized light. The effect is proportional to the square of the electric force. It is not instantaneous in solids, but requires an appreciable time (about one minute) to reach its maximum value, and disappears at a corresponding rate when the electric force is removed.

The volume of a Leyden jar increases when the jar becomes charged.

214. Dielectric Strength.

If the strain to which a dielectric is subjected becomes greater than a certain value the material breaks down. If a Leyden jar is made of very thin glass, and is charged to a high potential-difference, a disruptive discharge may occur, the glass being perforated. The power of resisting disruptive discharge—termed dielectric strength—may be expressed in terms of the electric force required to produce the disruption. The dielectric strength of solids and liquids is much greater than that of gases.

215. Dielectric Polarization.

When a dielectric is subject to a field of electric force it becomes polarized. According to the corpuscular theory, the polarization consists of a relative displacement of the positive and negative electricities of which the atoms of the dielectric are composed. It is analogous in some respects to the polarization of the molecules in a magnetizable substance, but there are important differences; the stresses in a polarized dielectric

cannot be entirely accounted for by quasi-magnetic polarization without the assumption of special stresses at the surfaces of conductors in contact with the dielectric. In respect of total mechanical action, the comparison may be applied: a charged body attracts a mass of dielectric (not independently charged) just as a magnetic pole attracts a mass of soft iron. Assuming a quasi-magnetic polarization, Boltzmann has determined the inductive capacities in certain cases by comparing the attractions exerted by a charged body on a ball of the dielectric, and on an equal ball of conducting material. The results, except for a few substances, agree fairly well with those obtained by other methods.

216. **Methods of Charging Insulators.**

A mass of insulating substance may be readily charged by holding it near a pointed conductor which is giving off an "electric wind". This method is used in the Holtz influence machine.

The charge on the surface of an insulator is readily removed by passing the substance through a flame. But if a charged body is held near the insulator whilst the latter is held over a flame, and the flame removed before the charged body, an induced charge will be found on the insulator. The flame covers the substance for the time being with a conducting layer. (See also Art. 111.)

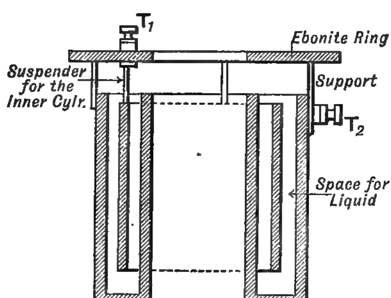


Fig. 124. –Condenser for Liquid Dielectrics

FLUID DIELECTRICS

217. The specific inductive capacities of a number of liquids were determined by Hopkinson, the method of mixtures described above being adopted. In place of the condenser AB a pair of cylinders (as shown in

fig. 124)[1] was used, one of which served as a vessel to contain the liquid. The capacity of this was determined in terms of the sliding condenser reading: first, with air as the dielectric; secondly, with the liquid as dielectric. Then K = ratio S_2/S_1.

Other methods have been used, some of which are mentioned below. The results of different observers differ considerably for certain materials. The surface of the liquid must be quite clean in order to avoid surface leakage.

The following table gives the values of K for the more important insulating liquids (Hopkinson)[1]:—

Liquid.	Specific Inductive Capacity.
Petroleum spirit...	1·92
Petroleum-oil.......	2·10
Turpentine..........	2·23
Castor-oil...........	4·78
Sperm-oil...........	3·02
Olive-oil............	3·16

218. Mechanical Force in Fluid Dielectrics.

Let two metal plates forming a true condenser be immersed in a liquid of inductive capacity K. Let the area of the plates be a, distance apart d, potential difference V, and surface density σ.

Then the energy stored per unit volume $= \dfrac{2\pi\sigma^2}{K}$ by Eqn. (4).

The energy in the condenser $= \dfrac{2\pi\sigma^2}{K} \times ad$. Let both plates be insulated, and one moved a small distance x towards the other. The remaining volume is $a(d-x)$. The tubes have been merely shortened, and the decrease in energy is—

$$\frac{2\pi\sigma^2}{K}\,ax.$$

If f is the mechanical force exerted by the field on the moving plate, the work done is fx. Hence since there is no external

[1] *Philosophical Trans.*, 1881.

source of energy, this work is equal to the energy lost by the field.

$$\therefore fx = \frac{2\pi\sigma^2}{K} ax,$$

$$\frac{f}{a} = \frac{2\pi\sigma^2}{K} \quad \text{......................(5)}$$

The effective mechanical force per unit area is 1/Kth of that exerted when the dielectric is air, for the same density of induction tubes.

Other expressions may be obtained equivalent to (5). See Art. 223.

219. Experimental Confirmation.

The expressions for the effective mechanical force on a conductor immersed in a fluid dielectric were tested experimentally by Quincke. The method consisted in measuring the force f and calculating K from equation (5). The value of this attraction constant was then compared with the value of the inductive capacity obtained in accordance with its definition as the ratio of two capacities.

A thin brass disc A, about 8 cm. in diameter, is suspended from one arm of a balance and made exactly horizontal. A second somewhat smaller disc B is supported in a horizontal position below A, the distance between them being about 1·5 mm. The two discs are kept from actual contact by three adjustable stops s, which touch three corresponding projections on the edge of the upper disc as shown at a. The stops are earthed, and A is also earthed through the balance. B is insulated and connected with (1) the inner coatings of a Leyden battery, (2) a Holtz machine, (3) the disc of an attracted-disc electrometer. The plates are supported within a glass vessel.

The plate A is first counterpoised and then an extra weight w added to the counterpoise. B is now charged from the machine until the attraction is so great that A is pulled into contact with the stops and held there. The potential of the jars, and therefore of B, is now *slowly* let down by means of a dry cotton thread, and the electrometer screw is adjusted continuously so as to keep pace with the fall of potential. At a certain potential the pull of the

induction-tubes reaches the value w, and the plate A is immediately pulled away from the stops. At the same instant the electrometer reading is taken. In this way the force w, for a known potential-difference, is obtained.[1]

The experiment is first carried out with air as the medium. The vessel is then filled up with the liquid to be tested, the

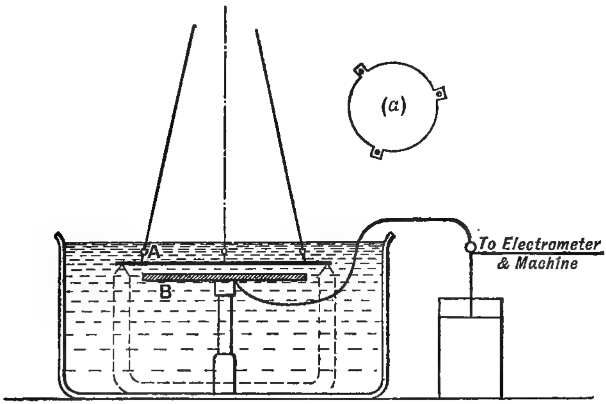


Fig. 125

counterpoise suitably altered to allow for the buoyancy, and the experiment repeated.

If w_1 and w_2 are the forces with air and liquid respectively, V_1, V_2 the potentials, then by equation (iv), Art. 223—

$$\frac{w_2}{w_1} = \frac{KV_2^2}{V_1^2}, \text{ or } K = \frac{w_2}{w_1} \cdot \frac{V_1^2}{V_2^2}.$$

The value of K so obtained is next to be compared with the inductive capacity obtained by the condenser method. The electrometer screw is set at about the same value as in the above experiment. The machine is then worked until the electrometer gauge overshoots the sighted position. The potential is now gradually let down as before. At the moment when the gauge regains the sighted position, a key q is put over so as to disconnect the jars and join the plate B through a ballistic galvanometer to

[1] Double readings and a method of differences are taken to avoid the unknown zero of the electrometer scale.

earth. The quantity of electricity in the condenser AB is thus obtained (for a known potential difference) in terms of the galvanometer throw. This is done for air and each of the liquids in turn.

If d_2, d_1 are the throws for liquid condenser and air condenser respectively; S_2, S_1 the capacities, we have—

$$\frac{Q_2}{Q_1} = \frac{d_2}{d_1}, \quad \therefore \frac{S_2V_2}{S_1V_1} = \frac{d_2}{d_1}.$$

$$\therefore K = \frac{V_1d_2}{V_2d_1}.$$

The values of K obtained with the two methods by Quincke agreed well in the case of most liquids, thus supporting the theory by which equation (5) is obtained.

The following numbers, obtained by Quincke,[1] illustrate this:—

K = specific ind. cap. obtained in accordance with the ordinary definition.

K_t = the constant in the formula for attraction.

Substance.	K.	K_t.
Ether	4·211	4·394
Carbon-bisulphide...	2·64	2·541
Benzol..................	2·359	2·36
Petroleum.............	2·025	2·073

220. Modification of the Law of Force.

Imagine two small spheres charged, say, oppositely, and suspended in air. The resultant force of attraction can be explained as due to the pull of the tubes of induction (Art. 131), and is given by—

$$f_1 = \frac{ee'}{d^2}.$$

Suppose now that the spheres are suspended in the middle of a large tank of oil, and let the inductive capacity = 3.

[1] *Wiedemann's Annalen*, vol. xix, N.S.; *Proc. Roy. Soc.*, vol. xli, p. 459.

The tubes of induction retain their shape and number, so that σ is unaltered. Thus by Eqn. (5) each tube pulls with one-third of the force that it exerts in air. The resultant attraction is therefore reduced to one-third. Thus—

$$f_2 = \tfrac{1}{3}\frac{ee'}{d^2}.$$

In the general case where the inductive capacity is K, we have by similar reasoning—

$$f_2 = \frac{1}{\mathrm{K}} \cdot \frac{ee'}{d^2} \quad \text{(6)}$$

> **The attraction or repulsion between two bodies immersed in a fluid dielectric is 1/Kth of the force exerted in air, provided the fluid occupies the whole field.**

The essential condition that this relation may hold is that the change of dielectric does not alter the distribution of the tubes of induction. The tension and pressure of the tubes of induction are then reduced in the ratio K : 1 at every point of the field by the substitution of the dielectric.

221. Force Exerted on the Dielectric.

The effective mechanical force exerted on the dielectric per unit area of induction tubes may be found thus—

Tension.—Suppose that a plate condenser is placed with its surfaces horizontal in a vessel containing a liquid dielectric, and so arranged that the lower plate only is covered by the fluid. Let the condenser be charged to density σ. Then pull on the upper plate (in air) per sq. cm. is $2\pi\sigma^2$. The pull on the lower plate is $2\pi\sigma^2/\mathrm{K}$. Hence by the equality of action and reaction there must be a resultant pull on the dielectric between the plates

$$= 2\pi\sigma^2\left(1 - \frac{1}{\mathrm{K}}\right) \text{ per sq. cm.} \quad \text{(7)}$$

The liquid is therefore heaped up slightly between the charged surfaces.

Pressure.—Similar reasoning may be applied to find the

lateral pressure. Let condenser plates be immersed edgewise in the liquid. Then since V is the same for the immersed portion and the part outside we have—

$$\text{Electric force in the air} = \frac{V}{d}$$
$$= \text{Electric force in the liquid.}$$

Thus, if σ is the surface density in air, and σ' the surface density for the immersed portion,

$$4\pi\sigma = \frac{4\pi\sigma'}{K} \text{ by equation (3).}$$
$$\therefore \sigma' = \sigma K.$$

The tubes of induction therefore collect more densely in the liquid.

Now the lateral pressure of the tubes in air $= 2\pi\sigma^2 = \dfrac{F^2}{8\pi}$.

Also, the effective „ „ „ liquid $= \dfrac{2\pi\sigma'^2}{K} = \dfrac{KF^2}{8\pi}$.

There is thus a greater effective pressure in the liquid owing to the greater density of the induction. The difference is exhibited as a mechanical pressure at right angles to the tubes of induction, tending to urge the liquid into the air space. The force per unit area is therefore—

$$\frac{F^2K}{8\pi} - \frac{F^2}{8\pi} = \frac{F^2}{8\pi}(K-1)\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Experimental Confirmation.

This formula is verified in an investigation by Quincke, in which the values of K obtained on the assumption of the truth of this formula are compared with those obtained from the ratio of capacities.

Two thin brass discs are supported horizontally, one just below the other, within a glass tank (fig. 126). The upper disc has a hole at the centre which leads into a tube attached to the plate at right angles. This tube communicates with a U-tube containing a liquid to serve as a manometer, and with a drying tube containing calcium chloride. The latter can be closed by a stop-cock. The

lower disc also communicates with a Leyden battery and electrometer.

The stop-cock is opened, the tank filled up with liquid, and then a little air is forced gently through the tube, after which the stop-cock is closed. The space between the plates then consists of a "disc" of air A, surrounded by a flat "ring" of the liquid L. The pressure of the enclosed air is indicated by the manometer.

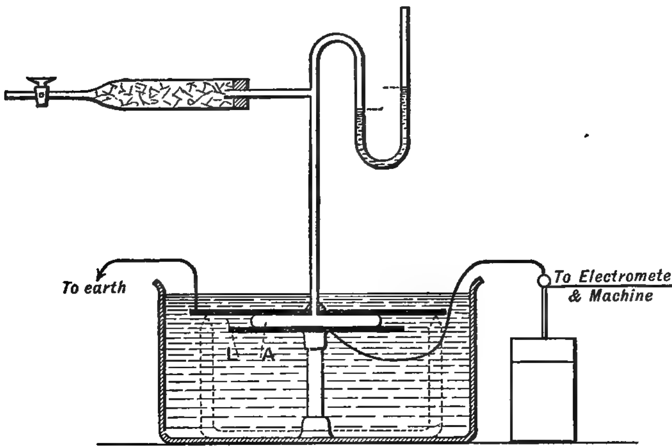


Fig. 126

The Leyden jars are now charged. The tubes of induction collect between the two discs, but most densely in the liquid ring. The greater lateral pressure of the tubes in the liquid causes the liquid to exert a mechanical pressure on the air bubble, which is thus compressed, and the liquid rises in the manometer. If p is the increase of pressure indicated by the manometer, we have by Eqn. (8) above—

$$p = \frac{F^2}{8\pi}(K - 1).$$

But the electric force F = potential difference V ÷ distance between the plates d. Thus—

$$p = \frac{V^2}{8\pi d^2}(K - 1),$$

from which K can be determined.

The following values obtained by Quincke show the accordance with the values obtained from ratio of capacities:—

Substance.	K (condenser).	K (pressure).
Ether..........	4·211	4·672
Benzol.........	2·359	2·375
Petroleum.....	2·025	2·149

In the above we have only considered the total mechanical effects on the dielectric. The discussion of the localization of dielectric stress is beyond the scope of this work.

222. Examples of the Mechanical Force.

The above equations show that the force exerted on a conductor per unit area depends on the two following quantities:—

(1) The density of the tubes of induction at the surface.

(2) The sp. ind. cap. of the medium **in contact** with the surface.

In applications of the formulæ it is necessary to observe carefully whether the conditions of the problem leave the charges on the conductors constant or the P.D.'s constant.

I. **Constant Charge.**—(*a*) If two metal balls are oppositely charged, and are suspended a short distance apart, they attract each other owing to the tension of the induction tubes. If a large block of wax is placed between the balls the tubes tend to collect in the wax. The density on the nearer side of the balls is therefore increased. The medium touching the conductors is air still, and therefore the attraction is *increased.*

(*b*) Suppose, however, that two oppositely charged conductors are immersed in the centre of a large bath of oil. Here there is no rearrangement of induction tubes. σ is unaltered, but the medium in *contact* with the conductors has greater inductive capacity. Hence the attraction is *diminished.*

(*c*) Let a charged conducting plate be held just over the

bottom of an earth-connected tray. The force on each (per sq. cm.) is $2\pi\sigma^2$. Let the tray be partly filled with dielectric liquid. The density of tubes is unaltered, and since the medium touching the upper plate is air, the force exerted on this plate is unaltered. The force acting on the tray is reduced to $2\pi\sigma^2/K$.

II. **Constant Potential-Difference.**—In each of the preceding examples we have supposed the charge or number of induction tubes constant. The results are, however, quite different if the two conductors are joined to the poles of a battery, or otherwise *maintained* at a constant difference of potential. In this case the introduction of the dielectric increases the capacity of the system, and the density is therefore increased. Considering the three examples given above, but introducing the condition of constant P.D., we have the following results:—

(*a*) When the block of wax is placed between the balls the attraction is *increased*, partly owing to the tendency of the tubes to collect in the dielectric, and partly to the increase of charge which maintains the potential-difference.

(*b*) If the oil has inductive capacity K, the tubes become K times as dense at every point (since owing to the increase of capacity an extra charge is drawn from the battery to restore the original potentials). Hence, since the force per unit area in air is $2\pi\sigma^2$, the new force per unit area is given by—

$$\frac{f}{a} = \frac{2\pi(K\sigma)^2}{K}; \text{ for } K\sigma \text{ is the new density.}$$

$$\therefore \frac{f}{a} = 2\pi\sigma^2 . K.$$

Comparing this with the result when the charge is constant, we see that here the force is *increased* in the ratio $1 : K$, whereas in the former case it was decreased in ratio $1 : \frac{1}{K}$.

(*c*) Here there is an increase of capacity and density. Hence the force acting on the plate in air is increased. For numerical example see below. (Art. 224, Ex. 3.)

223. Formulæ.

For convenience of reference, the principal formulæ modified by the specific inductive capacity are given below.

I. Electric force

$$F = \frac{4\pi\sigma}{K} \quad \text{(i)}$$

II. Mechanical force *per unit area* of conductor. (Energy per unit volume, and tension or pressure in induction tubes are given by the same expression.)

$$\frac{f}{a} = \frac{2\pi\sigma^2}{K} \quad \text{(ii)}$$

From equation (1) this becomes—

$$\frac{f}{a} = \frac{KF^2}{8\pi} \quad \text{(iii)}$$

For the uniform field between the plates of a condenser containing one dielectric, where d is distance apart of the plates and V the P.D., we have $F = V \div d$. Hence—

$$\frac{f}{a} = \frac{V^2K}{8\pi d^2} \quad \text{(iv)}$$

III. Mechanical force per unit tube (energy per unit length of unit tube). Dividing (2) by σ, we get—

$$\begin{aligned} f &= \frac{2\pi\sigma}{K} \quad \text{(v)} \\ &= \frac{F}{2} \\ &= \frac{V}{2d} \text{ (for condenser field, uniform dielectric)} \quad \text{(vi)} \end{aligned}$$

The student should carefully compare equations (ii) and (iii), and notice the meaning of the occurrence of K in the numerator in one case and denominator in the other.

224. Examples.

1. A condenser consists of two parallel plates, area a, separated by a small distance d, and contains a thin plate of dielectric of inductive capacity K and thickness t. Find the capacity.

Let σ = density of the tubes of induction.

$$\text{Electric force in the air-space} = 4\pi\sigma.$$
$$\text{Electric force in the dielectric} = \frac{4\pi\sigma}{\mathrm{K}}.$$

Work done in taking unit charge through the air-space

$$= \text{electric force} \times \text{distance} = 4\pi\sigma(d - t).$$

Work done similarly through the dielectric $= \dfrac{4\pi\sigma}{\mathrm{K}}t$.

$\therefore$ the potential difference of the plates $= 4\pi\sigma\left\{(d - t) + \dfrac{t}{\mathrm{K}}\right\}$

$$= \mathrm{V}.$$

Also $\quad \mathrm{Q} = \sigma \times a.$

Thus $\quad \mathrm{S} = \dfrac{\mathrm{Q}}{\mathrm{V}} = \dfrac{a\mathrm{K}}{4\pi(\mathrm{K}d - \mathrm{K}t + t)}.$

If the slab were removed, the capacity would be $\mathrm{S}' = \dfrac{a}{4\pi d}$.

The ratio in which the capacity is increased by the introduction of the dielectric is given by—

$$\frac{\mathrm{S}}{\mathrm{S}'} = \frac{\mathrm{K}d}{\mathrm{K}(d - t) + t}.$$

2. A condenser is formed of a metal tray with a plate 1000 sq. cm. area held 2 cm. from the bottom. It is charged to a P.D. 1500 volts. Find the pull on the plate.

$$1500 \text{ volts} = \frac{1500}{300} = 5 \text{ electrostatic units.}$$

Let σ = surface density. Then—

$$5 = 4\pi\sigma \times 2,$$
$$\sigma = \frac{5}{8\pi}.$$

Hence the pull per unit area $= 2\pi\sigma^2 = \dfrac{25}{32\pi}$.

Total force $= \dfrac{25000}{32\pi}$ dynes $= \dfrac{25000}{32\pi \times 980}$ grams wt.

$$= \cdot 253 \text{ gram wt.}$$

3. In Example 2, how will the pull on the upper plate be affected, if a dielectric fluid (K = 3) is poured into the tray to a depth of 1·5 cm., (*a*) if the charge is constant? (*b*) if the potential difference is constant?

(*a*) If the charge is unaltered, the pull is the same as before; since σ and K for the medium touching the plate (air) are both unchanged.

(*b*) If the potential difference is maintained constant, the battery or other source supplies more charge when the dielectric is introduced, so that σ is increased. Let σ' be the new value of the density of tubes.

$$\text{Then electric force in air} = 4\pi\sigma',$$
$$\text{and electric force in fluid} = \frac{4\pi\sigma'}{\text{K}}.$$
$$\therefore \text{V} = 4\pi\sigma' \times \cdot 5 + \frac{4\pi\sigma'}{3} \times 1\cdot 5,$$
$$i.e.\ 5 = 4\pi\sigma'.$$
$$\therefore \sigma' = \frac{5}{4\pi}.$$

Therefore pull on the plate in air ($2\pi\sigma^2 \times$ area)

$$= 2\pi \,.\, \frac{25}{16\pi^2} \,.\, 1000 \text{ dynes}$$
$$= \frac{25000}{8\pi \times 980} \text{ grams wt.}$$
$$= 1\cdot 012 \quad " \quad "$$

4. A small ball of shellac suspended by a long silk fibre is passed through the flame of a spirit-lamp, and an electrified ball is then brought near to it. Will it be attracted, and if so, why? What is the object of passing it through the flame? (1900.)

The ball of shellac will be weakly attracted. The field is distorted, as explained in Art. 210.

The object of passing the insulator through the flame in this case is to get rid of any initial charge the surface might accidentally possess, which would mask the weak effect to be observed.

QUESTIONS

1. Two uncharged brass plates, each metallically connected with the cap of a separate electroscope, are placed parallel to each other. One is charged, and then a plate of shellac is inserted between them. What effects are produced on the electroscopes during these operations? (1895.)

2. Two equal horizontal metal discs A and B are placed symmetrically one over the other and separated by air, A being insulated and B earth-connected. When A is charged the plates

attract each other. Will the attraction be the same when the space between them is filled with paraffin? Give reasons. (1896.)

N.B.—It may be assumed that a fluid paraffin (petroleum) is used.

3. Describe an experiment for comparing the specific inductive capacities of two non-conducting liquids. (1898.)

4. Two electrified balls are in presence of each other; in what way is their mutual action modified by the introduction of a thick glass plate between them? Give reasons for your answer. (1899.)

5. What do you understand by specific inductive capacity? Describe some experiments which establish the existence of this property of bodies. (1901.)

6. Explain carefully what is meant by *specific inductive capacity*. How would the indications of an electrostatic voltmeter be affected by immersing the whole instrument in oil, which completely fills it? (1904.)

N.B.—Assume that the instrument remains connected to the battery or circuit, and find in what sense the deflection for a *given potential-difference* will be affected.

7. A parallel-sided slab of dielectric, 5 cm. thick and of specific inductive capacity 2, is placed between the plates of an air condenser consisting of two very large flat plates at a distance 10 cm. apart. What will be the effect on the difference of potential, between the plates of the condenser, of the introduction of the slab, and to what distance apart must the plates be moved to bring the potential back to its original value? (1905.)

CHAPTER XVII

ELECTROSTATIC MACHINES

225. Electrostatic machines may be divided into two classes, according as their action depends on (*a*) **friction** or (*b*) **induction.**

The general principle of frictional machines has been given in Art. 112. These machines are now quite superseded by the more convenient and reliable induction machines. We

have seen that, given a small initial charge, such arrangements as the electrophorus and replenisher enable us to obtain repeated supplies of electrification by induction. On the same principle, machines are constructed with a view to producing and maintaining a very high potential at a roughly constant value. The main types are the following:—

(1) The Holtz machine.
(2) „ Voss „
(3) „ Wimshurst „
(4) „ Pidgeon „

In all these there are two prime conductors, which become charged positively and negatively respectively. These (by analogy with voltaic cells) are termed the "poles" or "terminals" of the machine. The prime conductors are provided with movable rods, which can be brought into sufficiently close proximity for a spark to pass, or into actual contact.

226. The Holtz Influence Machine.

The principle of this will be understood with the aid of figs. 127, 128. A glass plate is mounted on a horizontal axis. On one side of the plate, at opposite ends of a horizontal diameter, are two collecting combs C, D, connected to the terminals P, Q by metal rods. CP and DQ form the prime conductors. A and B are insulated metal pieces termed the *inductors.* We shall suppose that a positive charge is given to A and a negative charge to B in order to start the machine.

The + on A, acting through the glass, attracts − electricity to the points C, and repels + to P. The − electricity streams from the points, and is carried by the "electric wind" against the surface of the glass plate. The latter, though an insulator, thus becomes electrified (see Arts. 111, 216). At the other comb a similar action takes place, the plate here becoming + whilst the − electricity is repelled to Q. When P and Q are *in contact* or near enough to spark, these repelled charges neutralize each other.

If now the plate is rotated in the direction of the arrows,

the negative charge is carried away from A by the glass, and the positive charge away from B. At the same time the action explained above continues. When the negative charge on the glass comes opposite to D the following processes take place.

(i) A positive charge is attracted from the points, whilst the corresponding induced negative is repelled to Q.

(ii) The positive charge which leaves the points streams to

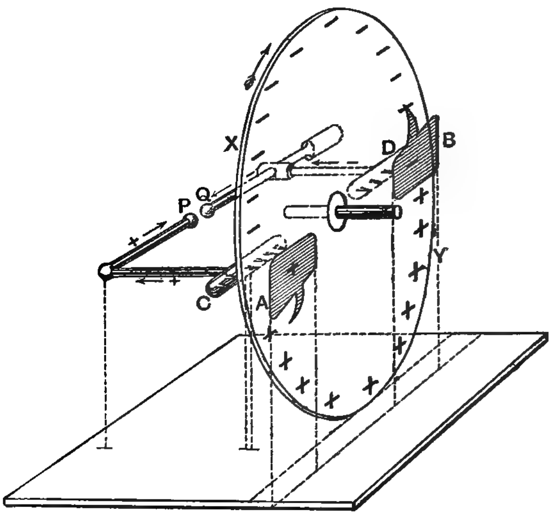


Fig. 127.—Holtz Machine—Theory

the plate, and neutralizes the negative charge which induced it. By these two processes the negative charge on the plate is virtually transferred to Q, and the glass is made neutral.

(iii) The negative charge on B now induces a further + at D and − at Q. By the agency of the points the + charge is transferred from D to the neutral glass.

The net effect of the above processes is that the charge on the glass is transferred to Q, and an opposite charge is induced in its place, so that the *charge on the glass changes sign in passing the comb.*

Notice also that the charge on the glass *before* reaching the comb is similar to that on the inductor. Advantage is taken of this to maintain the charge on the inductors against leakage. A long tooth projects from the inductor towards the approaching part of the plate, but does not quite touch the latter. By the usual action of points the inductor thus acquires a charge of the same sign as that on the glass.

Similar actions (with signs reversed) take place at the other comb and inductor.

The machine is started with P and Q in contact, so that no charge accumulates on the prime conductors. The initial charge is given to one of the inductors from a Leyden jar or other source. When the action is well started (indicated by a continuous luminous stream from the combs), the terminals P and Q may be separated; a torrent of sparks occurs in the gap.

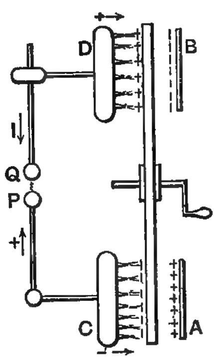


Fig. 128.—Holtz Machine—Theory

If the terminals P and Q are separated beyond sparking distance, the accumulation of charge on CP will continue until it is + charged right up to the points; and DQ will be − charged all over. Hence there will be no discharge from the points, and the glass will pass the comb, say C, without having its charge reversed. In this condition it reaches the tooth of the inductor B on the opposite side of the machine, to which it therefore gives the wrong kind of electrification. Thus the inductors soon become neutral, and may even have their charges reversed. To avoid this contingency, a "dummy" prime conductor is added, which resembles CPQD, except that there is no spark gap. The two combs are connected by a single brass rod, and the connection between the combs therefore cannot be broken. This rod is placed parallel to the diameter XY at an inclination of about 40° to the horizontal. Every portion of the glass plate comes opposite the conductors in the following order:—

(i) The collecting tooth of the inductor.
(ii) The comb of the prime conductor (C or D).
(iii) The comb of the "dummy" prime conductor.

The inductor is extended so that its influence shall reach to both (ii) and (iii). The action at (ii) and (iii) is of precisely the same nature, so that if reversal of charge fails at (ii), through the cause mentioned above, it will occur at (iii). With the extra pair of connected combs it is of course unnecessary to bring the terminals into contact in starting the machine.

Construction.—The inductors are not supported on rods, as indicated in fig. 127, but are attached to a large *fixed* glass plate. This is an indifferent insulator, and becomes inductively charged by the rotating plate. The induced charges prevent leakage of the charge on the rotating plate, so that the fixed sheet of glass serves much the same purpose as the flaps of an ordinary frictional machine. The inductor tooth on each side projects through a hole in the glass sheet, which is made fairly large and allows the ozonized air to escape. The inductors are made of *paper*, a partial conductor being found to act better than metal.

The chief disadvantages of the Holtz machine are that it always requires initial charging, and does not work well in damp weather.

227. The Voss Machine.

This differs from the Holtz merely in points of practical construction, not in principle.

1. In order to help the glass plate to carry the charge received from the combs, brass studs or *carriers* (usually eight in number) are fixed to the plate. A small wire brush in the middle of each comb transfers the charge from the prime conductor to the carrier, the comb doing the same duty for the glass.

2. To replenish the inductor charge from the carriers, a conduction method is adopted. The tooth is replaced by a stiff wire leading round the edge of the plate, and ending in a small wire brush which touches the carriers as they pass. The inductor charge is thus replenished from the same part of the plate as in the Holtz, but is obtained from the studs by conduction.

3. The inductors are mounted on a fixed sheet of glass as in the Holtz machine, but are made of tin-foil.

There is generally a minute initial charge which renders the machine self-starting except in unfavourable weather.

228. **The Wimshurst Machine** (fig. 129).

This has almost entirely supplanted the Holtz and Voss machines in recent years. It differs from these in that there are no fixed inductors. Two glass plates, some 45 cm. in diameter, are mounted so that they can rotate independently on a horizontal axle. They are driven by equal pulleys, but with crossed and open belts respectively, so that they rotate at equal speeds in opposite directions. Each plate is provided with twenty-four or more carriers consisting of radial strips of tin-foil or brass.

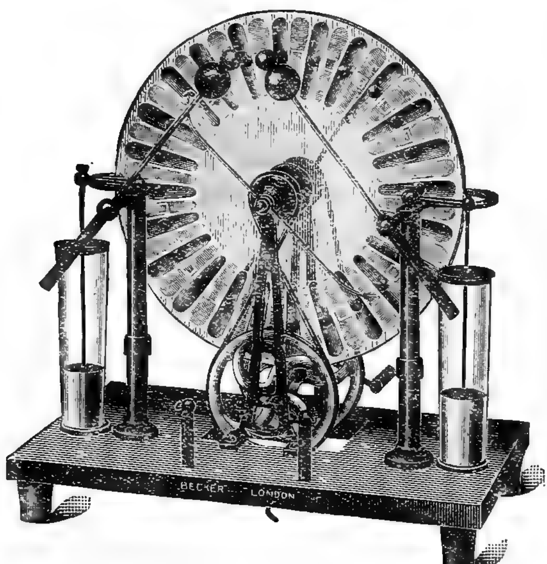


Fig. 129.—Wimshurst Machine

At each end of a horizontal diameter there is a pair of combs

connected with a prime conductor. Two brass rods terminating in wire brushes are fixed at an angle of 60° with the horizontal, one at the front and the other at the back. These "connecting rods" incline in opposite directions. They need not be insulated.

The action of the machine may be understood by reference to fig. 130, where the plates are represented as cylinders for the sake of clearness. The directions of rotation are shown by the arrows. Suppose that the back plate has an initial

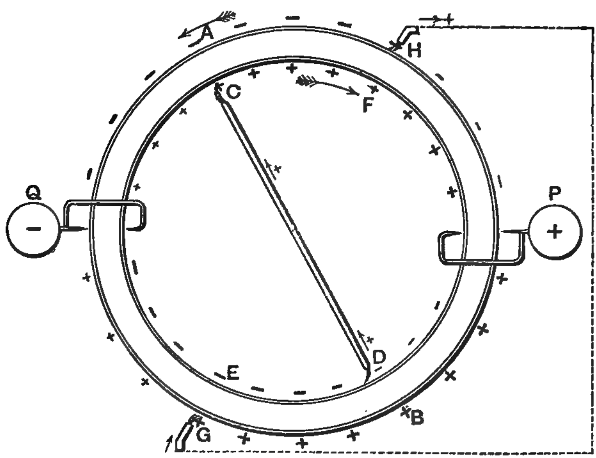


Fig. 130.—Wimshurst Machine—Theory

negative charge at A. This acts on the rod CD, attracting a positive charge to C, and repelling a negative to D. The induced charges stream from the brushes at C and D to the front plate (inner circle). The rotation of the front plate carries these charges round to the collecting combs P and Q, which will thus become charged positively and negatively respectively. But before reaching the collecting combs the induced charges must pass at E and F near the brushes G and H. The negative charge on reaching E acts inductively on the rod GH, attracting positive electricity to G and repelling negative to H. This effect is increased by the positive charge, which has

reached F at the same time. These induced charges are transferred to the back plate at G and H, and are carried round by the rotation to P and Q respectively. But again, before reaching the combs, the charges must pass near the brushes at C and D, so that the action we commenced with is repeated. The action of the machine may be summed up in the following way:—

"The charges received by the plate or sectors from the ends of one connecting rod are caused to act inductively on the other rod before they are allowed to pass to the prime conductor".

It is evident from this that the connecting rods must be set with reference to the direction of rotation.

The charges on the sectors or glass when the machine is in full action are shown by the signs in fig. 130. The small signs refer to sectors in a nearly neutral condition.

229. The Pidgeon Electrostatic Machine.

In the Holtz and Voss machines there is only one rotating plate, the charges being produced by fixed inductors. In the Wimshurst there are two rotating plates, and the carriers serve also as inductors during part of the revolution. By combining the two principles Mr. W. Pidgeon has produced a machine of very high efficiency.

In the figure for the Wimshurst we see that the charges induced at C and D are due to the influence of the moving sectors at A and B (which for the moment are behind C and D). If therefore a fixed − charged inductor is placed in front of C and a + charged one in front of D, these fixed inductors will assist the action of the moving sectors A and B. In the Pidgeon machine this is done.

Fig. 131 shows the action of the machine. The fixed inductors are shown at I, I, I, I. The connecting rods at P'—P_1', P'—P_2'.

The fixed inductors are replenished from the sectors which have not quite reached the collecting combs, by small brushes, as in the Voss machine. These are at the points marked P.

All the conductors are highly insulated. The sectors are separated by only $\frac{1}{8}$ in. This is rendered possible by the high insulation, the whole plate being covered with a layer of special insulating varnish consisting of a mixture of resin and paraffin-wax. Only the buttons of the sectors are left uncovered, so that they may make contact with the connecting

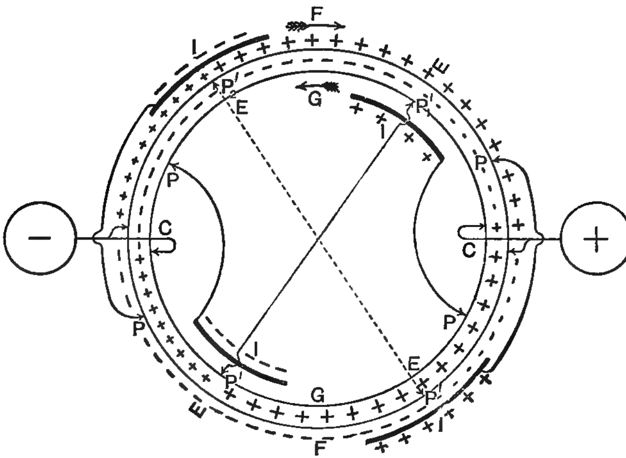


Fig. 131.—Pidgeon Electrostatic Machine—Theory

rods. The latter pass through holes in the inductors, being protected by ebonite tubes.

In a multiple plate machine of this type, the sectors are earthed at the moment of passing the inductors, and since the latter for the moment almost enclose the sector the induced charge is as great as possible.

230. Terminals of Induction Machines—Insulation.

The terminal ball of the prime conductor intended to be positive is usually smaller than that for the negative side. A positive charge escapes from a conductor less readily than a negative one, and to equalize the discharge the positive terminal must be more sharply rounded. The terminal balls

should be kept highly polished. The slightest corrosion or roughness tends to produce leakage by brush discharges.

The prime conductors are usually provided with Leyden jars. These are connected in cascade (see Art. 179). Their object is to increase the capacities of the terminals. The Leydens greatly modify the character of the discharge. The sparks obtained are much longer and brighter, but at the same time less frequent, than when the jars are disconnected. The presence of the jars does not increase the P.D. of the terminals. The discharges take place when the conductivity of the air has been sufficiently increased, the quantity of electricity in the Leydens being sufficiently great to produce the immediate supply of energy required by a long spark. If the gap is too long, the electrification is usually diffused by brush discharge.

If S is the capacity of each Leyden, the joint capacity is S/2. If V is the P.D. between the machine terminals in electrostatic units the energy stored in the jars is—

$$W = \frac{1}{2}\left(\frac{S}{2}\right)V^2$$
$$= \frac{SV^2}{4}.$$

Insulation.—The efficiency of an electrostatic machine depends largely on the insulation. The good results obtained with the Pidgeon machine are doubtless partly due to the high insulating properties of the varnish used. In Mr. Tudsbury's Patent Influence Machine the plates work within a steel case containing compressed air. A gas under pressure insulates more highly than under ordinary conditions, and very long sparks can thus be obtained.

231. Path of the Continuous Discharge.

When a continuous discharge is passing between the terminals of a machine a comparison may be drawn between the machine and a voltaic cell producing a steady current. Taking the Holtz machine, for example, we may trace the path of the positive electricity as follows.

The + electricity from the comb at C (fig. 127) flows round the prime conductor, across the spark gap, and reaches D. From D it passes with the electric wind to the plate. It is then carried round through the lower half of the revolution, until it comes opposite the points at C. The + electricity does not, however, flow from the plate to the points. Instead, the particles of air give up a + charge to the comb, and then pass to the plate, where they receive repayment for the electricity which they gave up. The circuit is thus completed.

The above takes account of only one-half of the current. The upper half of the plate (fig. 127) carries a negative charge round to D. If $-Q$ units are carried round per second, the points at D give up $+Q$ units to neutralize these. But since the charge is reversed they must give up a further quantity $+Q$ to charge the plate. Thus $+2Q$ units flow from D to the plate per second, and $-2Q$ from C to the plate. The current through the machine consists of a negative flow from C to D in the upper half of the revolution, and a positive flow from D to C in the lower half. But if we imagine the negative flow replaced by a positive one in the reverse direction we see that the upper and lower halves of the flow are in parallel, both being from C to D. Neglecting leakage, the flow across the spark gap $= 2Q$ units per second.

The student should trace out the circuit of a Wimshurst machine in a similar manner, remembering that part of the path of the current is along the connecting rod, and that the two plates are in parallel.

The mean current yielded by a machine is very weak, usually less than one-thousandth of an ampere. It may be measured with a suspended-coil galvanometer of suitable sensitiveness. Thus the current which a voltaic cell (P.D. say 1 volt) can produce in a wire is enormously greater than the current which a Wimshurst (P.D. say 100,000 volts) can produce through the same wire. This at first sight may seem opposed to Ohm's Law, but we must remember that the internal resistance of the generator must be taken into account. The effective internal resistance of an electrostatic machine is

extremely high. This is partly due to the air gaps at the combs, but more to the fact that the current has to be carried round artificially by the movement of the plates. The charge on the comb has to "wait" until the charge already on the plate has been moved out of the way by the rotation. If the speed of rotation is increased the charge on the plate is moved out of the way of that on the comb more rapidly, and the effective resistance is diminished. It has been found that the "internal" resistance of a Holtz machine is nearly inversely proportional to the speed of rotation. The speed does not affect the potential difference V on open circuit, but since the current is proportional to the speed, the effective resistance V/C varies inversely as the speed.

232. **Output and Activity.**

The "output" of an electrostatic machine at a given speed is defined as the quantity of electricity it supplies per second.

This may be determined as follows:—A condenser of high insulation and known capacity is connected across the terminals A and B (fig. 132) of the machine (the jars being removed). The condenser terminals are also connected to two polished metal spheres, which can be placed at an accurately measurable distance apart. On working the machine, sparks occur between the spheres at regular intervals. In each interval the machine produces enough electricity to raise the condenser coatings to the P.D. which produces the spark. Hence if S is the condenser capacity, V the potential difference, and n the number of sparks per second; then the output for the given conditions of sparking

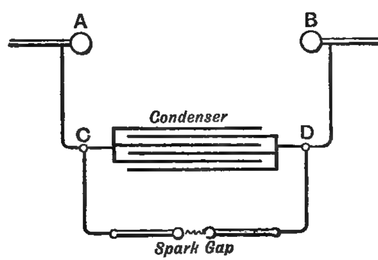


Fig. 132

$$= n\text{SV}.$$

Measurements of this kind are not susceptible of great accuracy on account of the unavoidable leakage at the high potentials employed. But if the spark length is kept below 1 cm., consistent results may be obtained. The P.D. may be ascertained from the spark length by reference to a table showing the relation between these quantities.

(Such tables have been drawn up from the results of special measurements with an electrometer. It is convenient to remember that a 1 mm. spark is produced by a P.D. of 4500 volts, and that for small sparks the P.D. is proportional to the spark length.)

The sparking distance must be accurately measured. This is done by mounting one of the knobs on a slide which is

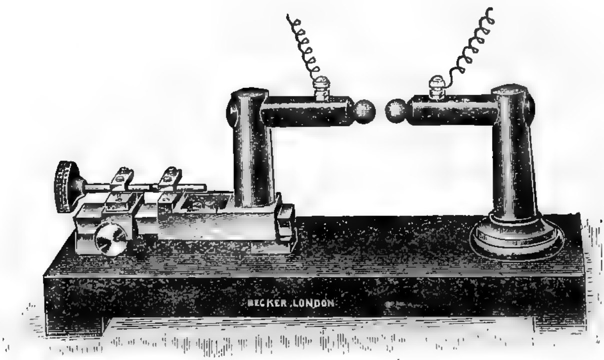


Fig. 133.—Riess Spark Micrometer

actuated by a micrometer screw. Fig. 133 illustrates the Riess spark micrometer, constructed on this principle. The knobs are mounted on polished ebonite pillars, the right-hand knob being fixed.

The "activity" of a machine is the energy it converts into the electrical form per second.

An estimate of this may be made by the condenser method given above. The energy communicated to the condenser in each interval between the sparks is $SV^2/2$. Hence the activity $= nSV^2/2$.

If S is in microfarads, V in volts, the energy per charge is $SV^2/(2 \times 10^6)$ joules, and the activity in watts $= \dfrac{nSV^2}{2 \times 10^6}$.

233. Comparison of Capacities.

The capacities of Leyden jars may be roughly compared by the aid of an electrostatic machine in good working order. The jars are connected one at a time, as shown in fig. 132. The outer coating of the jar may be kept to earth. The number of revolutions required to charge the condenser to sparking P.D. is determined by observing the total number of revolutions for (say) 20 sparks. This is done for each jar in turn, the spark gap being constant. The capacities are directly proportional to the number of revolutions per spark. The laws for series and parallel grouping may be illustrated in this way.

234. Example.

A condenser is formed of 41 parallel plates, each 30 cm. square, the alternate plates being connected as in a standard condenser. The distance between consecutive plates is 3 mm. The condenser is joined across the terminals of a Wimshurst machine and a 1 mm. spark gap. On working the machine at a certain speed sparks are produced at the rate of 50 per minute. Neglecting leakage, find the output and activity of the machine. (1 mm. spark is equivalent to 4500 volts.)

Capacity of the condenser

$$= 40 \times \frac{a}{4\pi t} = \frac{40 \times 900}{4\pi \times \cdot 3} = \frac{30000}{\pi} \text{ (e.s. units)}$$

$$= \frac{30000}{\pi} \times \frac{1}{900000} \text{ microfarad}$$

$$= \frac{1}{30\pi} \text{ microfarad.}$$

Charge collected in condenser and neutralized at each spark

$$= SV = \frac{1}{30\pi} \times 4500$$

$$= \frac{150}{\pi} \text{ microcoulombs.}$$

$$\text{Output} \quad = \frac{50}{60} \times \frac{150}{\pi} = 40 \text{ microcoulombs per second (very approx.).}$$

$$\text{Activity} \quad = \frac{n\text{SV}^2}{2 \times 10^6} = \frac{5}{6} \times \frac{1}{30\pi} \times \frac{(4500)^2}{2 \times 10^6}$$

$$= \cdot 089 \text{ watt.}$$

QUESTIONS

1. Describe and explain the action of either a Voss or a Wimshurst electric machine. (1905.)

2. Describe some apparatus by which an indefinitely large quantity of electricity may be obtained by means of electrostatic induction from a minute initial charge. (1902.)

3. Describe and explain the action of a Holtz machine. Does the velocity of rotation affect the quantity or the difference of potential produced? Give reasons for your answer. (1901.)

PART III.—CURRENT ELECTRICITY

CHAPTER XVIII

INTRODUCTORY PHENOMENA

235. It has already been shown that a steady current or continuous discharge of electricity may be maintained by the agency of voltaic cells or electrostatic machines. We have now to consider the phenomena accompanying the current and the forms of cells specially suitable for the production of strong currents. The three principal properties of the current are shown by its chemical, thermal, and magnetic effects.

236. **The Chemical Effect.**

Many conducting liquids, particularly acids and solutions of metallic salts, are decomposed by the passage of an electric current.

(*a*) Dip the wires joined to a cell into dilute sulphuric acid. Hydrogen is evolved at the wire joined to the zinc; the wire itself (if copper) is unaltered. The other wire is slowly dissolved so that in time the solution becomes blue through the formation of copper sulphate.

(*b*) Attach platinum plates to the wires leading from the poles of a battery and dip the plates into dilute acid. Hydrogen is evolved at one plate and oxygen at the other. The water is decomposed into its constituent elements.

(*c*) Soak a piece of blotting-paper in a solution of potassium iodide. Clean and wash the ends of the wires joined to the cell. Place the cleaned ends in contact with the paper but not far apart. A brown stain forms on the paper round the wire forming the + terminal. We have thus a convenient *pole-testing paper*.

The process of decomposing a substance by the action of an electric current is called *electrolysis.* The substance decomposed is termed the *electrolyte.* The plates or other terminals which convey the current to or from the electrolyte are termed *electrodes.* The cell with the liquid and plates forms a *voltameter* or electrolytic cell.

237. The Heating Effect.

When a current passes through a conductor (liquid or solid) the conductor becomes heated. This may be readily proved by passing the current through a coil of wire enclosed within the bulb of an air-thermometer (fig. 105). If a thin wire is used to connect the poles of a bichromate or secondary cell, it speedily becomes hot to the touch, or even red-hot.

238. The Magnetic Effect.

It was not until some twenty years after the discovery of the chemical action of a current that the magnetic properties became known. Oersted of Copenhagen found, in 1820, that a current would deflect a magnetic needle. The movement may be produced with the wire in almost any position near the needle. If the direction in which the needle is turned is compared with the direction of the current, it will be found that the following rule applies:—

> **Hold the right hand so that the fingers point along the wire in the direction of the current, and turn the palm of the hand towards the needle; then the outstretched thumb shows in which direction the N. pole of the needle is urged.**

This is called the "right-hand rule".

If a wire is bent to form a rectangle, and placed, in the plane of the meridian, round the needle, the deflection is increased; by the right-hand rule it will be evident that all parts of the rectangle urge the pole of the needle in the same direction. A rectangular frame wound with many turns of

wire and placed round the needle of a deflection magnetometer forms a *galvanoscope* or *multiplier*. Very weak currents may be detected in this way.

Reaction on the Circuit.—When a current exerts a force on a magnet, the latter—in accordance with Newton's Third Law—exerts an equal and opposite reaction on the circuit. De la

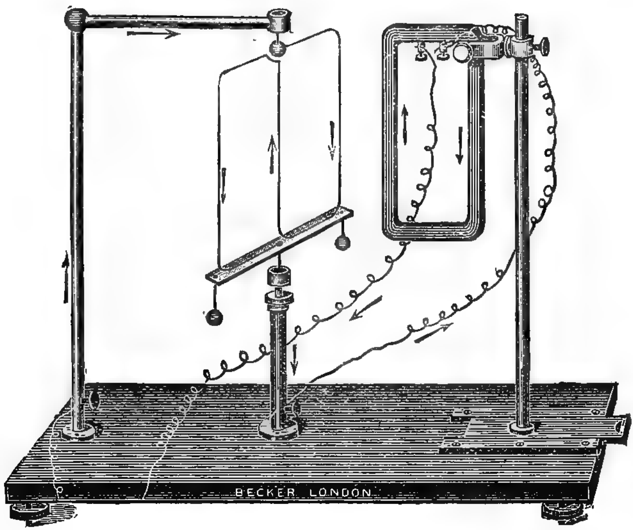


Fig. 134.—Mutual Action of Currents

Rive's "floating battery" affords an illustration. A small cell is attached to a circular coil, and the whole arrangement is floated, the cell being fixed through a wooden disc for this purpose. A magnet brought near the coil will attract or repel the floating system, according as the pole of the magnet would be visibly attracted or repelled if the coil were fixed and magnet movable.

Mutual Action of Circuits.—In the year immediately following that in which Oersted discovered the magnetic effect of a current, Ampère found that one current attracts or repels a

neighbouring one, and he made many experiments to investigate this action.

Fig. 134 shows a form of apparatus known as "Ampère's Stand". A wire is bent into a double rectangle. One end of the wire is suspended by copper fibres from the arm of a brass upright. The other end dips into a cup of mercury supported on a second upright. A current may then be sent through the wire, whilst the rectangle can rotate without breaking contact. A second circuit can be brought into various positions and its effect on the suspended wire ascertained. It is found that the following laws hold:—

1. **Parallel currents in the same direction attract each other.**
2. **Parallel currents in opposite directions repel each other.**
3. **Inclined currents tend to become coincident in position and direction of flow.**

The mechanical forces observed of course act on the *conductors.* It is merely for convenience that we speak of the *current* as being attracted or repelled.

239. Current Strength (Electrostatic Measure).

If we vary the number of cells in the battery, or change the connecting wires, we may increase or diminish the amount of chemical action produced in a given time. The quantity of heat produced, and the deflection of a galvanoscope included in the circuit also increase or diminish. In the ordinary theory of electricity we assume that these changes are quantitatively related to the amount of electricity passing per second. The assumption leads to consistent results, and is therefore adopted.

The strength of a conduction current in electrostatic measure is defined as the quantity of electricity which passes through any complete section of the circuit per second.

Thus if a quantity Q passes through the section in t seconds, at a uniform rate, and if C is the current strength,

$$C = \frac{Q}{t} \qquad \text{(1)}$$

An alternative and more frequently applied measure of current strength, which is termed the *electromagnetic* measure, is given later. In electromagnetic measure equation (1) must be regarded as expressing the definition of electric charge instead of current.

240. **Resistance.**

By the use of a galvanoscope and equal wires of (say) copper and iron, we may show that a weaker current passes through the circuit when the iron wire is substituted for the copper. Thus, other conditions being equal, the rate of flow of electricity through iron is less than through copper; iron has therefore the greater resistance. By similar experiments we may arrange the metals in order of increasing resistance. For the more important metals the order is—

Silver, copper, gold, zinc, platinum, iron, lead, mercury, bismuth.

It may also be shown that conducting liquids—electrolytes—have a resistance much greater than metallic conductors.

Notice that with the low potential difference of a voltaic cell distinctions can be drawn between the conducting powers of metals which are quite unnoticeable in frictional experiments. With the high potentials of the latter and the limited charge, the discharge is practically instantaneous through all metals. On the other hand, these high potentials enable us to arrange partial conductors and insulators in order of conducting power.

241. **Identity in Nature of Currents from Different Sources.**

Faraday established the identity of frictional and voltaic processes by many experiments.

(i) Moisten a piece of paper with potassium iodide solution, and place it in contact with the terminals of an electrostatic machine. When the machine is worked a brown stain appears on the portion touching the positive terminal.

(ii) Pass the discharge from a Leyden jar through a coil in the bulb of an air thermometer (fig. 105). The air expands, showing the heating effect of the discharge.

(iii) Make a galvanoscope with a large number of turns of fine wire (well insulated). Join the terminals to the prime conductors of a Wimshurst machine. When the machine is worked the needle is deflected, showing that a weak current passes.

Similar tests may be made with currents derived from other sources, such as the dynamo.

242. **Conduction through an Electrolyte.**

When the circuit of a Volta's cell is closed, the hydrogen liberated in the cell does not appear at the surface of the

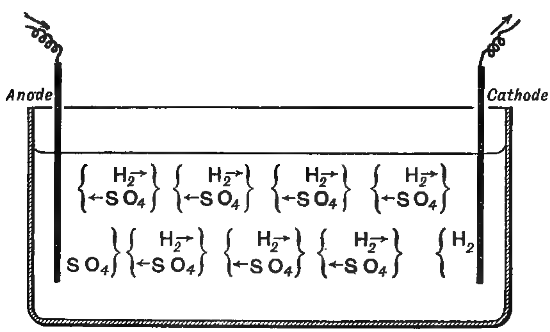


Fig. 135.—Grotthus's Hypothesis

amalgamated zinc. It is first liberated in the form of bubbles at the surface of the copper, although the copper itself is unacted on. To explain this and similar phenomena in electrolysis, Grotthus put forward a theory of transference of the hydrogen from molecule to molecule by repeated decompositions and recombinations.

In fig. 135 four molecules of sulphuric acid are represented

by their chemical formula. According to Grotthus' theory, an atom of Zn from the zinc plate combines with the (SO_4) of the first molecule, driving out the hydrogen. This combines with the SO_4 of the next molecule, driving the H_2 from this; and so on, all through the series, until the copper plate is reached. Here the H_2, being unable to combine with copper, is liberated.

The two parts of the molecule which travel through the liquid are termed *ions* (= "travellers"). The ion which goes with the current—that is, down the potential gradient—is called the *cation*, or positive ion. The other, which travels up the potential gradient, is the *anion*, or negative ion.

The electrode at which the current leaves the liquid is called the *cathode*. That by which the current enters the liquid is termed the *anode*.

In acids and most solutions of metallic salts the *cation* is hydrogen or a metal. When such solutions are decomposed by electrolysis the theory of Grotthus explains the main phenomena observed.

When copper sulphate solution $(CuSO_4)$ is electrolysed the SO_4, arriving at the anode, will combine with this plate if, as is usual, the plate is a copper one. For every atom of copper deposited on the cathode, an atom is taken from the anode and the solution remains of constant mean strength.

In recent years the theory of electrolytic conduction has undergone important modifications. The hypothesis of Grotthus can only be regarded as a first approximation.

243. Polarization.

The early investigators, following Volta, obtained the current from a "Crown of Cups" or from a Volta's pile. It was soon found that the current given by these arrangements rapidly falls off in strength, and after a few seconds from the time of making contact attains a low but constant value. The change in the electromotive force of the cell is termed *polarization*. It is due to the layer of hydrogen which covers the copper plate.

It is known that such a layer sets up a *back* E.M.F. This probably consists of (*a*) a reverse E.M.F. due to the chemical properties of hydrogen, (*b*) a reverse E.M.F. due to the energy required to detach the hydrogen in the form of bubbles. The second part of the polarization E.M.F. may be reduced by using as a positive pole some material with a rough surface, *e.g.* gas carbon; the sharp points and edges facilitate the liberation of the hydrogen bubbles. The same effect is obtained in Smee's cell by using a positive pole of silver coated with a deposit of platinum crystals.

Cells used for yielding a strong current of constant value are prevented from polarizing by the use of a substance which acts chemically or electro-chemically on the hydrogen.

244. **Compound Voltaic Cells.**

These may be conveniently grouped in two classes—

(1) *Replacement Cells.*—In these polarization is avoided completely, the hydrogen being replaced electro-chemically by the metal which forms the positive pole.

(2) *Oxidation Cells.*—In these the hydrogen is converted into water by the use of strong oxidizing agents.

245. **The Daniell Cell.**

This is the best-known type of replacement cell. It contains—

Zinc in a solution of zinc sulphate; and
Copper ,, copper sulphate.

The zinc sulphate is the exciting liquid and the copper sulphate the depolarizer. The two liquids are usually kept from mixing by a porous partition. When the circuit is closed the zinc is caused to dissolve in the zinc sulphate. The atoms of zinc—the positive ions—travel with the current as far as the partition; here they are exchanged for copper ions, which carry the current on to the copper plate. The changes may be represented as shown below, the second line indicating

the result of the changes and the dotted line representing the partition.

$$\begin{array}{l|ll:ll|l} Zn & (Zn\,SO_4) & (Zn\,SO_4) & (Cu\,SO_4) & (Cu\,SO_4) & \\ & (Zn\,SO_4) & (Zn\,SO_4) & (Zn\,SO_4) & (Cu\,SO_4) & Cu \end{array}$$

In the Minotto type of gravity Daniell, used on the Indian Telegraph Service, the porous pot is dispensed with, the solutions being kept separate by gravity, owing to their different densities. A disc of copper lies at the bottom of a glass jar. Above this is placed a layer of copper sulphate crystals. These are covered with a cloth; over this is a layer of clean sand, then another cloth, and lastly a thick zinc disc. The jar is filled with *water*, and the cell brought into working condition by putting it on short circuit for some hours.

The E.M.F. of a Daniell cell is about 1·07 volt.

246. **The Bichromate Cell.**

This is an oxidation cell, and is useful where a strong current is required. It contains—

Zinc in a solution of sulphuric acid.
Carbon in an acid solution of potassium bichromate.

Fuller's mercury bichromate cell is shown in fig. 136. The zinc is in the form of a stout rod with a broad foot. This stands in a pool of mercury at the bottom of a porous pot. The mercury creeps over the surface of the zinc, and keeps the latter well amalgamated. The pot contains dilute sulphuric acid, and stands in a larger vessel which contains a mixture of sulphuric acid and potassium bichromate, along with a carbon plate.

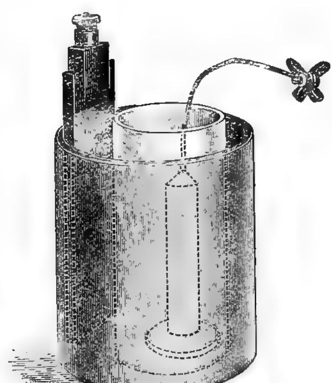


Fig. 136.—Bichromate Cell

The exciting liquid is sulphuric acid. The bichromate is the depolarizer, the chemical reaction being—

$$K_2Cr_2O_7 + 4H_2SO_4 + 3H_2 = K_2Cr_2(SO_4)_4 + 7H_2O.$$

As the depolarizer becomes used up the solution turns greenish, owing to the formation of chrome alum.

Chromic acid may be used in place of the bichromate. The porous pot may be dispensed with, as in the form known as the "bottle" bichromate. In this case the zinc must be removed when not in use, to prevent the local action, and crystallization of chromium salts on its surface.

247. The Leclanché Cell.

This is also an oxidation cell, and contains—

Zinc in a solution of ammonium chloride.
Carbon in a mixture of manganese dioxide and carbon.

The zinc dissolves in the solution, forming zinc chloride—

$$Zn + 2NH_4Cl = ZnCl_2 + 2(NH_4)'.$$

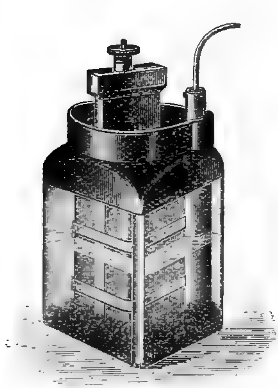


Fig. 137.—Agglomerate Leclanché Cell

The group $(NH_4)'$ probably attacks the water, forming ammonium hydrate and liberating hydrogen.

$$2(NH_4)' + 2H_2O = 2NH_4OH + H_2.$$

The hydrogen ions travel with the current to the manganese dioxide, which oxidizes the hydrogen, forming water.

$$2MnO_2 + H_2 = Mn_2O_3 + H_2O.$$

Since the depolarizer is a solid, it cannot act on the hydrogen so quickly as a liquid, and the cell runs down if allowed to give a strong current. It recovers, however, if allowed to rest, and is useful for intermittent work such as occurs on bell-circuits and telephones.

One of the best types is that known as the "agglomerate Leclanché" (fig. 137). On each side of a carbon plate is a block of "agglomerate". This consists of about equal proportions of manganese dioxide and carbon, mixed with a little gum-lac resin, and compressed into a hard mass. The blocks are laid on the carbon plate, then wrapped in canvas and bound with rubber bands. This is placed in a jar containing ammonium chloride solution and a cylinder of zinc.

An improved form—the "six-block agglomerate"—has a central carbon rod with six longitudinal grooves, in each of which a rod of agglomerate is laid, the whole being bound with rubber bands.

The E.M.F. of a Leclanché may reach 1·6 volt, but the average value is more nearly 1·4.

248. Dry Cells.

These are modifications of the Leclanché. The cell consists generally of a zinc cylinder, which serves as the negative pole and also as the containing vessel. A central rod of carbon forms the positive pole. The space between is filled with a paste which contains sal-ammoniac. The other substances present may be manganese dioxide, carbon, lime, magnesia, flour, zinc chloride, plaster of Paris, zinc oxide, etc., the composition of the paste being the chief feature which distinguishes one make of dry cell from another.

Dry cells give a strong current when fresh, but if allowed to become really dry, the internal resistance is so great that they become useless as sources of current.

249. Edison-Lalande Cell.

This consists of a plate of zinc and two plates formed of compressed copper oxide immersed in a strong solution of caustic potash. When the current passes the zinc dissolves, and the oxide is reduced to metallic copper:—

$$2KOH + Zn = Zn(KO)_2 + H_2$$
$$H_2 + CuO = Cu + H_2O.$$

The cell has a very small E.M.F. (·75 volt), but it is made of

large size, and has a small resistance. It can yield a strong current, and there is very little polarization, or local action.

Other efficient oxidation cells are the "Grove" and the "Bunsen". They contain zinc in dilute sulphuric acid and platinum or carbon in strong nitric acid. The E.M.F. is 2 volts, and resistance low, but they emit corrosive and irritating gases, which greatly militates against their use.

250. Local Action.

Common zinc readily dissolves in dilute sulphuric acid, but pure zinc does not. In the former case multitudes of voltaic circuits, each of microscopic size, are formed over the surface of the zinc by the particles of impurity present.

Such *local action* is almost entirely prevented by amalgamation. The cause of this is not yet understood. If the cell is placed under the receiver of an air pump, then when the pressure is reduced, hydrogen is given off even from amalgamated zinc.

When zinc is used in an *acid* solution amalgamation does not entirely prevent the local waste. Hence when a bichromate cell is not in use the zinc should always be removed. If the solution is not acid, there is very little effect on the zinc so long as the cell is idle.

Deposits of finely divided copper occur sometimes on the zinc plate of a Daniell cell, owing to diffusion of the copper sulphate, and quantities of botryoidal copper may form on the porous pot, this being due to a species of local action.

The Leclanché cell is almost entirely free from local action.

251. Keys and Commutators.

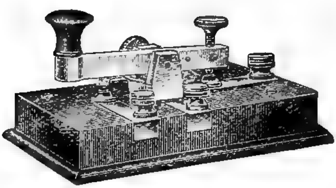


Fig. 138.—Morse Key

For convenience in making or breaking a circuit various forms of key are used. Fig. 138 shows a Morse "tapper", used for rapid make and break, and fig. 139 a "plug" key, which is used where a good contact is required for a considerable period.

A commutator is used to reverse the direction of the current in a portion of a circuit without altering the battery connections. Ruhmkorff's commutator is shown in figs. 140, 141.

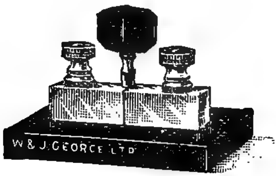


Fig. 139.—Plug Key

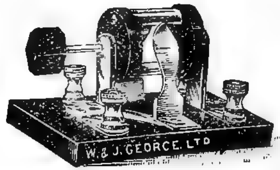


Fig. 140.—Ruhmkorff Commutator

Two brass bars G, H are mounted on a cylinder, so that they may be rotated about an axis EF. In one position (fig. 141 (*a*)) two springs or tongues C, D touch the bars G, H respectively.

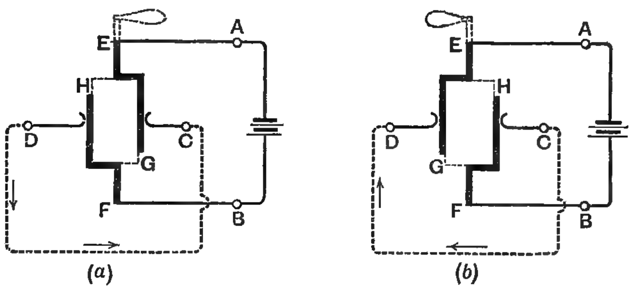


Fig. 141.—Ruhmkorff Commutator—Connections

When the cylinder is rotated through 180°, as shown in fig. 141 (*b*), the connections of G and H to C and D are reversed.

252. **Rheostats.**

The term *rheostat* is generally applied to a resistance which can be varied gradually for the purpose of adjusting the current strength to a desired value. Varley's carbon rheostat consists of a pile of discs of carbonized cloth between two metal plates. The pile can be subjected to various degrees of pressure by means of a screw. The variable resistance shown in fig. 142

is suitable for strong currents. It consists of carbon slabs about 10 cm. square and 1 cm. thick. These can be squeezed

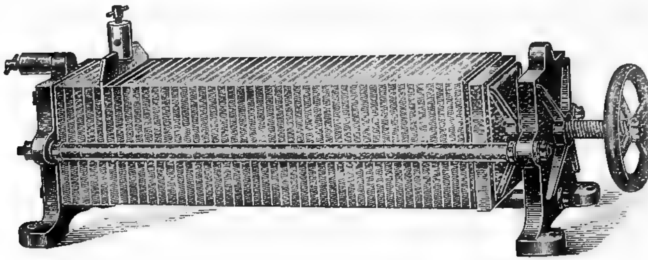


Fig. 142.—Carbon Plate Rheostat

to various degrees of good or bad contact, so varying the resistance.

A column of liquid (*e.g.* copper sulphate between copper plates) can also be used as a continuous rheostat.

QUESTIONS

1. A Leclanché cell is connected by long thin wires to a galvanometer, the needle of which is deflected. The poles of the cell are then bridged across for a short time by a piece of thick copper wire. After the removal of the thick wire the galvanometer deflection is much less than before, but gradually rises to its former value. Explain this. (1898.)

2. What kinds of cell would you choose for the following purposes:—

(*a*) To maintain a constant weak current for a long period?
(*b*) To maintain a strong current for a short period?
(*c*) To provide a current where little attention can be given to keeping the cell in order?

Give reasons for your choice.

CHAPTER XIX

CONDUCTION

253. The flow of electricity in a conducting circuit may be either steady or variable. We shall consider here the laws relating to steady currents only.

From the experimental fact that the potential at each point of a circuit carrying a constant current remains steady, we conclude that there is no continual accumulation of electricity at any part of the circuit. Hence, if A B (fig. 143) is a portion of a circuit, the flow across the section at A per second is equal to the flow across the section at B (whatever the cross-sectional areas at these sections may be), for there is no variation in the amount of either kind of electricity between A and B. Therefore—

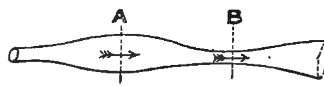


Fig. 143

The total current has the same strength at every part of the circuit.

The total current is that which flows through the complete section of the conductors. If the circuit divides, we must include both branches in the complete section. The quantity which reaches the junction per second equals the sum of the quantities flowing away along the branches.

The flow of electricity resembles that of an incompressible fluid. If the strength of a current of water through a pipe were measured by the *quantity* of water passing per second (not by the velocity), the current would be the same in the wide parts of the pipe as in the narrow parts.

If a number of conductors are joined end to end so that the same current flows through each, they are said to be connected in *series*. If joined so that the current divides between them, they are said to be joined in *parallel* (or multiple arc.)

254. Comparison of Currents.

For this purpose we may use a voltameter, a convenient form being the copper voltameter. This consists of two thin plates of copper immersed in a vessel containing a saturated solution of copper sulphate. One of the plates is first carefully weighed, and the voltameter is then joined in the circuit so that the current to be measured passes through it, the weighed plate being the kathode. After a definite interval, say one hour, the kathode is washed, dried, and reweighed. The difference gives the weight of copper deposited. To show that this weight is proportional to the current, we may proceed thus:—

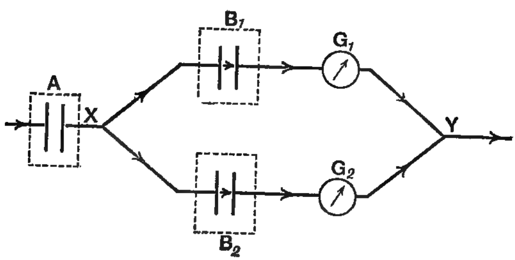


Fig. 144

Arrange two branches of a circuit XB_1Y, XB_2Y in parallel (fig. 144), each containing a voltameter and galvanoscope. Place a third voltameter at A in the main circuit. Complete the circuit through a Daniell battery and plug key. Close the circuit, and notice if the galvanoscopes (these must be of similar construction) indicate equal currents. If not, adjust the currents to equality by altering the resistance in the branches. Determine the weight of copper deposited in 1 hour in each voltameter. The weight of copper deposited in B_1 or B_2 will be found to be just one-half of that deposited in A. But since the current divides equally, the current flowing through B_1 or B_2 is one-half of that which flows through A. Thus the weight of copper deposited is proportional to the current strength.

The weight of copper deposited may therefore be taken as a convenient measure of current strength.

Calibration of Galvanoscope.—A voltameter may be used to ascertain the relative current strengths indicated by the readings of a galvanoscope.

If the two instruments are connected in series, they carry the same current. The reading of the galvanoscope can be observed, and the quantity of copper deposited in, say, 1 hour can be determined. If this is repeated with different strengths of current, a curve may be plotted with weights of copper as abscissæ and deflections as ordinates. The relative current strengths indicated by different readings of the galvanoscope may then be ascertained from the curve.

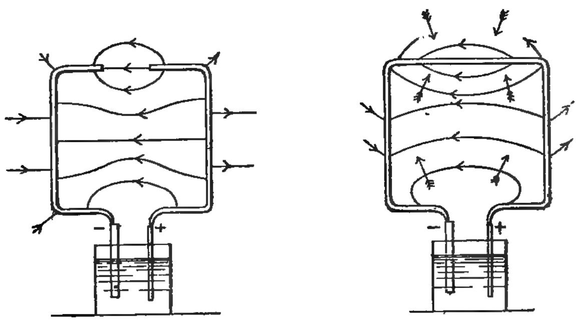
(The experiment may be carried out more quickly by using the hydrogen voltameter (Art. 315). The galvanoscope here referred to is one suitable for reading in amperes—not a sensitive galvanoscope.)

255. Electric Field of a Circuit.

It has been mentioned that the circuit carrying a current is surrounded by a *magnetic* field, but we must observe that generally an *electric* field also accompanies the current. This field is usually very weak, and can only be shown by sensitive instruments such as the quadrant electrometer; but the magnetic field can usually be detected with an ordinary compass-needle. We must also observe that the lines of electric force end on or in the conductors forming the circuit, whereas the lines of magnetic force are closed curves linked with the circuit. A third point of difference is that the electric field exists when the circuit is open as well as when it is closed; but the magnetic field only exists when the current is running. Fig. 145 (*a*) and (*b*) will give an idea of the general shape of the lines of electric force of a simple circuit. In (*a*) the circuit is open. One side is at a uniform potential, and the other at a lower potential, also uniform. The lines of force end on the wire in directions at right angles to the surface of the latter. The charges producing this static field are maintained by the E.M.F. of the cell. In (*b*) the circuit is closed. The electric field is again steady, but since there is now a current the shape of the

lines is altered: *they slope forwards in the direction of the current.* This gives rise to a component electric force parallel to the wire, which urges the current along.

It should always be remembered that the current is urged along the wire by the electric force. When we say that a current is produced by a potential-difference we mean that it



(*a*) (*b*)
Fig. 145.—Electric Field of a Circuit

is produced by a distribution of electric force along the wire, the effect of which is expressed *in terms of* potential-difference.

256. Effective Electromotive Force.

In the widest sense of the term, "electromotive force" means any cause tending to set electricity in motion. Hence we must regard potential-difference as one kind of E.M.F. Another kind is the E.M.F. generated in a voltaic cell. This tends to send the electricity in one direction through the cell. The potential-difference at the terminals tends to send the electricity in the reverse direction through the *cell*, and on open circuit these opposite forces balance. Since there is no current on open circuit, we may say that the *effective* E.M.F. is zero.

Although potential-difference is in the widest sense an E.M.F., it is useful for practical purposes to make a restricted use of the latter term. A potential-difference is always an E.M.F. of a *secondary* nature; that is, it is due directly or indirectly to a displacement of electricity arising from some

other cause, such as frictional production, voltaic cells, thermo-junctions, dynamos. We may therefore distinguish between these primary causes and the potential-difference which they usually produce.[1] Thus we have two kinds of electromotive force—

(1) **E.M.F. generated in cells or other sources of current.**

(2) **E.M.F. due to electric fields,** *i.e.* **potential-difference.**

In what follows the term *electromotive force* will be used for the first kind only, the term *potential-difference* for the second kind, and where the two exist together, the algebraic sum will be termed the *effective* E.M.F.[2]

Thus if E is the generated E.M.F., V the potential-difference produced, and E′ the effective E.M.F.,

$$E' = E + V.$$

An example will make these distinctions clearer. Let a cell be on open circuit, and let the potential-difference at its poles be 2 volts. There is no current, and the effective E.M.F. is zero. Calling the direction from zinc to copper positive,

$$0 = E - 2,$$

Therefore $$E = 2.$$

Next let the poles be connected. A current flows, and the potential-difference at the poles is found to be reduced. Let the value be now $1\frac{1}{2}$ volts. Then—

$$E' = 2 - 1\tfrac{1}{2}$$
$$= \tfrac{1}{2}.$$

Thus the effective E.M.F. sending the current through the cell is $\frac{1}{2}$ volt.

[1] The terms potential-difference and electric field, as here used, refer to fields of sensible (that is, not atomic) dimensions.

[2] The distinction between P.D. and E.M.F. is sometimes based on the reversibility of the processes involved. The discussion of this is, however, beyond our present scope.

257. Resistance.

The strength of the current flowing in a conductor depends on the resistance to the flow as well as on the effective electromotive force, and we may now assign an exact quantitative meaning to the former term. Since, with a given conductor, there is a definite value of the current corresponding to each value of the effective E.M.F., we may make the following definition:—

> **The resistance of a conductor is the ratio of the effective electromotive force in the conductor to the current strength.**

Hence by definition—

$$R = \frac{E'}{C}, \qquad (1)$$

$$\text{or } C = \frac{E'}{R}.$$

If the conductor does not contain a source of E.M.F., then $E' = V$ and $R = V/C$.

258. Ohm's Law.

This, the principal law of conduction, was first enunciated by Dr. G. S. Ohm in 1827. It may be stated thus:—

> **The current in a given conductor is proportional to the effective electromotive force, if the temperature is maintained constant.**

The following alternative statement expresses the essential feature of the law:—

> **The resistance of a solid or liquid conductor is independent of the current strength, if the temperature is maintained constant.**

The proof of this statement, like that of other physical laws, lies in its agreement with the results of experiment, when it is tested under as many conditions as possible.

We shall describe two tests,[1] one for a solid and the other for a liquid conductor.

Proof of Ohm's Law for a Solid Conductor.

Coil up a long thick wire A B (fig. 146), and connect it in series with a calibrated galvanometer and a battery of several cells (large Daniell or secondary cells). Join the terminals of the wire to a quadrant electrometer. When no current flows, the readings of both instruments are zero. If the key K is now depressed, a current flows through the circuit K G A B, and the galvanometer indicates its strength. At the same time static charges are produced on the wires joined to A B and on the quadrants; the deflection of the electrometer needle indicates the potential-difference between A and B. Note both deflections. Now remove one of the cells; a smaller deflection is obtained on each instrument. Remove the remaining cells one at a time, and take the readings after each removal. The ratio of the electrometer deflection to the galvanometer deflection is a measure of the resistance, and this ratio should be constant. The following are the results of an experiment (No. 18 manganin wire):—

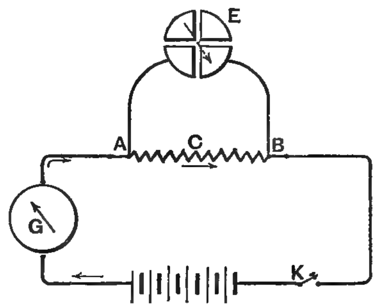


Fig. 146

Current (Galvanometer).	Potential-Difference (Electrometer).	Ratio V/C.
2	6·90	3·45
3	10·40	3·46
4	13·95	3·49
5	17·33	3·46
6	20·75	3·46

[1] The tests here given are useful on account of their directness. But our belief in Ohm's Law rests mainly on the *consistency* of the results obtained in experiments of the highest accuracy, when the law is *assumed* to be true.

In a very exact test the coil should be maintained at a constant temperature by immersing it in an oil bath surrounded by melting ice, or by using a thermostat.

Proof of Ohm's Law for an Electrolyte.

An electrolyte may be tested by the arrangement shown in fig. 147. Two platinum wires A and B are fused through a glass tube about 1 cm. in diameter at points about 20 cm. apart. The tube is provided with a funnel at each end. It is filled with the electrolyte, and the electrodes — two platinum plates—are suspended in the liquid, one in each funnel. The column of liquid is arranged in series with battery

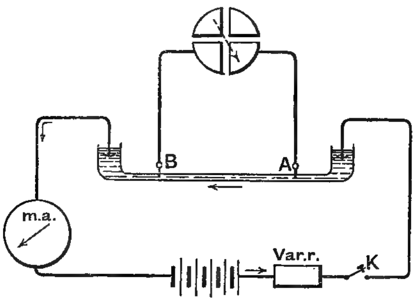


Fig. 147

and galvanometer. The platinum wires A, B are joined to a quadrant electrometer. The experiment is then carried out as described above for a wire. The following are the results of an experiment. (KCl solution, 23·3 grams per litre. Current in milliamperes.)

Current (Galvanometer).	Potential-Difference (Electrometer).	Ratio V/C.
4	17·3	4·32
5	21·6	4·32
6	25·8	4·30
7	30·1	4·30
8	34·5	4·31

Observe carefully that the test on an electrolyte cannot be made with the electrometer connected to the terminal electrodes. These generally become polarized by the current, and are then the seat of an unknown electromotive force. The electrometer only gives the potential-difference V, and this is not the *effective* electromotive force if the conductor is the source of a generated E.M.F. For

the same reason the first test above (fig. 146) refers to the portion A C B of the circuit, not to the portion B K G A.

259. Practical Units of Resistance and Current.

Resistance.—In the early days of electrical measurement arbitrary units were adopted, one of the most important being that devised in 1860 by Siemens. This was the resistance of a column of mercury 100 cm. long and 1 square millimetre in cross-section at 0° C. For theoretical reasons the Siemens' unit was afterwards replaced by the resistance of a column of mercury 106·3 cm. long and 1 sq. mm. in cross-section. This unit is termed the **Ohm.**

Current.—The practical unit is defined as the current produced by an E.M.F. of 1 volt through a resistance of 1 ohm. This unit is called the **Ampere.**

A current of 1 ampere will deposit ·001118 gram of silver per second from a silver voltameter, or ·0003287 gram of copper from a copper voltameter.

260. Resistivity.

Ohm also established the laws expressing the relation of the resistance of a conductor to its dimensions, namely—

The resistance of a conductor varies—
(1) directly as the length;
(2) inversely as the area of cross-section.

Both laws are expressed in the formula—

$$R \propto \frac{l}{a}.$$

If ρ is the resistance of a rod of the substance 1 cm. long and 1 sq. cm. cross-sectional area, the direction of flow being parallel to the length, then—

$$R = \rho \frac{l}{a}.$$

ρ is called the **resistivity** of the substance.

The unit of resistivity is 1 sq.-cm.-ohm per centimetre.

Laws (1) and (2) may be proved experimentally with an

electrometer (compare experiment, p. 341); or they may be deduced from the laws of resistances in series and parallel: for doubling the length of wire is equivalent to arranging two equal resistances in series, and doubling the cross-sectional area may be considered equivalent to putting two equal wires side by side, or in parallel.

EXAMPLE.—The resistance of 60 metres of copper wire 1 sq. mm. in cross-sectional area is 1 ohm. Find the resistivity of copper.

$$R = \rho \,.\, \frac{l}{a},$$

$$1 = \rho \,.\, \frac{60 \times 100}{\frac{1}{100}} \therefore \rho = \frac{1}{600000} = \cdot 00000166.$$

The resistivity is therefore 1·66 millionths of an ohm for a centimetre cube, or 1·66 microhm. The following table gives the resistivities of the more common conductors numerically in sq.-cm.-microhms per cm., at 0° C.:—

Substance.	Resistivity.	Temperature Coefficient.
	Microhms.	
Copper	1·6	·0040
Iron	9·6	·0048
Platinum	8·2	·0032
Aluminium	3·0	·004
Mercury	94·0	·00072
Alloys—		
German-silver	21·2	·00044
Platinum-silver	26·8	·00018
Phosphor-bronze	8·5	·0006
Platinoid	43·6	·00025
Manganin	42·0	·00002
Non-metal—		
Carbon (gas-retort)	670,000 at 0°	— ·0005

For the meaning of *temperature coefficient* see Art. 313.

The values for different specimens of the same material vary according to the mode of preparation. The above may be taken as mean values.

261. Resistances in Series and Parallel.

1. *Series.*—If a number of wires are arranged in series the *same current* passes through each. Let three wires, AB, BC, CD, be arranged in series. Let R_1, R_2, R_3 be the resistances, C the current, and V_0, V_1, V_2, V_3 the potentials at A, B, C, D respectively. Then—

Fig. 148

$$\left.\begin{aligned} R_1 &= \frac{V_0 - V_1}{C} \\ R_2 &= \frac{V_1 - V_2}{C} \\ R_3 &= \frac{V_2 - V_3}{C} \end{aligned}\right\} \quad \therefore\ R_1 + R_2 + R_3 = \frac{V_0 - V_3}{C}.$$

But, by definition, the total resistance is the total potential-difference divided by the current (since there is no source of E.M.F. in the wires). Hence, if R is the joint resistance—

$$R = R_1 + R_2 + R_3.$$

The same reasoning applies to any number of conductors.

If there are n conductors in series, each of resistance r, then—

$$R = nr.$$

2. *Parallel (or Multiple Arc).*—In this case the wires all have the *same potential-difference* at their ends. If C is the total current, this divides into portions C_1, C_2, C_3, etc., along the parallel branches. If there are three conductors, we have, with the usual notation—

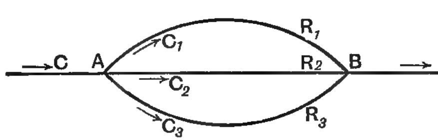


Fig. 149

$$R_1 = \frac{V}{C_1},\quad R_2 = \frac{V}{C_2},\quad R_3 = \frac{V}{C_3}.$$

If R is the joint resistance, we have, by definition, $R = V/C$.

$$\therefore \frac{1}{R} = \frac{C}{V} = \frac{C_1}{V} + \frac{C_2}{V} + \frac{C_3}{V},$$

$$\text{or } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

The same reasoning applies to any number of conductors in parallel.

If there are n conductors, each of resistance r, arranged in parallel, the joint resistance is given by—

$$\frac{1}{R} = \frac{n}{r}, \quad \text{or } R = \frac{r}{n}.$$

If there are **only two** conductors in parallel, then from—

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2},$$

$$\text{we have} \quad R = \frac{R_1 R_2}{R_1 + R_2}.$$

Hence—

> **The joint resistance of two conductors in parallel is the product of their resistances divided by the sum.**

EXAMPLES.—1. Two conductors in parallel have resistances 5 and 6 ohms respectively; what is their joint resistance?

Here $$R = \frac{5 \times 6}{5 + 6} = 2{\cdot}72 \text{ ohms.}$$

2. Three wires of resistances 2, 3, 6 ohms are arranged in parallel. Find their joint resistance.

Here, since there are *more than two* conductors, we must use the reciprocal formula—

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}. \quad \therefore R = 1 \text{ ohm.}$$

Observe that—

(1) The joint resistance of a number of wires in parallel is less than the *least* of the separate resistances.

(2) The drop of resistance produced by joining a wire in

parallel with another is greater, the smaller the resistance of the wire added. *E.g.* a wire has a resistance of 10 ohms. If 1000 ohms are joined in parallel with it, the resistance drops only by one-tenth of an ohm. But if 1 ohm is joined in parallel, the resistance falls by more than 9 ohms.

262. Division of Current in Parallel Branches.

To find the currents in the several branches, we have—

$$C_1 = \frac{V}{R_1}, \quad C_2 = \frac{V}{R_2}, \text{ etc.}$$

If there are two conductors in parallel, then—

$$\frac{C_1}{C_2} = \frac{R_2}{R_1},$$

or the current divides inversely as the resistances. Also—

$$\frac{C_1}{C_1 + C_2} = \frac{R_2}{R_1 + R_2}, \text{ and } \frac{C_2}{C_1 + C_2} = \frac{R_1}{R_1 + R_2}.$$

This result is of frequent use, and may be stated thus—

"If there are **only two** conductors in parallel, the current through one branch is the same fraction of the total current that the resistance of the *other* branch is of the sum of the resistances".

Examples.—1. A current of 5 amperes splits along wires of 8 and 7 ohms respectively arranged in parallel; what is the current in each branch?

$$\text{Here, along the 8 ohms, } C_1 = \frac{7}{15} \text{ of } 5 = 2\tfrac{1}{3} \text{ amp.}$$

$$\text{,, ,, 7 ,, } C_2 = \frac{8}{15} \text{ of } 5 = 2\tfrac{2}{3} \text{ ,,}$$

2. A current of 5·2 amperes splits along three wires of resistances 2, 5, 6 ohms. Find the current in each.

The required currents are proportional to the fractions—

$$\frac{V}{R_1}, \frac{V}{R_2}, \frac{V}{R_3}.$$

Since the value of V does not affect the relative magnitudes, the fractions are proportional to—

$$\frac{1}{2}, \frac{1}{5}, \frac{1}{6}; \text{ that is, to } \frac{15}{30}, \frac{6}{30}, \frac{5}{30}, \text{ or to } 15, 6, 5.$$

Hence if the whole current is supposed to be divided into 26 parts, 15 parts go through the first wire, 6 parts through the second, and 5 parts through the third. Hence the required currents are—

$$\frac{15}{26} \text{ of } 5{\cdot}2, \ \frac{6}{26} \text{ of } 5{\cdot}2, \ \frac{5}{26} \text{ of } 5{\cdot}2,$$

or 3 amperes, 1·2 ampere, and 1 ampere.

3. Wires of resistances 8, 7, 9 ohms are arranged in parallel; if a current of 5 amperes passes along the 9-ohm wire, what current passes in the main circuit feeding the wires?

Here $\frac{V}{R_3} = C_3. \quad \therefore V = 5 \times 9 = 45$ volts.

Then $C_1 = \frac{45}{8} = 5{\cdot}625.$

$$C_2 = \frac{45}{7} = 6{\cdot}428.$$

$$\therefore C = C_1 + C_2 + C_3 = 5{\cdot}625 + 6{\cdot}428 + 5$$
$$= 17{\cdot}053 \text{ amperes.}$$

263. Conductance.

This term is used to denote the reciprocal of resistance. The word "mho" has been suggested as a name for the unit of conductance. Thus a conductor of resistance 2 ohms has a conductance of "half a mho".

The conductance of a rod 1 cm. long and 1 sq. cm. cross-sectional area is termed the *conductivity* of the material. Conductivity is therefore the reciprocal of resistivity.

264. Application of Ohm's Law to a Complete Circuit.

If we consider a complete circuit, the total resistance will be the sum of the battery resistance and that of the external circuit, for these are necessarily in series. If b is the battery resistance, and r the external resistance,

$$R = b + r.$$

Since the potential has only one value at each point, the total

rise or fall of potential, taken from any point round the whole circuit and back to the same point, must be zero.

Thus in the equation $E' = V + E$, we have $V = 0$, and therefore $E' = E$. That is, the effective E.M.F. in a *complete* circuit is equal to the generated E.M.F.

Thus $$R = \frac{E'}{C} = \frac{E}{C}.$$

Therefore $$C = \frac{E}{b + r}.$$

265. Use of Different Modes of Grouping Cells.

If it is desired to send as strong a current as possible through a high resistance the cells must be joined in series; this makes the E.M.F. as great as possible with the cells available, whilst the addition made to the total resistance is negligible. If, however, a strong current is required through a wire of very low resistance, say a short length of copper wire, the cells must be arranged in parallel: for now the resistance of the battery is all-important, and although putting the cells in parallel makes no addition to the E.M.F., yet it decreases the battery resistance.

When the external resistance is neither very high nor very low, but is comparable with the resistance of one cell, a series-parallel arrangement often yields the strongest current.

A rule which applies to all cases is:

"Arrange the cells so that the resistance of the battery is as nearly as possible equal to the external resistance."

To prove this, let there be n cells in the whole battery arranged in y parallel rows, each containing x cells in series. Let b be the resistance of one cell, R the external resistance, e the E.M.F. of one cell, C the total current.

We have $$n = xy.$$

$$\text{Battery resistance} = \frac{xb}{y}.$$

$$\text{Total resistance} = \frac{xb}{y} + R.$$

$$\text{Total E.M.F.} = xe.$$

Hence the current C

$$= \frac{xe}{\frac{xb}{y} + R} = \frac{ne}{xb + Ry} = \frac{\text{a constant}}{xb + Ry}.$$

Now C will be greatest when $xb + Ry$ is least; that is, when—

$$(\sqrt{xb} - \sqrt{Ry})^2 + 2\sqrt{xbRy} \text{ is least.}$$

The second term is constant for a given number of cells and given external resistance. Hence, since the square cannot be negative, the least value occurs when $(\sqrt{xb} - \sqrt{Ry})^2$ is zero; that is, when—

$$xb = Ry,$$

or

$$R = \frac{xb}{y};$$

that is, when external resistance = battery resistance.

Example.—Find the best arrangement of 12 similar cells which are required to send as strong a current as possible through a wire of 9 ohms resistance, each cell having E.M.F. 2 volts and a resistance 3 ohms.

(1) All in series. $C = \frac{12 \times 2}{9 + 36} = \frac{8}{15}$ amp.

(2) Six in series—two rows. $C = \frac{6 \times 2}{9 + 9} = \frac{8}{12}$ „

(3) Four in series—three rows. $C = \frac{4 \times 2}{9 + 4} = \frac{8}{13}$ „

Thus the second arrangement gives the strongest current. When the cells cannot be arranged so as to make the internal resistance equal to the external, one of the two arrangements which make the resistances most nearly equal must be chosen.

The variations of current and internal resistance with different groupings may be shown graphically.

Draw a horizontal line OB to represent the resistance of the battery, and BC at right angles to it to represent the total E.M.F., the cells being all in series in both cases. Through O and C, with OB as axis, draw a parabola. Divide BC into n parts, and mark points on the parabola at the same heights as the points of division. Produce BO to R, making OR equal to the external resistance. Then if R is joined to any one of the points marked

on the curve, say P, the tangent of the angle $P\hat{R}B$ represents the current strength for the corresponding arrangement of cells.

For by a property of the parabola, if N is the foot of the perpendicular from P on OB, we have—

$$\frac{PN^2}{ON} = \frac{CB^2}{OB}. \quad \therefore ON = \frac{PN^2}{CB^2} \cdot OB.$$

Let the number of divisions in the height PN = number of cells in series in each row. Then—

$$PN = xe,$$
$$ON = \frac{x^2e^2}{n^2e^2} \cdot nb$$
$$= \frac{x^2b}{n} = \frac{x^2b}{xy} = \frac{xb}{y}.$$

Thus
$$ON = \text{the internal resistance,}$$
$$OR = \text{,, external ,,}$$
$$NP = \text{E.M.F.}$$
$$\therefore \text{current} = PN/RN = \tan P\hat{R}N.$$

The current is a maximum when RP is a tangent to the curve.

The change in the steepness of the line PR as we increase the number of cells in series, shows the steps by which the current increases before the maximum, or diminishes after the maximum.

266. Examples.

1. A circuit is made up of (1) a battery with terminals AB, its resistance being 3 ohms and its E.M.F. 2·7 volts; (2) a wire BC of resistance 1·5 ohm; (3) two wires in parallel circuit CDF, CEF, with respective resistances 3 and 7 ohms; (4) a wire FA of resistance 1·5 ohm. The middle point of the last wire is put to earth. Find the potential at the points A, B, C, F. (1897.)

$$\text{Joint resistance of the two wires in parallel} = \frac{3 \times 7}{10} = 2{\cdot}1.$$
$$\text{Total resistance of circuit} = 1{\cdot}5 + 2{\cdot}1 + 1{\cdot}5 + 3 = 8{\cdot}1.$$
$$\text{E.M.F.} = 2{\cdot}7,$$
$$\text{Current} = \frac{2{\cdot}7}{8{\cdot}1} = \tfrac{1}{3}.$$

Let A be the negative pole of the battery.

$$\text{Potential drop from B to C} = \tfrac{1}{3} \times 1{\cdot}5 = {\cdot}5.$$
$$\text{,, \quad ,, \quad C to F} = \tfrac{1}{3} \times 2{\cdot}1 = {\cdot}7.$$
$$\text{,, \quad ,, \quad F to A} = \tfrac{1}{3} \times 1{\cdot}5 = {\cdot}5.$$

But the mid-point of FA is connected with the ground. Hence the potentials with respect to the earth are—

$$V_A = -\frac{\cdot 5}{2} = -\cdot 25,$$
$$V_F = +\frac{\cdot 5}{2} = +\cdot 25,$$
$$V_C = \cdot 25 + \cdot 7 = +\cdot 95,$$
$$V_B = \cdot 25 + \cdot 7 + \cdot 5 = +1\cdot 45.$$

2. A battery of twelve equal cells in series screwed up in a box, being suspected of having some of the cells wrongly connected, is put into a circuit with a galvanometer and two cells similar to the others. Currents in the ratio of 3 to 2 are obtained according as the introduced cells are arranged so as to work with or against the battery. What is the state of the battery? Give reasons for your answer. (1895.)

The reversal of the cells added does not affect the resistance of the circuit. Hence the currents are proportional to the E.M.F.'s. Let E be the E.M.F. of the battery and e that of one cell.

Then—

$$\frac{E + 2e}{E - 2e} = \tfrac{3}{2}. \quad \therefore E = 10e.$$

Hence one cell is connected in opposition to the others, leaving only 10 cells out of the 12 effective.

QUESTIONS

1. State Ohm's Law, and explain its meaning as carefully as you can. Apply the law to prove that the conductivity of any number of coils placed in parallel is equal to the sum of the conductivities of the separate coils. (1902.)

(*N.B.*—Conductivity is here used with the same meaning as conductance.)

2. How would you connect two equal constant cells of internal resistance 5 ohms each, if you wished to deposit copper as rapidly as possible in a voltameter of 7 ohms resistance?

3. A, B, C, D, E are five points in a circuit. A and B are joined by a battery of 4 volts E.M.F. and ·5 ohm resistance; A and C by a wire of ·5 ohm; C and E by wire of 5 ohms; C and D by two wires, resistances 6 and 4 ohms, in parallel; D and E by a wire of ·6 ohm; E and B by wire of ·125. A—the negative pole—is earthed. Find the current and potentials at C and E.

CHAPTER XX

MAGNETIC FIELD OF A CURRENT—GALVANOMETERS

267. Magnetic Field of a Current.

The deflection of a compass-needle by a current is an indication that the circuit is surrounded by a magnetic field. We may obtain a map of this field with the aid of a short compass-needle or filings, in the manner already explained for a magnet. The filings method will generally necessitate a strong current (say 10 amperes), and stout wires must be used.

It will be found that the *shape* of the lines of force does not depend on the strength of current, but only on the shape of the circuit. The general distribution of the lines of force in the following important cases should be obtained experimentally by the student, and carefully observed:—

(1) A straight wire.
(2) A circular coil.
(3) A solenoid.
(4) A current sheet.

1. **Field for a Straight Wire.**—The lines are circles concentric with the wire (see fig. 8). (In diagrams they should be drawn nearer together close to the wire to indicate the greater strength of field there.) A compass-needle will show that the direction of the lines is related to the direction of the current as stated in the following rule:—

> **I. Look along the wire so that the current flows away from you: the direction of the lines of force round the wire then appears clockwise.**

This is the primary rule, from which those given below can be derived.

2. **Circular Coil.**—The lines are oval curves, except close to the wire, where they are practically circles. The field is

nearly uniform over a small region at the centre of the coil (see fig. 150). The following rule is sometimes convenient:—

II. Face the plane of the circuit; then if the direction of the current appears clockwise, the lines of force point away from you through the circuit.

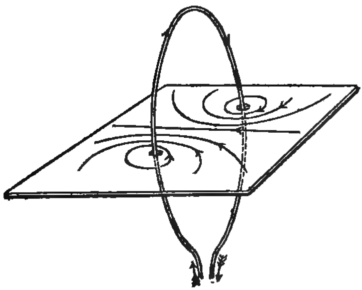


Fig. 150.—Field of Circular Coil

If two circular coils are placed coaxially a short distance apart the field half-way between them is sensibly uniform over a larger region than that at the centre of a single coil. This fact is made use of in certain kinds of galvanometer and in the current balance.

3. **Solenoids.**—If a helix is closely wound and is fairly long in comparison with its diameter, the field just outside it is negligible (fig. 151). The field inside is uniform, and may be strengthened by increasing (*a*) **the current; or** (*b*) **the closeness of the windings.**

To find the direction of field we have the following rule:—

III. Look at the spiral "end on": the lines of force enter at the end where the direction of the current appears clockwise.

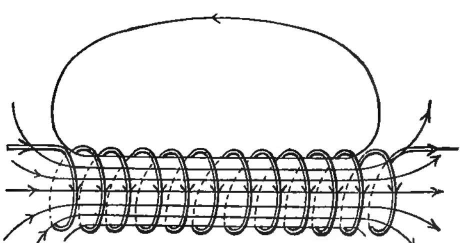


Fig. 151.—Field of Solenoid

This rule applies to either a right-handed or left-handed helix.

The end where the lines of force enter the solenoid acts like the S. pole of a magnet, and the end where they leave like the N. pole. There is no field outside a closed solenoid, such as that shown in fig. 38.

4. **Current Sheet.**—If a current is fed into a metallic sheet by a number of equidistant wires carrying equal portions

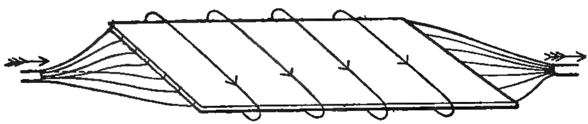


Fig. 152.—Field of Current Sheet

of the current, the lines of *flow* of the current are parallel and the field of force just above and below the sheet is uniform.

The influence of soft iron on a magnetic field has already been considered. We shall now deal with the application of magnetic fields to the measurement of current strength; we shall generally find it necessary to assume that the field is free from iron—chiefly on account of irregularities which would otherwise be introduced through hysteresis.

268. The Tangent Galvanometer—Law of Magnetic Force for a Current.

The strength of the field at any given point depends on the strength of current, the position of the point, and the shape of the circuit. The general laws of current action are dealt with in Ch. XXIX. For the present we shall only consider the field at the centre of a circular coil, in which case the field-strength depends on the following factors:—

(1) **The current strength.**
(2) **The radius of the coil.**
(3) **The length of wire in the coil.**

The relations between these quantities and the magnetic

force may be proved experimentally with the aid of the following instrument, termed a *tangent galvanometer.*

Two circular frames of wood, of different radii, are supported in the same vertical plane, and so that their centres coincide. A horizontal disc supported centrally within the inner frame is provided with a divided circle and a *short* compass-needle balanced on a pivot at the centre of the circle, as in the deflection magnetometer. Coils are wound on the frames and attached to binding-screws on the base of the instrument.

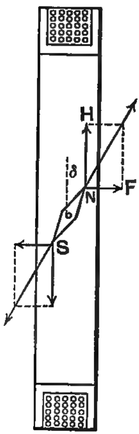


Fig. 153.—Tangent Galvanometer—Theory

The instrument must be set up with the plane of the coils in the magnetic meridian. When a current is passed through one of the coils there is a fairly uniform field at the centre, which will act in an E.–W. direction (magnetic), since the plane of the coil coincides with the meridian. The needle is deflected through an angle δ (see fig. 153). Under these circumstances the deflecting force F, due to the current, and the restoring force H, due to the earth, act at right angles to each other, and are therefore connected by the relation—

$$F = H \tan \delta.$$

Since H is constant, we have—

$$F \propto \tan \delta.$$

The magnetic force due to the current may therefore be represented for purposes of *comparison* by the tangent of the deflection.

The experiments are carried out as follows:—

Arrange an accumulator in series with a key, a galvanoscope, a variable resistance, and the tangent instrument.

(1) Take a coil of one turn round the inner frame, and note the deflection δ_1. Next take a coil of two turns round the same frame. If necessary, adjust the variable resistance until the accessory

galvanoscope shows that the current is the same as before, and note the second deflection δ_2. Repeat with more turns. It will be found that the *tangent* of the deflection is proportional to the number of turns of wire.

Hence, if l is the length of wire in the coil—

$$F \propto l,$$

the current and radius being constant.

(2) (We shall suppose that the frames on which the coils are wound have radii in the ratio 1 : 2.) Wind a number of turns on the inner frame so that a fairly large deflection is obtained (say 50°). Now take the same *length* of wire on the outer frame. Pass a current of the same strength as before and note the deflection. The tangent of the second deflection will be found to be one-fourth of the tangent of the first. Thus if r is the radius—

$$F \propto \frac{1}{r^2},$$

the current and length of wire being constant.

Combining the above laws, we have—

$$F \propto \frac{l}{r^2}.$$

(3) The magnetic force (F) also depends on the battery and circuit wires, and according to the *electromagnetic* definition, the current is the quantity to which the magnetic force is proportional in a circuit of a given shape. Thus, by *definition*—

$$F \propto \frac{l}{r^2} C \qquad (1)$$

The force due to a current is therefore directly porportional to the length of the conductor, inversely proportional to the square of the distance, and directly proportional to the current. These relations are known collectively as **Ampère's Law.**

In Art. 239 we have defined current strength as the quantity of electricity which flows past any point of the circuit per second. That this is in agreement with the above definition may be proved as follows:—

Arrange the tangent instrument in series with a copper voltameter. Note the deflection, and ascertain the strength of the current (say γ) in terms of the copper deposited per hour. Repeat,

with a stronger current. Let δ_1 and δ_2 be the deflections. It will be found that if w is the weight of copper deposited—

$$w_1 : w_2 = \tan\delta_1 : \tan\delta_2.$$

$$\text{But } w_1 : w_2 = \gamma_1 : \gamma_2. \quad \text{(Art. 254.)}$$

$$\text{Also, } \tan\delta_1 : \tan\delta_2 = F_1 : F_2 = C_1 : C_2 \text{ by the definition above.}$$

$$\therefore C_1 : C_2 = \gamma_1 : \gamma_2.$$

Thus the current, measured in terms of the magnetic force it produces, is proportional to the quantity of electricity transmitted in unit time.

269. Units of Current.

To change the proportion given by (1) into an equation, we must introduce a constant. Thus—

$$F = (\text{a constant})\,\frac{Cl}{r^2} \qquad \text{(2)}$$

The constant depends on the units in which we choose to express C, l, r, and F.

The C.G.S. unit of current strength is defined as follows:—

The unit current is that which flowing in a circle of 1 cm. radius, produces at the centre of the circle a magnetic field of strength 2π units.

Substituting, from this definition, in Equation (2) we have—

$$2\pi = (\text{constant}) \times \frac{2\pi \times 1}{1^2}.$$

$$\therefore (\text{the constant}) = 1.$$

$$\text{Hence in C.G.S. units } F = \frac{Cl}{r^2} \qquad \text{(3)}$$

The practical unit (the ampere) is related to the C.G.S. unit thus—

1 C.G.S. unit = 10 amperes.

Consequently if C is in amperes, and the remaining quantities are expressed in C.G.S. units,

$$F = \frac{C}{10} \cdot \frac{l}{r^2} \qquad \text{(4)}$$

270. Absolute Measurement of Current.

The tangent instrument described above may be applied to determine the strength of a current in C.G.S. units. Only one coil is necessary, but its radius and number of turns must be accurately known, and to ensure accuracy the radius should be very large. The needle should be formed like that of the *reflecting* magnetometer, the deflections being observed with a scale and lamp. The instrument so arranged is termed an **absolute** tangent galvanometer.

Since the circumference of a circle $= 2\pi r$, we have $l = 2\pi rn$, where n is the number of turns. Hence—

$$F = \frac{C \cdot 2\pi rn}{r^2} = \frac{2\pi nC}{r}.$$

But since F is at right-angles to the meridian,

$$F = H \tan \delta.$$

Thus
$$\frac{2\pi nC}{r} = H \tan \delta;$$

or
$$C = \frac{r}{2\pi n} \cdot H \cdot \tan \delta \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

If H is determined by a preliminary experiment (Art. 49), the value of C may be calculated.

Example.—The coil of a tangent galvanometer has a radius of 20 cm. and carries 30 turns. A current produces 30° deflection at a place where H = ·18 gauss. Find the current in C.G.S. units and in amperes.

$$C = \frac{20 \times \cdot 18 \times 1}{2 \times 3\cdot 142 \times 30 \times \sqrt{3}} \quad \left(\tan 30^\circ = \frac{1}{\sqrt{3}}\right)$$
$$= \cdot 011 \text{ C.G.S. unit,}$$
$$= \cdot 11 \text{ ampere.}$$

271. Comparison of Current Strengths.

The quantity $2\pi n/r$ is termed the *coil constant.* Denoting this by z, we have—

$$F = zC.$$
$$C = \frac{H}{z} \cdot \tan \delta.$$

In comparative measurements the controlling field H is unaltered, and H/z is then the *working constant* or *reduction factor* of the instrument. If

$$\frac{H}{z} = k,$$

we have

$$C = k \tan \delta. \qquad (6)$$

EXAMPLES.—1. Two currents passed in turn through a tangent galvanometer produce deflections 20° and 60° respectively. Find the relative strengths.

From a table of tangents we find—

$$\text{Tan } 20^\circ = \cdot 364, \text{ and } \tan 60^\circ = 1\cdot 732.$$

Hence

$$\frac{C_1}{C_2} = \frac{k \tan 20^\circ}{k \tan 60^\circ} = \frac{\cdot 364}{1\cdot 732} = \cdot 21,$$

$$C_1 = \cdot 21\, C_2.$$

2. A current, strength ·5, if passed through a tangent galvanometer produces deflection 30°. What current must be passed through a galvanometer with a coil of twice the radius and half the length of wire to produce 60° deflection under the same controlling force?

Here the coil constants differ.

$$z_1 = \frac{l_1}{r_1^2} \text{ and } z_2 = \frac{l_2}{r_2^2}.$$

$$\therefore \frac{z_1}{z_2} = \frac{l_1}{l_2} \cdot \frac{r_2^2}{r_1^2} = \frac{2}{1} \cdot \frac{4}{1}.$$

Since H is the same in both cases—

$$k_1 : k_2 = 1 : 8.$$

$$\therefore \frac{C_1}{C_2} = 8 \times \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1}{8} \cdot \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} = \frac{1}{24}.$$

$$\therefore C_2 = 24\, C_1 = 12\cdot 0.$$

272. The Tangent Galvanometer—Ordinary Form.

In practice the tangent galvanometer is used more frequently for comparative measurements than as an absolute standard of current measurement. It may then be made in a more convenient form than the absolute instrument described above.

Fig. 154 shows the usual pattern of the instrument.

The coils are brought nearer the needle, thus giving greater sensitiveness. A controlling magnet, which slides on a vertical rod screwed into the top of the frame, is also provided. The controlling force H in the formula is then the *resultant* of the earth's field, and that due to the magnet. The sensitiveness is varied by varying the distance of the magnet from the needle. Since neither H nor the coil constant is known, the working constant (k) must be determined for each chosen position of the controlling magnet. This is done by sending a known current C_1 through the instrument, and noting the deflection δ_1 produced. Then—

$$k = \frac{C_1}{\tan \delta_1}.$$

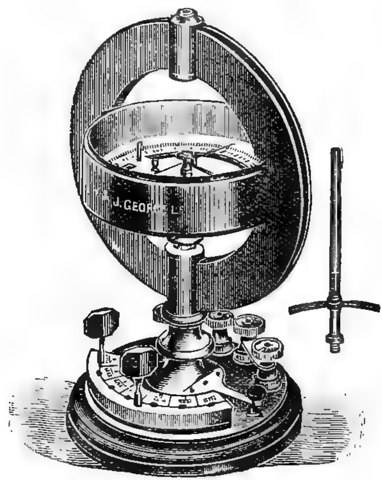


Fig. 154.—Tangent Galvanometer for Comparative Measurements

The current C_1 is determined either with an absolute instrument or its substitutes—a copper voltameter, or a standard cell and resistance.

It is usually more convenient to balance the needle on a pivot than to use a fibre suspension.

For the measurement of very strong currents a single strap of copper is provided instead of coils of wire. Shunts are arranged on the base.

To avoid reference to a table of tangents, the instrument may be provided with a *tangent scale*. Draw the circle for the scale of the instrument (fig. 155). Draw the tangent at right angles to the diameter AB, and construct on it a scale of *equal* divisions. Join each point of division with the centre of the circle. The distance along the line to any point, say X, divided by AO, is the tangent of the angle AOX. But since AO is con-

stant, AX is *proportional* to tan δ. Hence if we mark the points where the radial lines meet the circle with numbers corresponding to those of the scale, the *relative* values of the tangents of the deflections can be read off directly.

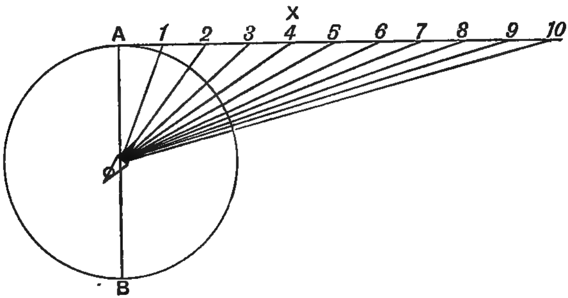


Fig. 155.—The Tangent Scale

273. The Skew Position.

The tangent scale shows clearly how the sensitiveness decreases as the deflection increases. If an increase of 1 ampere produces 10 *degrees* deflection near OA it produces much less increase near OX; and however strong the current the deflection can never really be 90°. To obtain a more uniform sensitiveness with strong and weak currents a "skew" scale is frequently used, the plane of the coil being fixed at an angle with the meridian (or direction of controlling force), but still vertical.

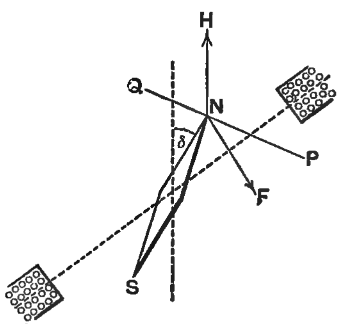


Fig. 156.—Skew Pattern Galvanometer—Theory

Let θ be the angle between the coil and meridian, and let the current be passed in such a direction that it deflects the needle towards the plane of the coil. Let δ be the deflection from the *meridian*. Draw PNQ through the pole of the needle

at right angles to NS. Resolve the forces at N along PNQ, and at right angles to it. The resolved parts which act along PNQ are alone effective in displacing the needle. For equilibrium,

$$\left.\begin{array}{l}\text{Effective component of H}\\ \quad\text{along NQ}\end{array}\right\} = \left\{\begin{array}{l}\text{Effective component of F}\\ \quad\text{along NP.}\end{array}\right.$$

Now angle HNQ $= 90° - \delta$; and angle FNP $= \theta - \delta$.

Hence $\quad H \cos (90° - \delta) = F \cos (\theta - \delta).$

$$\therefore \frac{F}{H} = \frac{\sin \delta}{\cos (\theta - \delta)}.$$

The advantages of a skew zero are (1) increased range in the possible deflections of the needle, (2) greater sensitiveness for the higher values of the deflections. If the deflection is less than $\frac{1}{2}\theta$ the skew instrument is less sensitive than the ordinary instrument.

274. Sine Galvanometers.

If the coil of a galvanometer can be rotated about a vertical axis, it may be adjusted to the same vertical plane as the needle *when the latter is deflected.* The relation of F to H is then—

$$F = H \sin \delta.$$

But F is proportional to C—

$$\therefore C \propto \sin \delta.$$

The instrument is therefore termed a *sine* galvanometer. Thus a current which gives a deflection of 45° on a tangent galvanometer gives 90° on the same instrument arranged as a sine galvanometer ($\tan 45° = \sin 90° = 1$). Sine galvanometers have the advantage of greater sensitiveness.

Since the coil and needle are always adjusted to the same *relative* positions, the needle always lies in the same part of the field of the coil. Hence the coil may be of any shape, and a long needle may be used if *comparative* measurements only are required. The instrument is, however, not direct reading, *i.e.* it requires a preliminary adjustment before every reading — a disadvantage which outweighs the other advantages.

275. Sensitive Galvanometers.

In highly sensitive galvanometers containing a suspended magnetic needle the following principles are observed:—

(1) The deflecting force must be as great as possible with a given current. The coils are therefore brought as close to the needle as practicable. The number of turns adopted depends on the resistance of the circuit for which the galvanometer is intended to be suitable.

(2) The controlling force is made as small as is consistent with definite action. (If the controlling field is too weak the position of the zero becomes uncertain, for the friction of the pivot, or torsion of the silk suspending fibre introduces irregularities.) The controlling field may be reduced by the use of a magnet having its N. pole turned towards the north and placed above or below the needle. In practice, however, the controlling magnet is used to adjust the direction of the field, so that it is unnecessary to set up the instrument with the plane of its coil in the meridian.

(3) The needle system is the same as in a reflecting magnetometer, and is used with a lamp and scale, usually at a distance of 1 metre.

The deflection of the needle of a sensitive galvanometer is usually so small that it is almost exactly proportional to the strength of the current.

Resistance of Sensitive Galvanometers.—The sensitiveness of a galvanometer depends, *inter alia,* on the number of turns of wire in the coil. If there are many turns the wire must be thin, or the coil would become too bulky, and the outer turns would be too far from the needle. Thus owing to both increase of length and decrease of thickness the resistance increases with the number of turns. Very delicate galvanometers have usually a high resistance (5000 to 10,000 ohms).

It does not follow, however, that the galvanometer, which is most sensitive to a given current, will be most effective in a given circuit. The resistance of the circuit outside the gal-

vanometer must be considered. Let G be the galvanometer resistance, and R the resistance of the remainder of the circuit. Then if E is the E.M.F.,

$$C = \frac{E}{R + G}.$$

If R has a very low value compared with G, then practically—

$$C = \frac{E}{G}.$$

In this case if we double the number of turns we make the instrument (roughly) twice as sensitive; but owing to the necessity of using thinner wire G would be increased about eight times, and the current reduced to one-eighth. Hence it would be disadvantageous to increase the number of turns, and a low resistance galvanometer is most effective. On the other hand, *if R has a very large value*, as in testing the insulation resistance of a cable, the current is not much dependent on the value of G, and a high resistance galvanometer is most effective.

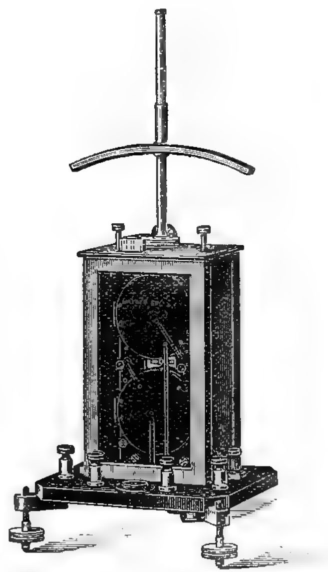


Fig. 157.—Sensitive High-resistance Galvanometer

The general arrangement of a high resistance needle galvanometer is shown in fig. 157. There are two needles attached to the same rod, the latter being suspended vertically by a silk thread attached at its upper end. The needles are suspended in the centres of the two coils. The current circulates round the coils in opposite directions, but since the N. poles of the needles point in opposite directions, the deflecting forces acting on the needles assist each other and the deflection is increased. The

controlling magnet acts unequally on the needles. The coils are of fine copper wire highly insulated on ebonite bobbins, the coil ends being brought to terminals on the ebonite base.

A low resistance galvanometer is similar in construction, but there is usually only one coil, and it need not be so highly insulated. Galvanometers of low and medium resistance are now usually of the suspended coil pattern (Art. 283).

276. **Ammeters.**

This term, or its equivalent, *ampere-meter*, is applied to instruments of very low resistance, and arranged to be of almost invariable sensitiveness. The latter condition is attained by placing the needle (which may be of soft iron) between the poles of a permanent horse-shoe magnet. The needle turns on a pivot, and is provided with a pointer and scale. The coil through which the current passes is placed very close to the needle. The instrument is only suitable for fairly strong currents.

Needle ammeters are now almost entirely supplanted by instruments of the suspended coil type.

277. **Galvanometer Shunts.**

Sensitive galvanometers are generally used in connection with *shunts*. These are coils arranged in *parallel* with the galvanometer, so that a portion of the main current may be diverted from the galvanometer (fig. 158). The deflection would otherwise frequently go beyond the limits of the scale, and a sensitive instrument might be injured by too strong a current.

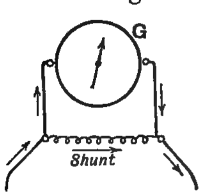


Fig. 158

Let G = resistance of galvanometer coil.
s = resistance of shunt coil.
C_g = current through the galvanometer.
C_s = current through the shunt.
C' = total current in the leads.

Then by the laws of parallel resistances we have—

$$\frac{C_g}{C_s} = \frac{s}{G}.$$

$$\therefore \frac{C_g}{C'} = \frac{C_g}{C_s + C_g} = \frac{s}{s + G} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

This result may be expressed thus—

> "The current in the galvanometer is the same fraction of the total current that the resistance of the *shunt* is of the sum of the resistances."

Example.—If the galvanometer resistance is 100 ohms and shunt resistance 4 ohms, $\frac{1}{26}$th part of the current goes through the galvanometer.

If we wish to send only $\frac{1}{1000}$th of the total current through the galvanometer, we may suppose the whole current divided into 1000 parts; 999 parts go through the shunt and 1 part through the galvanometer. Hence the shunt resistance must be $\frac{1}{999}$th of the galvanometer resistance.

To find the current in the main circuit from the current in the galvanometer, we must know the **multiplying power** of the shunt; that is, the number of times that the main current is stronger than the galvanometer current.

If n is the multiplying power,

$$n = \frac{C'}{C_g} = \frac{s + G}{s} = \left(1 + \frac{G}{s}\right).$$

$$\therefore \frac{G}{s} = (n - 1).$$

$$s = \frac{G}{n - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

From the last equation we may calculate the resistance of the shunt for any multiplying power, with a given galvanometer.

Example.—Find the shunt resistance to give a multiplying power of 300 with a galvanometer of resistance 5000 ω.

$$s = \frac{5000}{300 - 1} = 16{\cdot}7\ \omega.$$

278. **Compensating Resistance.**

If G is practically the only resistance in the circuit a simple shunt is useless.

For without the shunt the current is $C = E/G$, and since this all goes through the galvanometer, $C_g = E/G$.

With the shunt the joint resistance is $\dfrac{sG}{s+G}$, and the current is—

$$C' = \frac{s+G}{sG} \cdot E.$$

But only a portion of this goes through the galvanometer.

$$\therefore C_g = \frac{s}{s+G} \text{ of } C' = \frac{s}{s+G} \cdot \frac{s+G}{sG} \cdot E = \frac{E}{G},$$

which is the same as before.

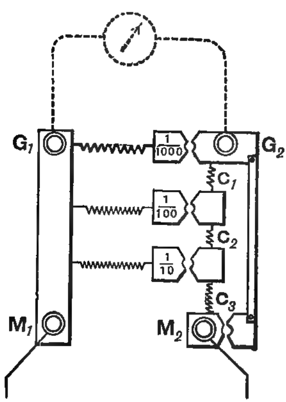


Fig. 159.—Constant-Resistance Shunt

The introduction of the shunt diminishes the resistance of the circuit, and therefore increases the total current; and in the case considered this increase of total current is equal to that diverted through the shunt, which therefore fails to produce the required effect.

A similar effect is produced, though in diminished degree, if the galvanometer resistance is comparable with that of the remainder of the circuit.

In order to maintain the power of the shunt, it is necessary to introduce, into the main circuit, a *compensating* resistance, equal to the drop caused by the introduction of the shunt coils.

To calculate the compensating resistance—

Resistance of galvanometer alone $= G$.

" " " and shunt $= \dfrac{sG}{s+G}$.

$$\text{Drop in resistance} = G - \frac{sG}{s+G}$$
$$= G\left\{1 - \frac{s}{s+G}\right\}$$
$$= \frac{G^2}{s+G},$$

which is the compensating resistance required.

Fig. 159 shows how a number of shunt coils and compensating resistances may be included in the same shunt-box. Such an arrangement is called a **constant resistance shunt**, but the term is, of course, only applicable when the shunt is used in connection with its proper galvanometer.

Universal Shunt.—If we wish to find the actual value of the main current we must know the constant of the galvanometer and the multiplying power of the shunt; and for this purpose each shunt must be used with its own galvanometer.

But more frequently we require to compare the current strengths without knowing their actual values. If one current is much stronger than the other it may be necessary to shift the plug in the shunt-box to keep the deflection on the scale, and if the box is used with its proper galvanometer, the relative currents may be found as follows:—

Let C_1 be the first current, c_1 the current through the galvanometer, δ_1 the deflection, m_1 the multiplying power. Let C_2, c_2, δ_2, m_2 be the corresponding quantities for the second current.

Then $$C_1 = m_1 c_1, \text{ and } C_2 = m_2 c_2,$$
$$\therefore \frac{C_1}{C_2} = \frac{m_1}{m_2} \cdot \frac{c_1}{c_2},$$
$$\text{or, } \frac{C_1}{C_2} = \frac{m_1}{m_2} \cdot \frac{\delta_1}{\delta_2}.$$

If an ordinary shunt is used this formula will only apply when the shunt is used with its own galvanometer. But for relative measurements of this kind it is possible to arrange the coils so that the shunt is applicable to *any* galvanometer. This is the main feature of the Ayrton-Mather *universal* shunt.

The principle of the universal shunt is shown in fig. 160.

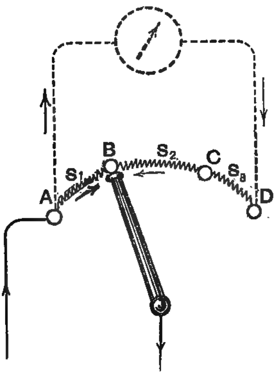


Fig. 160.—Principle of Universal Shunt

A set of coils, s_1, s_2, s_3, in series is joined across the galvanometer terminals. One of the circuit wires is brought to A, and the other to the axle of a movable arm which can be placed at the points of division B, C, D of the shunt resistances.

Suppose the arm is at D. Then the shunt resistance

$$= s = s_1 + s_2 + s_3.$$

The multiplying power

$$= n_1 = \frac{s + G}{s}$$

$$= \frac{s_1 + s_2 + s_3 + G}{s_1 + s_2 + s_3}.$$

Next suppose the arm moved to C. This cuts the resistance s_3 out of the shunt, and puts it in series with the galvanometer resistance. The galvanometer resistance must now be reckoned as $(G + s_3)$, and the shunt as $(s_1 + s_2)$.

The multiplying power

$$= n_2 = \frac{(s_1 + s_2) + (G + s_3)}{s_1 + s_2}.$$

Similarly, $$n_3 = \frac{s_1 + (G + s_3 + s_2)}{s_1}.$$

The numerators of the fractions expressing the multiplying powers are all equal. Hence the relative values of n_1, n_2, n_3 are—

$$\frac{1}{s_1 + s_2 + s_3}, \quad \frac{1}{s_1 + s_2}, \quad \frac{1}{s_1}.$$

These values are independent of the galvanometer resistance, and if these fractions are marked on the box opposite the positions of the arm, the instrument may be used for comparative measurements with any galvanometer.

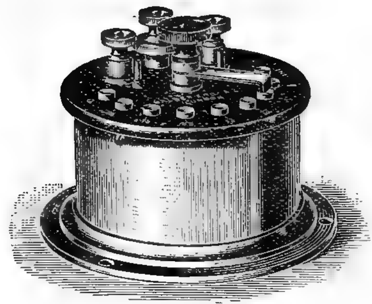


Fig. 161.—Universal Shunt—Dial Pattern

Fig. 161 shows form of the box as made by Messrs. Nalder Bros. & Co.

279. Examples.

Find an expression for the magnetic force at any point on the axis of a circular coil carrying a steady current. (1901.)

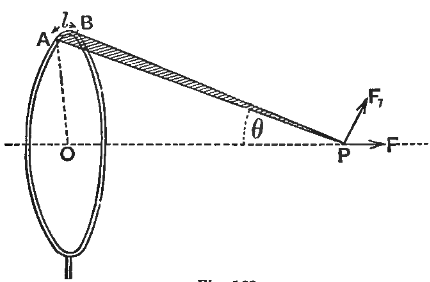


Fig. 162

Let AB be a small portion of the coil (fig. 162), and P the point at which the force is required. Let OP be the axis.

Let l = length of AB,
r = radius OA,
and d = distance of P from centre of coil.

The magnetic force at P *due to the portion* AB *alone*

$$= \frac{C \cdot l}{AP^2} = \frac{C \cdot l}{OA^2 + OP^2} = \frac{C \cdot l}{r^2 + d^2}.$$

This force, F_1 say, acts at right angles to the line joining P to the centre of AB. If this line is inclined θ to the axis, the direction of F_1 is inclined $(90° - \theta)$ to the axis. The component of F_1 along the axis

$$= F_1 \cos (90 - \theta) = F_1 \sin \theta.$$

But $$\sin \theta = \frac{OA}{AP} = \frac{r}{\sqrt{r^2 + d^2}}.$$

$$\therefore \text{ component along OP } = \frac{C \,.\, l}{(r^2 + d^2)} \cdot \frac{r}{\sqrt{r^2 + d^2}}.$$

Now the whole coil may be imagined as divided into elements similar to AB, and therefore the total force acting along the axis

$$= \frac{C}{(r^2 + d^2)} \cdot \frac{r}{\sqrt{r^2 + d^2}} \times (\text{total length of coil}).$$

If the coil consist of n turns, its length is—

$$2\pi r \times n.$$

$$\therefore F = \frac{2\pi r^2 n C}{(r^2 + d^2)\sqrt{r^2 + d^2}},$$

which is the formula required.

(It should be noticed that for complete turns the components *perpendicular* to the axis will neutralize each other.)

QUESTIONS

1. Describe the construction and use of a tangent galvanometer. Calculate the strength of current in C.G.S. units, and also in amperes from the following data:—Radius of coil, 12 cm.; number of turns in coil, 10; deflection of needle, 45°; value of earth's horizontal force, 0·18. (1902.)

2. Define unit magnetic pole and unit electrical current in the electromagnetic system, and state the relation of the ampere to the latter. (1895.)

3. A current flows through two tangent galvanometers in series, each of which consists of a single ring of copper, the radius of one ring being three times that of the other. In which of the galvanometers will the deflection of the needle be greater? If the greater deflection be 60°, what will the smaller be? (1897.)

4. Discuss the several forces or moments which act on the needle of a tangent galvanometer, when deflected by the action

of a current passing through the coil of the galvanometer, and deduce the law of action of the instrument. (1900.)

5. Two long wires of unequal resistance are joined in series in a circuit. How would you show experimentally that the current-strength (electromagnetic measure) is the same in each wire?

6. A tangent galvanometer having a coil of one turn and radius 10·5 cm. is arranged in series with a copper voltameter. A current is passed which deposits ·8 gram of copper in 30 minutes and produces a deflection 30°. Given that a current of 1 ampere deposits ·000328 gram copper in one second, find the strength of the controlling field.

7. Calculate the resistance of the shunt which will allow $\frac{1}{50}$ of the total current to pass through a galvanometer of 120 ohms resistance. Also find the compensating resistance required.

8. A universal shunt is required to give the ratios $1:\frac{1}{3}:\frac{1}{10}$. Find the resistance of each section of the shunt if the lowest of the three resistances is 200 ohms.

CHAPTER XXI

MAGNETIC FIELD—ELECTRODYNAMICS

280. **Mechanical Force acting on a Current Conductor.**

Since a current in the coil of a tangent galvanometer produces a deflecting force on the needle, there must be at the same time an equal reaction on the coil itself, in accordance with the general law that "action and reaction are equal and opposite". (See also Art. 238.) We may calculate the law of reaction on the circuit by considering the force due to a point pole at the centre of a circular coil (fig. 163).

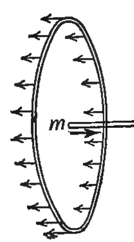


Fig. 163.—Reaction on the Circuit

Let m be the pole-strength, C the strength of current, and r the radius of the circle. The force acting on the pole is—

$$(\text{field-strength}) \times m,$$

and this by Art. 269 is

$$= \frac{lC}{r^2} \cdot m.$$

There is an equal and opposite reaction on the wire, and by consideration of symmetry it is evident that the force is uniformly distributed round the circumference. Hence the force per *unit length* is—

$$\frac{Cm}{r^2}.$$

Note.—The reaction due to each element dl of the wire is a force df acting on the element, together with a couple of moment $df \times r$. In general the mutual action of a magnet and a circuit involves a couple; but in the case considered the couples balance.

Experiment shows that the mutual force between a current and a magnet is independent of the nature of the medium. Now in any medium the expression m/r^2 is the induction [1] due to a point pole m at a distance r. Hence, denoting the induction at the wire (due to the pole) by B, we have—

Force per unit length $= \mathrm{CB}.$
Force exerted on length $l = \mathrm{CB}l.$

(This result is quite general, since we must regard the magnet field as possessing similar properties at all points.)

Since the permeability of air and most fluid media is practically unity, we may substitute magnetic force for magnetic induction in the above formula without altering the numerical relations, and B will denote simply the *field-strength.*

In the case considered the magnetic field is at right angles to the current. If the field and current are inclined at any other angle, we may still apply the formula, but in this case we must *either*—

(*a*) take C as the component of current at right angles to the field;

or (*b*) take B as the component of induction at right angles to the current.

[1] In air m/r^2 is also the magnetic force. But in other media this expression represents the induction only, the force being $m/\mu r^2$.

Example.—A vertical wire 1 metre long carries a current of ·5 C.G.S. unit. If the earth's horizontal component is ·18 unit, find the force acting on the wire.

Here the given field is at right angles to the wire. Hence—

$$f = CBl = \cdot 5 \times \cdot 18 \times 100$$
$$= 9 \text{ dynes.}$$

281. Direction of the Force.

The physical cause of the mechanical pressure on the conductor and the method of finding its direction will be better understood if we consider the resultant field in the neighbourhood of the wire.

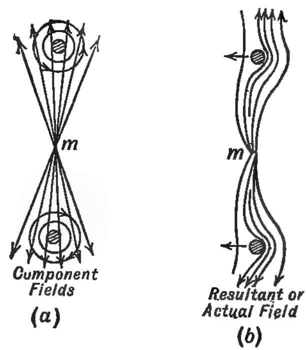


Fig. 164

Taking the case of a point-pole and circular wire, the component fields are as shown in fig. 164 (*a*). The circles are the lines of force due to the current, and the radial lines those due to the pole. On the left of the wire these fields are opposed to each other and the resultant is weak. On the right the fields agree in general direction and the resultant is stronger. The general form of the *resultant* field is as shown in fig. 164 (*b*). Now the tubes of induction tend to expand sideways. We may therefore expect the wire to be urged away from the side where the tubes are more densely packed—a conclusion which agrees with experimental results.

The **direction** of the mechanical pressure exerted on a conductor carrying a current and placed in a magnetic field may therefore be found from the following rule:—

> **Imagine the field due to the current and the deflecting field to be superposed; then the conductor is urged away from the side where the fields directly assist each other towards the side where they oppose each other.**

After a little practice the student will find it unnecessary to actually draw the diagram. The fields may be pictured mentally, the relation between the directions of a straight current and its field (Art. 267) being kept clearly in mind.

The mechanical force which acts on a conductor carrying a current and placed in a magnetic field is termed the *electro-dynamic* or *amperian* force.

282. Examples of the Mechanical Force.

The attraction and repulsion of two conductors carrying currents may be explained in the same way. Diagrams of the

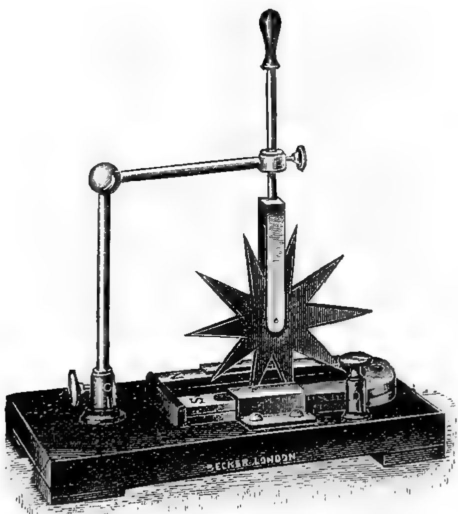


Fig. 165.—Barlow's Wheel

resultant fields at once show the difference between the conditions of attraction and repulsion.

If we take two circuits in given relative positions, then the force f which each circuit exerts on the other is proportional to the *product of the current strengths*, or—

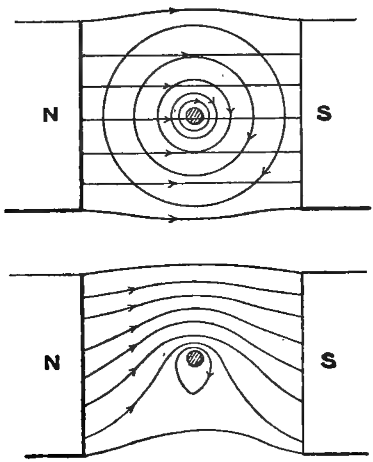
$$f \propto C_1C_2.$$

This will be evident from the formula—

$$f \propto CBl.$$

For C in the latter formula, being the *current acted on*, is equivalent to C_1, and B being the field which acts on it, is proportional to the current producing this field, namely C_2.

Barlow's Wheel.—This arrangement (shown in fig. 165) forms a simple electric motor. The star-shaped wheel is of



(a)

(b)

Fig. 166

copper, and as it rotates the points dip into a cup of mercury between the poles of a strong permanent magnet. The current enters by the brass upright, passes to the axle of the wheel, down the spoke of the wheel into the mercury cup, and thence to the terminal joined to the latter.

The resultant field when the current travels downwards through the spoke is shown in fig. 166 (*b*). The electro-dynamic pressure on the conductor, being greatest where the tubes are most dense, urges the conductor in a direction at right angles to the line joining the poles. This action may be readily imagined if we compare the lines shown in fig. 166 (*b*)

with india-rubber bands. These, tending to straighten, would urge the conductor in the direction found.

The momentum acquired maintains the motion of the wheel until the next spoke reaches the mercury, and a continuous movement is thus kept up. If the direction of current is reversed, the direction of motion is reversed also.

The star shape is not essential. A simple disc of copper may be substituted. The current spreads through the disc, but most of it is concentrated near the radius connecting the axle with the mercury.

Rotation of a Current Conductor round a Magnetic Pole.—Another arrangement which may also be described as

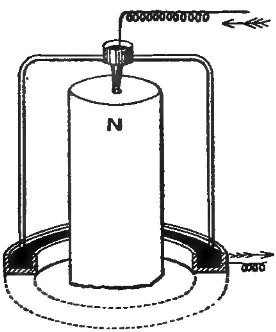


Fig. 167.—Rotation of Wire round Pole

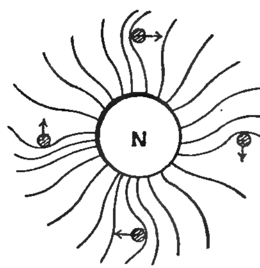


Fig. 168

a simple type of motor is shown in fig. 167. The bent wire and mercury cup are balanced by a pivot on the upper end of a vertical magnet. A circular trough of mercury is placed round the magnet so that the ends of the wire just dip into the mercury. The wire can rotate round the magnet without breaking connection. (A better balance is obtained by using two bent wires.) The direction of rotation for a downward current will be perceived from the plan of the resultant field (fig. 168).

A spiral suspended round the pole revolves in a similar manner. The force exerted on the spiral may be resolved into vertical and horizontal components, the latter causing rotation.

Action of a Current on its own Circuit.—Referring to the lines of force for simple circuits, we see that there is a difference of density of the tubes of induction on each side of the wire. The tendency of this is always to make the circuit *expand*.

We may show this experimentally by floating a portion of a circuit on two mercury troughs (fig. 169). The bent wire moves so that the total area of the circuit increases.

A solenoid tends to contract lengthwise and expand sideways. The former property is well shown by Roget's spiral (fig. 170). This is a helix fixed at its upper end, and having its lower end just dipping into a cup of mercury. When a current is passed the spiral contracts and breaks contact with the mercury. The weight and elasticity of the spiral cause it to quickly regain its original length and re-establish the contact. Thus the spiral is set in rapid oscillation, an intermittent current passing through the circuit.

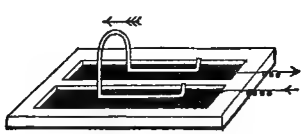


Fig. 169.—Expanding Circuit

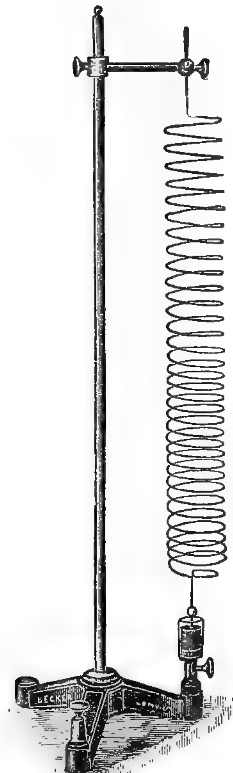


Fig. 170.—Roget's Spiral

By introducing an iron core we may increase the amplitude of the vibrations. Nodes are easily produced and the spiral illustrates in a striking manner the various modes of vibration of the air in an open organ pipe.

The mechanical pressure on a current-carrying conductor (or the "electrodynamic force") is made use of in many important measuring instruments. The chief of these are—

(*a*) **Suspended-coil galvanometers and ammeters;**
(*b*) **Current balances and electrodynamometers.**

SUSPENDED-COIL GALVANOMETERS

283. The suspended-coil galvanometer—as a laboratory instrument—was first designed by D'Arsonval. It is now constructed in many forms, but the principle is the same in all; fig. 171 shows one pattern. A rectangular coil of fine copper wire is suspended by means of a wire from a brass upright. One end of the coil is joined to the suspending wire, and the other end to a wire connected with a spring on the base of the instrument. The spring and the upright are joined respectively to the two terminals. Hence when the instrument is joined in a circuit a current passes round the coil. The coil swings between the poles of a powerful permanent magnet, and when no current flows the plane of the coil is parallel to the lines of force between the poles. When the current passes, the coil tends to set with its plane at right angles to the lines of force of the magnet; that is, so that its own lines of force coincide in direction with those of the magnet. The movement of the coil puts a twist on the suspending wire, and the torque so called into play forms the controlling influence. The deflecting influence is the moment of the couple formed by the mechanical forces

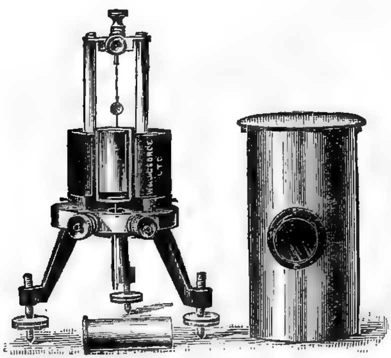


Fig. 171.—Suspended-Coil Galvanometer

acting on the vertical sides of the rectangular coil. The force acting on each side is given by—

$$f = \mathrm{CB}ln,$$

where C is the current, B the strength of the magnet's field, l the length of the side, and n the number of turns of wire in the coil. Since C is very small, to make the instrument very sensitive we must increase B as much as possible. For this purpose a soft-iron cylinder is supported independently within the coil (but clear of the latter), and this concentrates the field on the coil.

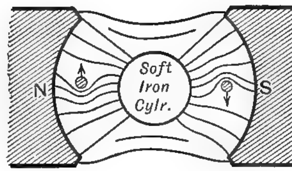


Fig. 172.—Field of Suspended-Coil Galvanometer

With the form of pole-pieces shown in section in fig. 172 a practically *radial* field is produced, and the deflection is very accurately proportional to the current.

Owing to the strength of the deflecting field, the earth's field has in comparison no appreciable effect.

The best form of suspending fibre is a strip of phosphor-bronze.

284. **Advantages of Suspended-Coil Galvanometers.**

(1) The galvanometer is free from disturbing influences due to stray fields, and is therefore especially serviceable where a galvanometer is required in the vicinity of dynamos, tramway circuits, etc.

(2) The movements of the coil can be rendered dead-beat. If the coil is wound over a metallic frame the transient currents induced in this as the coil swings in the field check the movement. In galvanometers of low resistance the movements may also be checked by short-circuiting the instrument. The suspended coil is usually of low or medium resistance.

(3) The galvanometer can be made very sensitive, though not so sensitive as the best forms of needle galvanometer. A galvanometer with pole pieces of the form shown in fig. 172

and a coil of 800 ohms resistance may give a deflection of 1 scale division with a cell of 1 volt E.M.F. through 200 million ohms resistance.

(4) The sensitiveness is constant. The field of the permanent magnet remains constant for long periods, so that the instrument always gives the same deflection with the same current.

(5) The coil may be more readily clamped than a needle, and the instrument is therefore more portable.

285. Moving-Coil Ammeters.

These are dead-beat moving-coil galvanometers arranged to indicate the current directly in amperes.

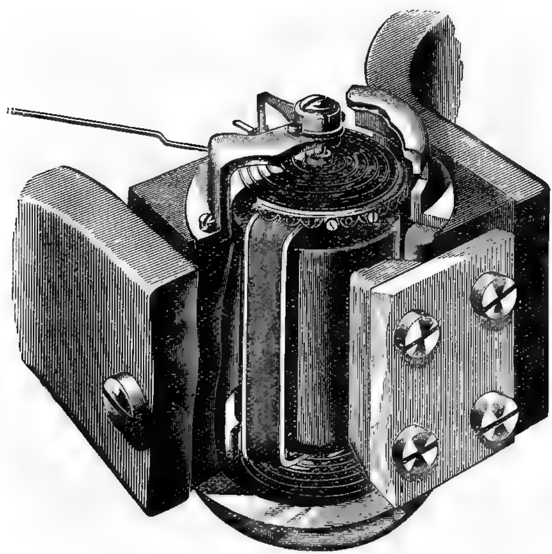


Fig. 173.—Weston Permanent-Magnet Ammeter

The working parts of the Weston ammeter are shown in fig. 173. The coil is rectangular, and rotates between the poles of a strong permanent magnet. A soft-iron cylinder is

supported within the coil space, and the gap between the iron core and the magnet poles is thus occupied by a practically radial field (fig. 172). The controlling torque is supplied by two delicate phosphor-bronze springs like the hair-spring of a watch. The coil is wound on an aluminium frame, and the currents induced in this make the instrument very dead-beat.

When used for very weak currents (milliamperes) the instrument may take all the current through the springs. But for currents of ordinary strength the coil must be joined up as a shunt across the terminals of a very low resistance, so that the whole current is measured in terms of the small proportion tapped off through the coil. (See Art. 308.)

ELECTRODYNAMOMETERS

286. Electrodynamometers are instruments in which the current is measured without the aid of magnets; that is, by the mutual forces between coils carrying the current.

An instrument for measuring current by the attraction or repulsion between parallel coils is termed a **current balance.**

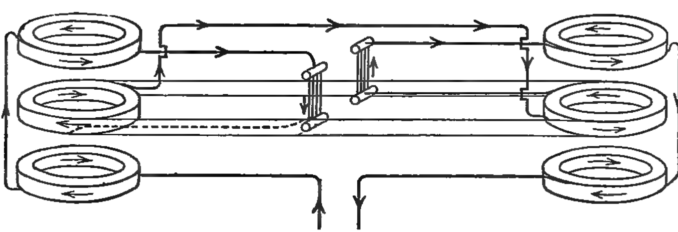


Fig. 174.—Kelvin Current Balance

This type of instrument is capable of great precision, and has been designed in a convenient and accurate form by Lord Kelvin, whose *ampere balances* are now standard instruments for the direct measurement of current. By slight variations in the construction, the instrument can be arranged for measuring various ranges of current. The general principle is shown in fig. 174, which represents the connections of the

centiampere balance (·01 to 1 amp.). Our Frontispiece shows the general arrangement of the same instrument.

There are four *fixed* coils through which the current passes in series. Two other coils are attached to the ends of a light frame, which we may call the "beam". The beam is not balanced on knife-edges, but is suspended by two small "ligaments", each of which consists of a number of fine copper wires arranged parallel to one another, so that they form a short narrow ribbon. The ligaments form a special feature of the Kelvin balances; by their means the current can be conducted from the fixed to the movable parts of the instrument without interference with the freedom of movement, or introduction of loose contacts. An aluminium scale is engraved on the beam, along one side, and a small rider can be moved along the scale by the aid of threads and a sliding-piece. A **V**-shaped pan is attached to the right-hand end of the beam, and receives a small counterpoise. This balances the slider when the latter is at the zero (the left-hand extremity) of the scale. The horizontal position of the beam is indicated by pointers at each end.

When a current is sent through the coils the right-hand end of the beam is displaced upwards by the repulsion of the lower fixed coil on the right and attraction of the upper fixed coil; there are corresponding forces at the other end, but in the reverse direction, so that the total deflecting force is four times as great as that due to one fixed coil. The forces form a couple with a constant arm, the moment of the couple being proportional to the force. This couple is balanced by adjusting the slider until the beam returns to the normal position. The change in the moment of the sliding weight is proportional to the distance d through which it is moved. Hence—

$$\text{(the force due to current)} \propto \text{(change in moment of slider)},$$

or by the result of Art. 282, if C_1, C_2 are the currents in the fixed and movable coils respectively, and d the distance the slider is moved—

$$C_1C_2 \propto d.$$

But since the current flows through all the coils, $C_1 = C_2$. Thus—

$$C^2 \propto d.$$

or,

$$C = (\text{a constant}) \times \sqrt{d}.$$

The current is proportional to the square root of the scale reading.

Besides the scale attached to the beam, a second scale is provided. This is supported in front of the beam in a *fixed* position. The readings on the fixed or "inspectional" scale are proportional to the square roots of the corresponding readings of the beam scale, and enable the current to be read off directly with sufficient accuracy for most purposes.

Since the instrument obeys a known law, the value of the readings in amperes is known when the current corresponding to any one reading has been determined. This "standardizing" of the balance may be carried out with a copper or silver voltameter.

The chief ranges of the centi-, deci-, deca-, hecto- and kilo-ampere balances are respectively ·01 to 1; ·1 to 10; 1 to 100; 6 to 600; and 100 to 2500 amperes. The construction of the instrument is modified in several details in the balances used to measure strong currents.

The "composite" balance is a useful form of the instrument, and possesses a wide range. It resembles the above, except that the two fixed fine wire coils on the left are replaced by two fixed coils of thick wire rope brought to separate terminals. To use as a centiampere balance (·005 to 1 ampere), a switch is put over connecting all the fine wire coils, the thick coils being out of use. The action is then exactly as described above for the ordinary centiampere balance. To use as a hecto-ampere balance (1 to 500 amperes), the switch is put back, and a constant current (·25 ampere) is now passed through the *suspended* fine wire coils (the fixed fine wire coils being out of use). The strong current to be tested is passed through the wire rope. Then since—

$$C_1C_2 \propto d,$$

we have

$$C_1 \times \cdot 25 \propto d.$$

The current under test is directly proportional to the distance the slider is moved along the scale to balance.

The current balance can be used as a voltmeter and wattmeter, and forms a useful laboratory instrument for calibrating ammeters, wattmeters, and other instruments which do not obey any simple or known law.

287. Electrodynamometers—Torsional Control.

If a movable coil is suspended within a fixed one, and if currents are sent through both, the coils tend to set with their planes parallel and fields coinciding in direction. The deflected coil may be restored to its initial position (and the coils so maintained in the *same relative position*) by the use of a spring or wire suspension attached to a torsion head, the instrument so forming a torsion balance for the measurement of current. The torque is proportional to the product of the currents in the two coils. Hence if the same current passes through both coils the torsion on the suspending wire necessary to balance the deflecting torque is proportional to the square of the current strength. Instead of maintaining the coils in a constant relative position, we may allow the suspended coil to be deflected. In this case there will be no simple law connecting deflection and current strength, and the instrument must be calibrated.

The torsion balance principle is applied in Siemens' electrodynamometer. This instrument will be described later, since it is now chiefly used as a wattmeter.

288. Examples.

1. A circular coil traversed by a current is placed horizontally in the earth's field. What is the nature of the force acting on the coil, and how does it vary with (1) the strength of current? (2) the strength of field? (3) the number of turns of wire in the coil? (4) the size of the wire? (5) the radius of the coil? (1904.)

Resolve the earth's field into a horizontal component (H) and vertical component (V). Make diagrams showing the general form of the lines of force for—

(*a*) the resultant of the vertical field and the current's field;
(*b*) the resultant of the horizontal field and the current's field.

The diagrams will show that the mechanical action of (*a*) is, in England, a radial pressure tending to "burst" the coil if the current is clockwise as seen from above, or to "crush" the coil if anti-clockwise.

The mechanical action of (*b*) is a torque. The northern half of the coil is pressed upwards and the southern half downwards if the current is clockwise as seen from above. The points of maximum pressure being the N. and S. points of the coil.

As regards the magnitude of the pressures: The force per cm. of the wire is CB dynes where B is the component field at right angles to the wire. Thus in (*a*) the pressure is nCV dynes per cm., and is independent of the size of wire and the radius. In (*b*) the pressure varies from point to point, but the force per cm. at a given part of the circle is proportional to n, C, and H. The whole torque on the coil is proportional to these quantities and to the area of the coil. It is independent of the size of the wire.

2. What is the magnitude and direction of the force acting on a straight conductor, 10 cm. long, placed at right angles to a magnetic field of 50 lines per sq. cm., the current through the conductor being 5 amperes? In what unit is your result expressed. (1905.)

If C is in C.G.S. electromagnetic units, we have—

$$f = \mathrm{CB}l.$$

Hence if C is in amperes, and f, B, l in C.G.S. units—

$$\begin{aligned} f &= \frac{\mathrm{C}}{10} \,.\, \mathrm{B} \,.\, l \\ &= \frac{5}{10} \,.\, 50 \,.\, 10 \\ &= 250 \text{ dynes.} \end{aligned}$$

QUESTIONS

1. Explain the advantages and disadvantages of a suspended-coil galvanometer as compared with an instrument with a suspended magnetic needle. (1902.)

2. A rectangular coil of length 20 cm. and breadth 10 cm., contains 100 turns and is placed with its plane parallel to the lines of force of a magnetic field. The coil can turn about an axis passing through the middle of the shorter sides, and the couple acting on the coil is found to be 4000 dyne-cm. when the coil is traversed by a current of 10 amperes. What is the strength of the field? (1905.)

3. Describe the construction of some practical form of ammeter for measuring currents of the order of 1000 amperes. (1902.)

4. Describe a suspended-coil galvanometer. Under what conditions is such a galvanometer dead-beat? (1903.)

5. Describe and explain the mode of action of some form of sensitive galvanometer suitable for use in a place where the earth's field is much disturbed by the presence of variable electric currents. (1906.)

CHAPTER XXII

RESISTANCE AND JOULE HEAT

289. Work Done in Maintaining a Current.

If there is a drop of potential V from a point A to a point B in a wire or other conductor, it follows from the definition of potential that—

Work done by electric forces when *unit* quantity of electricity passes from A to B

$$= \text{V ergs;}$$

therefore the work done when Q units pass,

$$= \text{W} = \text{VQ ergs} \quad \text{(1)}$$

If the quantity Q passes in time t seconds, the work done per second

$$= \frac{\text{W}}{t} = \frac{\text{VQ}}{t}.$$

Therefore

$$\frac{\text{W}}{t} = \text{VC} \quad \text{(2)}$$

290. Electromagnetic Units.

The definition of unit current in C.G.S. electromagnetic measure has already been given (Art. 269). From this we derive the definition of unit charge or quantity. Thus—

"The electromagnetic unit of charge is the quantity of electricity transferred per second by an electromagnetic unit of current."

The units of potential-difference (or electromotive force) and resistance then follow from this by the definitions given in Arts. 143, 257.

The **practical units** of resistance, potential-difference, quantity, current, and work, which we have already made use of are derived from the C.G.S. e.m. units.

First, the ohm and volt are chosen so that their actual magnitudes are convenient for the measurement of the resistances and potentials commonly occurring in practice, and at the same time so that they bear a simple numerical ratio to the C.G.S. units. Thus it has been agreed to make

1 **ohm** = 1,000,000,000 C.G.S. units of resistance;
1 **volt** = 100,000,000 C.G.S. units of potential.

Next, in order to avoid the introduction of useless constants into the equation $C = E/R$ we must make

$$1 \text{ ampere} = \frac{1 \text{ volt}}{1 \text{ ohm}} = \frac{10^8}{10^9} = \tfrac{1}{10}.$$

Thus— 1 **ampere** = $\frac{1}{10}$ C.G.S. unit of current.

Similarly, in order that the equation $Q = Ct$ may apply, we must have—

$$1 \text{ coulomb} = 1 \text{ ampere} \times 1 \text{ second} = \tfrac{1}{10} \times 1.$$

Thus— 1 **coulomb** = $\frac{1}{10}$ C.G.S. unit of quantity.

Again, the practical unit of work follows from the equation $W = QV$; therefore—

$$1 \text{ joule} = 1 \text{ coulomb} \times 1 \text{ volt} = \tfrac{1}{10} \times 10^8 = 10^7.$$

Thus— 1 **joule** = 10^7 ergs.

Observe that, in consequence of the above mode of derivation, equations which involve the quantities C, V, R, W, Q, *only*, will apply whether these quantities are expressed all in C.G.S. units or all in practical units.

291. Work in Relation to Heat.

When work is expended in moving a body against frictional resistance, heat is always produced; thus a brass block rubbed

on the table soon becomes heated, and the bearings of machinery run hot unless they are kept well lubricated. It has been shown by careful experiments that the quantity of heat produced bears a definite relation to the amount of work expended in this way. The amount of work required to produce a unit quantity of heat is termed the **mechanical equivalent of heat.**

As a mean result of a number of measurements it has been found that 41,900,000 ergs are required to produce 1 calorie of heat; or, adopting practical units,

4·19 joules are equivalent to 1 **calorie.**

The unit of heat—the calorie—is the amount of heat required to warm 1 gram of water through 1 degree centigrade. Hence, if h units of heat are required to warm m grams of water through θ degrees, we have—

$$h = m\theta.$$

For other substances the heat required will usually be less than this, and to find the heat required, we must take into account the specific heat of the material. Hence m grams of a substance, of specific heat s, require for a rise of θ degrees a quantity of heat given by—

$$h = ms\theta.$$

The quantity ms for any body is termed the *water value*: for it is the mass of water which has the same capacity for heat as the given body. If v is the water value—

$$h = v\theta.$$

The mechanical equivalent is usually denoted by J. Thus if W units of work are expended in producing h units of heat—

$$Jh = W \quad \text{..........................} (3)$$

292. Heat Due to Electrical Resistance.

The resistance which a conductor offers to the flow of current may be considered to be of a frictional nature, for the conductor always becomes heated by the passage of a current.

The work necessary to produce this heat is derived from that done by the electric forces which urge the current along. If the conductor does not contain a source of electromotive force, the *whole* of the work done by the electric forces is converted into heat. Under these circumstances we combine Equations (2) and (3).

Thus— $\frac{W}{t} = CV$ and $W = Jh$.

Hence $\frac{Jh}{t} = CV$ ....(4)

Also, when there is no source of E.M.F. in the wire—

$$V = CR.$$

Hence by substitution—

$$J \cdot \frac{h}{t} = C^2R \text{ ....(5)}$$

Equation (5) expresses what is known as **Joule's Law**—

> **The heat produced in a conductor per second is proportional to the square of the current strength when the resistance is constant, and to the resistance when the current strength is constant.**

The heat produced by frictional resistance is unaltered if the direction of the current is reversed. To distinguish this *irreversible* heating effect from certain reversible effects produced when a current crosses the junction of different metals, we shall frequently refer to it as "Joule heat". (The reversible effects are dealt with in the chapter on Thermo Electricity.) Joule heat is always produced when a current flows through a solid or liquid conductor, and must be carefully allowed for in designing a conductor to carry a steady current without undue heating.

If we substitute the value of C from V = CR in Equation (4), we obtain—

$$\frac{Jh}{t} = \frac{V^2}{R} \text{ ....(6)}$$

Comparing (5) and (6), we see that the heat produced in a conductor is directly proportional to the resistance *if the current is constant*, but inversely proportional to the resistance *when the P.D. is constant.*

Equations (4), (5), and (6) are all equivalent when the conductor is not a source of E.M.F.

EXAMPLES.—1. How much heat is produced in 1 hour in a wire of 15 ohms resistance which is traversed by a steady current of 5 amperes?

$$J \cdot \frac{h}{t} = C^2R$$

$$4{\cdot}2 \cdot \frac{h}{3600} = 25 \times 15$$

$$h = \frac{15 \times 25 \times 3600}{4{\cdot}2} = 321{,}430 \text{ calories (approx.).}$$

2. Two wires, resistances 4 and 12 ohms, are arranged (*a*) in series, (*b*) in parallel. Compare the amounts of heat generated in the two wires in each case.

(*a*) In series, the wires carry the same current. Hence by (5)

$$h_1 : h_2 = 4 : 12 = 1 : 3.$$

(*b*) In parallel, the wires are subjected to the same P.D. Hence by (6)

$$h_1 : h_2 = \tfrac{1}{4} : \tfrac{1}{12} = 3 : 1.$$

In the latter case the wire of *less* resistance becomes more strongly heated. This is because under the given P.D. it carries a stronger current, and since the heat is proportional to the square of the current, the effect of the increase in current overbalances the effect of diminished resistance.

293. Measurement of Joule Heat.

The amount of heat developed in a wire by a current was first determined by Joule by means of an apparatus similar to that shown in fig. 175.

A stout wire is formed into a coil and soldered to two thick copper terminals A B. The coil is immersed in a calorimeter containing distilled water. The calorimeter is provided with

an ebonite cover which carries the terminals, a central hole being left for the introduction of the thermometer and stirrer. The coil is joined in series with a set of large accumulators, a variable resistance, an ammeter (or tangent galvanometer), and a switch.

The temperature of the water is first carefully noted and the switch then closed, the time being noted at the same instant. The current is allowed to run, say 2 minutes, and if it shows any

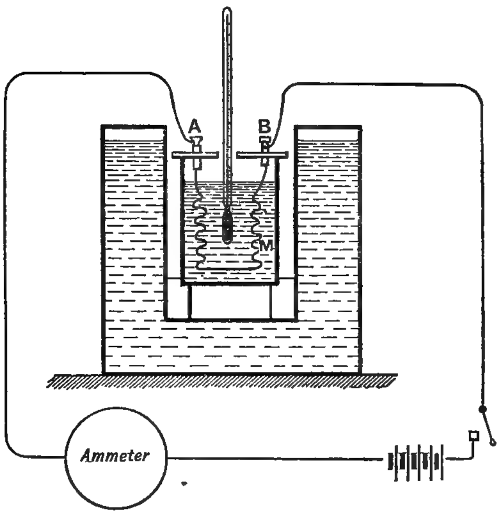


Fig. 175

tendency to vary, must be kept constant by means of the adjustable resistance. The switch is then opened, the time and temperature being again accurately observed.

If m is the mass of water in the calorimeter, v the water value of the calorimeter, stirrer, thermometer, and coil, and θ the rise of temperature, the heat received—

$$h = (m + v)\,\theta.$$

The accurate determination of the quantity of heat produced is an experiment of some difficulty. The chief sources of error are—

(*a*) Differences of temperature between the coil and water;

(*b*) Conduction of heat to or from the coil along the leads;

(*c*) Variation of resistance of the coil through heating;

(*d*) Loss by radiation and conduction from the calorimeter;

(*e*) Leakage of current through the water.

The liquid should be stirred continuously and a very accurate thermometer used, divided at least into tenths of a degree. The current should be fairly strong, 5 to 10 amperes, so that the rise may take place quickly. Leakage through the water is almost negligible, but may be avoided by the use of a non-conducting liquid such as petroleum. (The rise of temperature is also greater with such liquids owing to their small specific heat.) The wire is conveniently of manganin, which does not alter in resistance with ordinary change of temperature. The calorimeter should be silvered and polished, and should be surrounded by a vessel of cold water (as shown in fig. 175) to serve as a shield.

Proof of Joule's Law.—By conducting the experiment as described above, with various strengths of current, Joule's law may be confirmed. The quantities may be compared as shown in the table below, which gives the result of an average experiment (No. 18 manganin wire in 500 c.cm. water)—

Current.	Time.	Rise of Temperature.	$\theta \div C^2$.
4·5	2 min.	·88° C.	·0434
8·5	2 min.	3·11° C.	·0430

The second part of Joule's law may be proved by the use of different coils, the current strength being made accurately the same in each.

294. Measurement of P.D. and Resistance.

If the current is determined in absolute measure by the use of a tangent galvanometer (Art. 270), and the heat determined

in calories as described above, the resistance may be calculated in C.G.S. units from the equation—

$$\frac{J.(m+v)\theta}{t} = C^2R.$$

The potential-difference which exists (during the passage of the current) at the terminals of the coil is also incidentally determined. For—

$$\frac{J(m+v)\theta}{t} = CV.$$

If C is taken in amperes and J in joules per calorie, the resistance and potential-difference may be calculated in ohms and volts respectively.

On account of experimental difficulties connected with the heating experiment this method is not sufficiently exact for the accurate determination of the absolute standards of resistance. The methods adopted to ensure the highest accuracy depend on the use of induced currents.

295. Temperature of Wires Carrying a Current.

It must be carefully observed that the laws enunciated above refer to the *quantity* of heat generated. In order to determine the *temperature* acquired, we must take several other circumstances into consideration.

When a current is started in a wire, the heat produced raises the temperature. The heat at the same time begins to escape from the wire by radiation and by currents of heated air (convection). The hotter the wire becomes the more rapidly does the heat escape from it. As soon as the wire is so hot that the loss of heat by radiation just keeps pace with production of heat by the current, there is no further rise of temperature. The conductor thus acquires a **steady temperature.** A thick conductor may require some time to acquire the steady condition; on the other hand, the filament of a glow-lamp acquires its high temperature in a fraction of a second.

The steady temperature depends chiefly on the following conditions:—

(i) **The amount of heat produced per second.** $(\propto C^2R)$;

(ii) **The amount of surface per unit length of conductor;**

(iii) **The emissive power of the surface.**

If a thick wire and a thin one (of different materials) are chosen of equal resistance per unit length, then if they carry the same current, the amount of heat *produced* per second will be the same in the unit length of each. Hence in the steady state each *loses* the same amount of heat per second. In order that the thin wire may do this it must become hotter, so that its high temperature will compensate for its smaller surface. For a similar reason a brightly-polished wire becomes hotter than a dull one, all other conditions being equal.

Let a wire have a circular section of radius r; let the material be of specific resistance ρ at the steady temperature. Then the resistance per unit length

$$= \frac{\rho}{\pi r^2}.$$

The heat *generated* per unit length by a current C is (per second)

$$= \frac{C^2 . \rho}{J\pi r^2}.$$

The heat *lost* per second (for moderate rise of temperature)

$$= E . a . \theta,$$

where E = emissive power, a = area, θ = excess of temperature. Since $a = 2\pi r$ (per unit length) we have—

$$\frac{C^2\rho}{J\pi r^2} = E . 2\pi r . \theta,$$

$$\text{whence } \theta = \frac{C^2\rho}{2J\pi^2 r^3 E}.$$

Thus for a given material and current the rise of temperature (if small) is inversely proportional to the cube of

the radius (but only approximately, since ρ depends on the temperature).

Notice that the specific heat of the material is not involved in the expression for the *steady* temperature.

Effect of Insulation.—The above calculation refers to bare wires. In the case of insulated wires and cables the thick coating of vulcanized rubber acts as a lagging, which tends to prevent the escape of heat. It should be remembered that the bottom layers of a coil (*e.g.* in an electromagnet winding) may run very hot whilst the outer layers remain cool owing to radiation. For this reason the lower layers of bulky coils are often ventilated.

296. Hot-Wire Instruments.

The temperature to which a given exposed wire is raised depends only on the current strength. Again, a definite rise

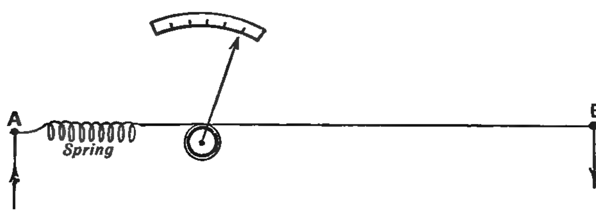


Fig. 176.—Principle of Hot-Wire Instruments

of temperature produces a definite amount of expansion in the wire. Hence an instrument may be constructed in which the current strength is measured in terms of the expansion of the wire.

The principle of construction is shown in fig. 176. A wire AB is clamped at B and passed over a pulley at A; it is kept tight by means of a spring. The pulley is provided with a pointer. When a current is passed the wire is heated and expands, but the increase of length is at once taken up by the spring, so causing a movement of the pointer. After a few seconds the deflection becomes steady.

In the hot-wire instruments designed by Messrs. Hartmann

and Braun an ingenious method is adopted for registering the expansion. It is based on the fact that when a stretched wire is heated there is a large amount of lateral sag for a very small increase in actual length. In the instruments of this type made by Messrs. Johnson and Phillips, a platinum-silver wire is stretched between two supports T_1, T_2 (fig. 177). A fine fibre of phosphor-bronze B is attached to the first wire

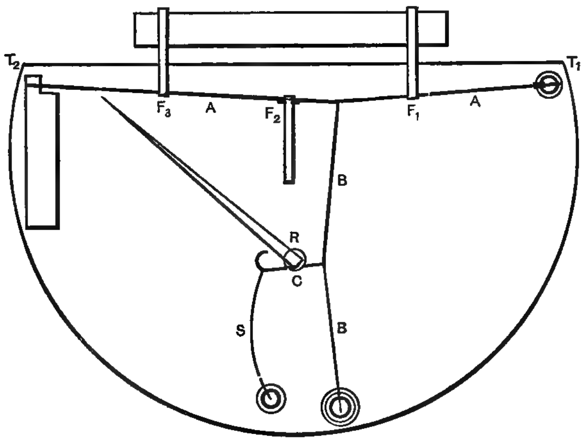


Fig. 177

in a direction at right angles, and is brought to a fixed support. A piece of silk fibre C is attached to the second wire, again in a direction at right angles to the wire, and is passed round a pulley, and finally attached to a steel spring S. The spring thus keeps all the fibres quite taut. When a current passes, the platinum-silver wire sags, but the pull of the spring keeps this and the phosphor-bronze wire stretched, and the consequent movement of the silk fibre causes the pointer to move.

In order to diminish the resistance of the instrument the current is fed into the wire so that it passes through different portions of the wire in parallel, the current being led to the wire by strips of silver foil F_1, F_2, F_3. The current taken may then amount to as much as 5 amperes. To compensate for

expansion due to ordinary atmospheric changes of temperature, the hot wire supports are attached to a semicircular plate, of an alloy having a coefficient of expansion about equal to that of the wire.

The instrument is rendered dead-beat both for deflection and return by an aluminium ring attached to the pulley and moving between the poles of a permanent magnet. The

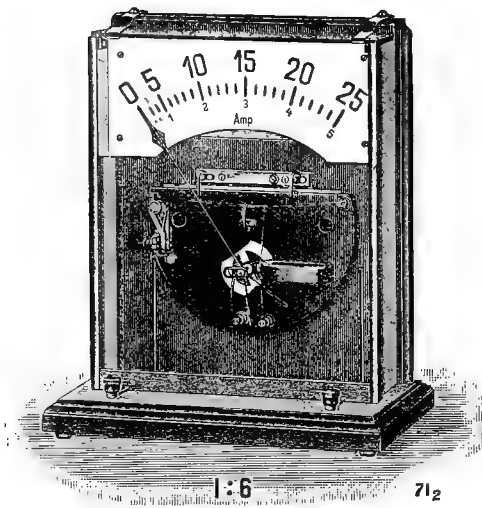


Fig. 178

working parts are shown in fig. 178 (a Hartmann and Braun hot-wire ammeter made by the Union Electric Co.).

When the instrument is used as an ammeter for the measurement of strong currents, the wire is connected across the terminals of a low resistance on the principle explained in Art. 308. When used as a voltmeter, a suitable resistance is placed in *series* with the wire. The current required by the wire may be ·25 of an ampere for the higher reading. Owing to the energy absorbed in the production of Joule heat, the hot-wire voltmeters are not suitable for very high voltages.

The instrument must be calibrated by comparison with some standard instrument, such as a current balance. It must be noticed that, owing to the variation of specific resistance with change of temperature, the sensitiveness of a hot-wire instrument cannot be varied by shunting the working wire unless the scale is recalibrated for the shunt used.

297. Examples.

1. An electric battery of constant E.M.F. having an internal resistance of 5 ohms is connected to resistance coils of 10 ohms and 20 ohms respectively arranged (1) in series, (2) in parallel. Neglecting the resistance of the connecting wires, compare the amounts of heat produced in the two cases, (*a*) in the whole circuit, (*b*) in the two coils. (1904.)

(1) Total resistance of the circuit $= 5 + 10 + 20 = 35$

(2) Resistance of two coils (joint) $= \frac{10 \times 20}{30} = \frac{20}{3}$,

and the total resistance $= 5 + \frac{20}{3} = \frac{35}{3}$.

Heat produced is proportional to C^2R. Distinguishing the two cases by suffixes, then—

(*a*) For whole circuit $\frac{R_1}{R_2} = \frac{35}{35/3} = \frac{3}{1}$; $\therefore \frac{C_1}{C_2} = \frac{R_2}{R_1} = \frac{1}{3}$.

Hence $\frac{h_1}{h_2} = \frac{C_1^2R_1}{C_2^2R_2} = \frac{1}{9} \times \frac{3}{1} = \frac{1}{3}$.

(*b*) For the two wires in parallel taken conjointly—

$$\frac{R_1}{R_2} = \frac{10 + 20}{\left(\frac{20}{3}\right)} = \frac{9}{2},$$

and $\frac{C_1}{C_2} = \frac{1}{3}$ as before.

Hence $\frac{h_1}{h_2} = \frac{C_1^2R_1}{C_2^2R_2} = \frac{1}{9} \times \frac{9}{2} = \frac{1}{2}$.

Thus the ratios required are—

(*a*) 1 : 3 and (*b*) 1 : 2.

2. A wire of resistance r connects A and B, two points in a

circuit, the resistance of the remainder of which is R. If, without any other change being made, A and B are also connected by $n-1$, other wires, the resistance of each of which is r, show that the heat produced in the n wires together will be greater or less than that produced originally in the first wire according as r is greater or less than $R\sqrt{n}$.

Let E be the E.M.F. of the battery, C the total current in the first case, and C_1 the total current in the second case.

$$\text{Then} \quad C = \frac{E}{R+r}$$

$$C_1 = \frac{E}{R+\frac{r}{n}} \left(\text{since joint resistance of } n \text{ wires is } \frac{r}{n}\right);$$

$$\therefore \frac{C}{C_1} = \frac{nR+r}{n(R+r)}.$$

If h_1 and h_2 are the quantities of heat to be compared—

$$\frac{h_1}{h_2} = \frac{C^2r}{C_1{}^2\frac{r}{n}} = \frac{(nR+r)^2}{n^2(R+r)^2}\cdot\frac{n}{1} = \frac{(nR+r)^2}{n(R+r)^2}.$$

Thus h_2 is greater or less than h according as $n(R+r)^2$ is greater or less than $(nR+r)^2$,

or $(nR^2+2nRr+nr^2)$ is greater or less than $(n^2R^2+2nRr+r^2)$,

or nR^2+nr^2 „ „ $n^2R^2+r^2$,

or nr^2-r^2 „ „ $n(n-1)R^2$,

or r^2 „ „ nR^2,

or r „ „ $R\sqrt{n}$.

QUESTIONS AND EXERCISES

1. A current of 10 amperes is sent through a platinum wire the resistance of which is 2 ohms. Find the mechanical equivalent in ergs of the heat generated per second. (1905.)

2. State the laws relating to the production of heat by an electric current. What will be the ratio of the currents which will produce in one second the same amount of heat in two wires of the same material and length, if the radius of one is twice that of the other? (1902.)

3. Two circuits whose resistances are respectively 1 ohm and 10 ohms are arranged in parallel. Compare the amount of current passing through each of these circuits with that through the

battery. Compare also the amount of heat developed in the same time in the two circuits. (1901.)

4. A current of 1 ampere flowing for one second through a resistance of 1 ohm produces ·239 gram-centigrade unit of heat. What current would have to flow for an hour through a resistance of 41·84 ohms in order that the heat produced might suffice to raise a kilogram of water from 0° C. to the boiling-point?

5. Explain how the mechanical equivalent of heat may be determined by measuring the electric energy spent in heating a resistance. What instruments would you require, and how would you perform the experiment? (1906.)

CHAPTER XXIII

POTENTIOMETRY

298. When a current flows through a conductor in which there is no source of E.M.F., the relation between the P.D., current, and resistance is—

$$V = CR \qquad (1)$$

Hence for a *given current* the potential-difference is proportional to the resistance.

If two conductors of resistance r_1, r_2 are arranged *in series*, they carry the same current. Hence if v_1 is the drop of potential along one conductor and v_2 that along the other—

$$v_1 : v_2 = r_1 : r_2 \qquad (2)$$

or the drop of potential is proportional to the resistance.

In the present chapter we shall deal with methods of measurement depending on the direct application of this formula.

299. Comparison of Resistances—The Wheatstone Bridge.

Let two wires be arranged in *parallel* between the points A, B (fig. 179). A current entering the wires at A divides, in general, unequally between them, but the total drop of potential along each wire is of course the same. Now choose any point C on the

upper wire. The parts AC and CB form two resistances in series. If v_1 is the drop of potential from A to C and v_2 the drop from C to B, then—

$$v_1 : v_2 = \text{resistance AC} : \text{resistance CB}.$$

It is evident that there must be a point somewhere on the lower wire at the same potential as C. If D is this point, the fall of potential from A to D is v_1 and the fall from D to B is v_2. Therefore—

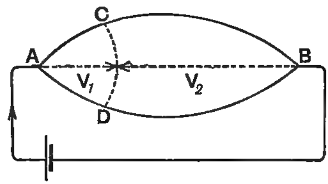


Fig. 179

$$v_1 : v_2 = \text{resistance AD} : \text{resistance DB}.$$

Thus when the points of division of the wires are at the same potential, the parts have resistances in the same ratio.

$$\text{Res. AC} : \text{res. CB} = \text{res. AD} : \text{res. DB} \ldots\ldots\ldots (3)$$

Now by using a galvanometer as a *bridge* across the two wires, points of division which are at the same potential may be found experimentally; for when the galvanometer is connected to such points there is no deflection. Thus with the aid of a galvanometer we may divide two wires in parallel so that the resistances of the parts are in the same ratio.

EXAMPLE.—A wire of 8 ohms resistance is arranged in parallel with a wire of 72 ohms resistance. The former wire carries the stronger current. The drop of potential in each ohm of the first wire is the same as that in every 9 ohms of the second. Points at the same potential may easily be located. Thus, for example, the point dividing the first wire into resistances 3 and 5 is at same potential as the point dividing the second wire into resistances 27 and 45, the smaller resistance being on the left in each case.

For convenience it is usual to represent the four resistances in the form of a parallelogram (fig. 180). If the four resistances in (3) are respectively P, R, Q, S, we have—

$$\frac{P}{R} = \frac{Q}{S},$$

or

$$\frac{P}{Q} = \frac{R}{S} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The arrangement of conductors represented in fig. 180 is of first importance in electrical testing, and is known as a *Wheatstone's net* or *bridge.* It is used to compare two resistances, or to measure a resistance. The resistance S is to be measured, the other three P, Q, R are known or variable. Thus P, Q, R may be three ordinary resistance boxes and S a length of wire under test. Connections are made to the battery and galvanometer as shown. The resistances P, Q, and R are adjusted until there is no deflection of the galvanometer. Then the relation (4) holds, and we have—

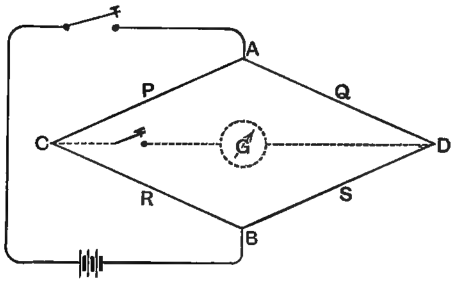


Fig. 180.—Wheatstone Bridge

$$S = \frac{Q}{P} . R \ldots\ldots\ldots\ldots\ldots\ldots \text{(5)}$$

It is evident that it is unnecessary to have the values of P, Q, R all in ohms. If we know one of these resistances, say R, in ohms, and the *ratio* of the other two P/Q, the value of S may be determined. This method is adopted in practice. The arms P, Q of the parallelogram are called the *ratio arms*, and the known resistance R is called the *rheostat arm.* The adjustment of resistances necessary to obtain the "balance" is made in one of the two following ways:—

(*a*) **The resistance R is kept constant whilst the ratio P/Q is varied;** *or*

(*b*) **The ratio P/Q is kept constant whilst the resistance R is varied.**

The two principal forms of Wheatstone bridge are—

(a) *The metre-wire bridge;* (b) *the Post-office box.*

300. The Metre-Wire Bridge.

A stout wire of German silver, platinoid, or platinum silver is stretched along a board and soldered to thick copper strips L F, M K (fig. 181). The wire is usually 1 metre long. A copper strip G H is attached to the board so that "gaps" F G and H K are left for the insertion of the resistances to be compared. Terminal screws are placed in suitable positions for the attachment of leads, and a scale runs the whole length of the metre wire. Battery and galvanometer are joined as shown.

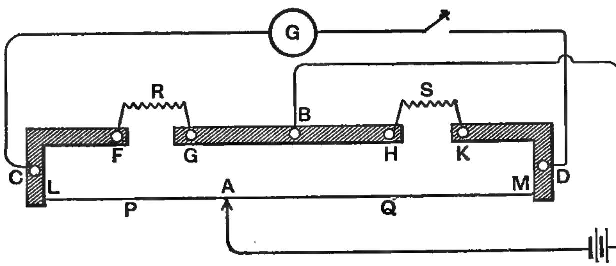


Fig. 181.—Metre-Wire Bridge

Comparing this bridge with the Wheatstone parallelogram, and remembering that the thick, copper strips have no appreciable resistance, we see that the strips L F, G H, K M correspond respectively to the *points* C, B, D of the parallelogram. If a known resistance R is connected across the gap F G, and an unknown resistance across H K, the latter resistance may be ascertained as follows:—

Make contact with the battery wire at some point A of the metre wire. Then close the key in the galvanometer branch. If a deflection is obtained, move the point of attachment of battery wire A to a new position. Again close the galvanometer key. Repeat the trials in this way, until a position is found for A where there is no deflection on closing the galvanometer key. Then by (5)

$$S = \frac{Q}{P} . R.$$

But the ratio $\frac{Q}{P}$ is the ratio $\frac{\text{length MA}}{\text{length LA}}$ since the wire is uniform. Thus S may be calculated.

For convenience in making the contact at A, a "jockey" or sliding contact is provided with the bridge. The positions of the galvanometer and battery shown in fig. 181 may be interchanged, but trouble is then likely to arise from thermo-electromotive forces developed at the junctions L, M, F, G, etc. The galvanometer should be sensitive, and kept shunted until the balance is nearly exact. The *battery key* must be closed *first*, then the galvanometer key. Otherwise, if the coil under test is inductive, as for example the coil of an electromagnet, an induced current will flash round the circuit when the battery key is closed, and cause a kick of the galvanometer needle.

The metre-wire bridge is more particularly adapted for the accurate comparison of small resistances — of the order of 1 ohm. For larger resistances the second form of Wheatstone bridge is more suitable.

301. Post-Office Bridge.

This pattern of bridge consists virtually of three boxes of resistance coils enclosed in one case, with the addition of two keys. The coils are wound as explained in Art. 313, and the resistances are brought into play by the removal of plugs. Fig. 182 shows the usual form of the box, and fig. 183 the relation of the parts to the Wheatstone parallelogram. AD, AC are sets of coils each 10, 100, and 1000 ohms. These form the ratio arms. The rheostat arm consists of the coils between C and B arranged in two or three lengths. Thus *three arms of the bridge are in the box*: the fourth arm is formed by the wire whose resistance has to be determined. This must be connected to the terminals B and D.

The galvanometer is joined to C and D, the battery to A and B. But the connections to A and C are not made directly. Permanent connections are made inside the box between the points A and C, and the contact points of the keys K and H respectively. Hence by making the actual connections to D and the galvanometer key, for galvanometer, and to B and the battery key, for the battery, we introduce a key into each of these connections.

In making the test, resistances are unplugged from the ratio

Fig. 182.—Post-Office Box

arms, thus giving the ratio P : Q some definite value. (This is usually 1:1, 10:1, 100:1, 1:10, or 1:100.) Resistances are then unplugged from the rheostat arm, and the resistance in this arm is

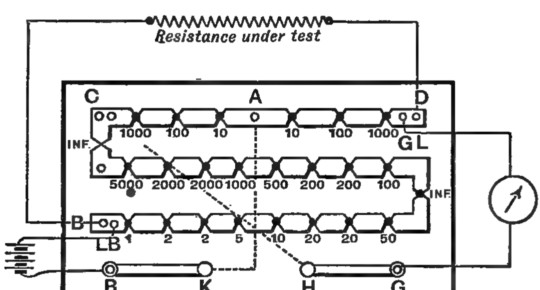


Fig. 183.—Post-Office Box—Connections

varied until on putting down first the battery key, and then the galvanometer key, no deflection is obtained. The unknown resistance S is then calculated from (5).

The rheostat arm contains the following coils:—1, 2, 3, 4; 10, 20, 30, 40; 100, 200, 300, 400, 1000, 2000, 3000, 4000. Hence the total resistance is 11,110 ohms. If P : Q = 1 : 100, the highest resistance for which a balance can be obtained is 1,111,000 ohms. If P : Q = 100 : 1 and 1 ohm is taken in the rheostat arm, a balance is obtained with ·01 ohm in arm S. The instrument thus has a wide range. It is, however, of little use for measuring resistances less than 10 ohms; for the smallest change that can be made in the rheostat arm is 1 ohm, and with P : Q = 100 : 1 this corresponds to ·01 in the unknown arm.[1] Thus to obtain the result correct to $\frac{1}{10}$ % the resistance S must not be less than 10.

(In many makes of Post-office bridge the resistances are 1, 2, 2, 5 and their multiples by powers of 10. These, however, give the same total as 1, 2, 3, 4, etc.)

302. Comparison of Electromotive Forces—The Potentiometer.

Let a wire AB of uniform material and uniform cross-section be connected to the poles of a constant cell so that a steady

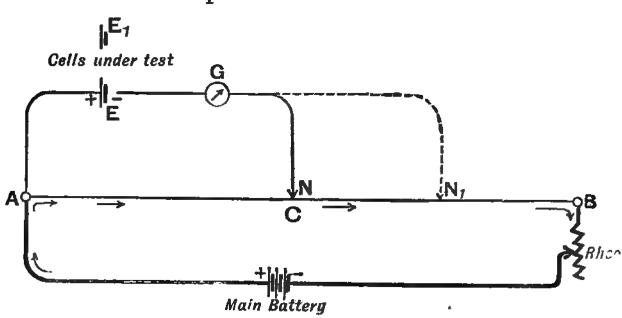


Fig. 184.—The Potentiometer

current flows from A to B (fig. 184). Then there will be a

[1] This refers to the use of the bridge coils only. But if an additional box of high resistance coils is joined *in parallel* with the rheostat arm it is possible to obtain a much higher degree of accuracy.

uniform rate of fall of potential along the wire. The fall of potential in any length of the wire is, by Eqn. (2), proportional to the resistance, and therefore, since the wire is uniform, to the length.

Now let the positive pole of a cell E be joined to A, and the negative pole through a galvanometer to a sliding contact N. Let the contact at N be broken. Then there is no current in the branch circuit. The potential at the positive pole of E is the same as the potential at A. There is a sudden drop of potential through the cell, and then a uniform potential through the galvanometer to N. Hence the potential of the

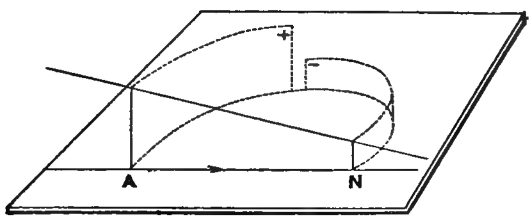


Fig. 185.—Potential Diagrams for the Potentiometer

end N of the branch circuit is as much below that of the end A, as the potential of the negative pole of E is below that of the positive pole. Thus, if the fall in potential along the main wire from A to C (due to the current) is equal to the fall of potential through the cell, the point C on the main wire and the end of the branch circuit which is about to touch it are at the same potential. If the contact is now made at C, there will be no current through the branch circuit and no deflection of G. The above reasoning will be better understood from fig. 185, in which the potentials are shown graphically by means of a space diagram. The steady fall of potential along the main wire is shown by the sloping line. The dotted line shows the potentials along the branch circuit, the poles of E being shown by $+$ and $-$.

Let R be the resistance of the wire from A to N, and C the current in the wire. Then the drop from A to C, $V = CR$.

If a second cell is now used in the branch circuit to replace the first, a different contact point N_1 must be obtained. If resistance of $AN_1 = R_1$, $V_1 = CR_1$.

Further, since (when the balance is obtained) no current flows through the branch, the cell under test is virtually on *open* circuit. Hence $E = V$ and $E_1 = V_1$.

Thus—
$$\frac{E}{E_1} = \frac{R}{R_1} = \frac{\text{length } AN}{\text{length } AN_1},$$

if the wire is quite uniform.

We have thus a means of comparing the E.M.F.'s of two cells. The arrangement described is called a *potentiometer*. It is an exceedingly accurate and convenient method of comparing maintained potential-differences. Indirectly, currents and resistances may also be compared (see below).

In order that it may be possible to find a point of contact so that no current flows through the branch circuit, it is necessary that the *total* fall of potential from A to B be greater than the E.M.F. of the cell E.

To ensure accuracy the following conditions must be observed:—

(1) The wire AB must be uniform, and of considerable length so that the drop of potential per cm. may be small and the point of balance thus found more exactly.

(2) The main battery E must be very constant.

An accumulator should always be used for the main battery.

(3) The galvanometer must be of the sensitive reflecting type; preferably a dead-beat suspended-coil galvanometer.

Observe carefully that the pole of the cell under test which is connected to A must be of the **same** sign as the pole of the main battery connected to this point.

In the more elaborate types of potentiometer the wire AB is divided into a large number of segments made up as separate

coils. The points of division are brought to brass studs, and the sliding contact N is represented by an arm arranged to touch the studs successively as it is rotated.

303. Comparisons of Potential-Difference, Current, and Resistance.

(1) *Comparisons of Potential-difference in Wires.*

The use of the potentiometer is not confined to the measurement of the E.M.F.'s of cells. The potential-differences between points in circuits, produced by *currents* flowing in the circuits, may be compared. The two points to be tested are connected up exactly as the two poles of a cell, care being taken to join

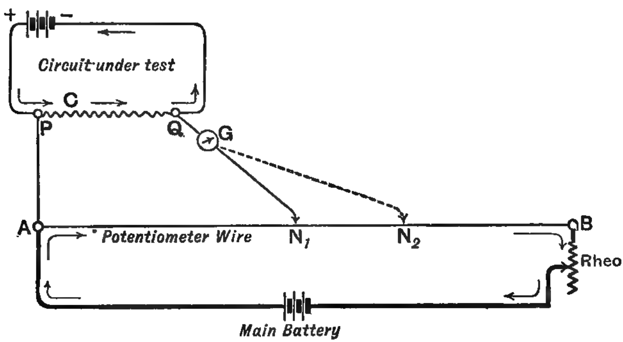


Fig. 186.—Comparison of Current Strengths

the point of higher potential to the same end A of the potentiometer wire as the positive terminal of the main driving battery.

(2) *Comparison of Current Strengths.*

Let PQ be a standard resistance = R, and let a current C_1 be sent through it. Then a potential-difference V_1 is produced between P and Q. The points P and Q may therefore be treated like the poles of a cell, and if these points are connected to the potentiometer, as shown in fig. 186, the P.D. may be balanced at a point on the potentiometer wire, say N_1.

If, now, a second current C_2 is sent through PQ, a new potential-difference V_2 will be produced between P and Q, and

a balance will be found at a new point N_2 of the potentiometer wire.

Now $\left.\begin{aligned} V_1 &= C_1R \\ V_2 &= C_2R \end{aligned}\right\}$ $\therefore \dfrac{V_1}{V_2} = \dfrac{C_1}{C_2}.$

But $\dfrac{V_1}{V_2} = \dfrac{AN_1}{AN_2}.$ $\therefore \dfrac{C_1}{C_2} = \dfrac{AN_1}{AN_2}.$

In fig. 187 the potentials along the main and branch circuits are shown by means of a space diagram. Observe that the

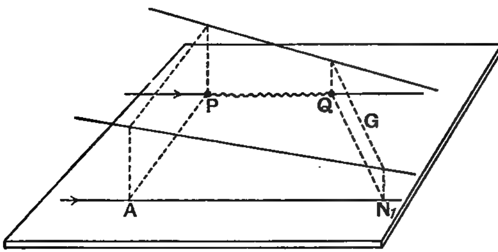


Fig. 187

potentials of AP and NQ are uniform. These connections carry only static charges.

(3) *Comparison of Resistances.*

Resistances may be compared by means of the potentiometer. They must be joined up in series with a constant cell, and the lengths of potentiometer wire required to balance their potential-differences must then be found. The resistances are proportional to these lengths.

304. Use of a Standard Cell and Standard Resistance.

The above measurements are all comparisons. But if we have a standard cell whose E.M.F. is known in volts, the E.M.F.'s of other cells may be determined also in volts.

Again, if the value of one resistance coil is known accurately in ohms, the comparison experiment enables us to determine other resistances in ohms.

With the aid of a standard cell and standard resistance,

we may measure a current in amperes. Join the standard resistance PQ in circuit so that the current to be measured flows through it (as shown in fig. 186). Find the P.D. at its terminals in terms of a length of the potentiometer wire (l_1). Next (with the arrangement shown in fig. 184) find the length (l_2) of the potentiometer wire required to balance the standard cell. Then if C is the current to be measured, R_s the standard resistance, V the potential-difference of P and Q, E_s the electromotive force of the standard cell—

$$\frac{V}{E_s} = \frac{l_1}{l_2}. \quad \text{But } V = CR_s; \therefore C = \frac{E_s}{R_s} \cdot \frac{l_1}{l_2}.$$

The potential-differences tested may be read off directly in volts from the potentiometer scale, provided we make a suitable adjustment of the main current in the potentiometer wire AB. This may be done by a variable resistance (rheostat) connected as shown in fig. 184. The experiment is conducted as follows:—

Suppose that E is a Clark cell, and the potentiometer wire has a total length of 2000 mm. Join up E as shown, and set the contact point N at a reading corresponding to the known E.M.F. of the cell. Since the Clark cell has an E.M.F. of 1·434 volt (at 15° C.), N may be set at the reading 1434. Now vary the current in the main circuit, by means of the rheostat, until there is no deflection when contact is made at N. The current is now such that there is a drop of ·001 volt in each millimetre of the wire. If this current is maintained constant, it is evident that readings for cells subsequently tested may be interpreted directly in volts.[1]

305. Increase of Range.

To avoid heating and consequent change of resistance the current in a potentiometer wire must be weak. Hence unless the wire is of inconvenient length, the potential-differences measured must be small, say less than 2 volts. But higher

[1] One of the most complete testing sets constructed on this principle is the potentiometer made by Messrs. Crompton & Co., Ltd.

voltages may be measured by the aid of a resistance coil divided into segments in a known ratio. Thus let AD be a coil of high resistance having intermediate terminals at B and C, and let the resistances of AB, AC, AD have the ratios 1 : 10 : 100. If A and C are joined to the points whose P.D. is to be determined, the current which flows through AC produces a small potential-difference between A and B, and this can be measured with the potentiometer. This potential-difference is one-tenth of that between A and C. Hence if the potentiometer reads up to 2 volts, P.D.'s between A and C as high as 20 volts may be measured; or if the P.D. under test is applied to A and D, a value as high as 200 volts may be tested, by measuring the P.D. between A and B with the potentiometer. A set of proportionally divided resistances thus greatly increases the usefulness of the potentiometer.

306. The Voltmeter Principle.

Let AB (fig. 188) be two points in a circuit carrying a current C. If r is the resistance between A and B, the potential-difference is Cr. Now let a shunt circuit of *high resistance compared with* r be joined across AB. This taps off a portion of the main current, but only a very small portion, say c. The current flowing through AB is now $(C - c)$, and the P.D. is—

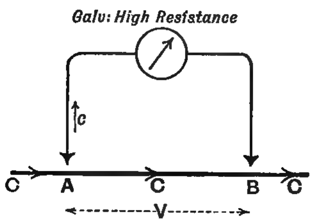


Fig. 188.—Voltmeter Principle

$$Cr - cr.$$

If now, owing to the high resistance of the shunt, c is so small that it may be neglected, the P.D. is Cr. This shunt therefore *is not regarded as forming a split in the circuit.*

> **"If a high resistance is connected as a shunt across a comparatively low resistance, the P.D. at the ends of the latter is practically unaltered."**

If a high-resistance galvanometer is connected as a shunt across different portions of a circuit in turn, the deflections of the galvanometer are proportional to the currents tapped off, and these are proportional to the potential-differences. Hence the deflections of the galvanometer may be taken as indicating potential-difference instead of current. *A high-resistance galvanometer may therefore be used as a voltmeter.*

Note.—A low-resistance galvanometer, of course, also takes a current proportional to the P.D. But the application of such a galvanometer to two points of a circuit would alter the potential-difference it is required to test. The following analogy from heat experiments may be useful:—If you plunge a small thermometer into a beaker of hot water, the temperature indicated will be practically what it was before the thermometer was introduced. But if you place a large thermometer in the hot water, the latter is cooled considerably: the thermometer indicates the temperature now existing, but this is not the temperature you required to test.

Just as in testing temperatures we must have thermometers with a small capacity for heat, so in testing P.D. we must have galvanometers with a small capacity for taking a current.

Any form of galvanometer or ammeter may therefore be used as a voltmeter if wound to have a suitable resistance; and we may therefore classify voltmeters as—

(1) Electromagnetic: (*a*) needle, (*b*) suspended coil;
(2) Electrodynamic.

Besides these we have Electrostatic Voltmeters (Chap. XV).

Observe carefully the following points of difference in ammeters and voltmeters:—

(1) An ammeter has a low resistance, whilst a voltmeter must have a high resistance in comparison with the conductors it is intended to test.

(2) An ammeter is joined in *series* with the circuit to be tested. Its indications represent the current actually passing, and are therefore unaffected by change in the resistance of its coil (or the strip taking the main current). A voltmeter is joined in *parallel* with the portion of the circuit tested. The deflections depend on the small current tapped off, but

the voltage represented by these deflections depends on the resistance of the coil as well. The resistance of the instrument should therefore be unaffected by changes of temperature. To attain this condition the working coil is made of fine copper wire, and the main part of the resistance of the instrument is then made up of a coil of manganin, constantin, or other material of small temperature coefficient.

Increase of range in voltmeters may be attained in the following ways:—

(1) An extra resistance may be placed in series with the voltmeter coil.

EXAMPLE.—A certain current-measuring instrument, to be used as a voltmeter, indicates ·005 ampere per scale division, and has a resistance of 38 ohms. There are 50 scale divisions. What resistances used in series with the instrument will enable ranges of 1 to 50 and 4 to 200 to be read?

Since 1 scale division and therefore ·005 ampere is to indicate 1 volt, we have—

$$C = \frac{V}{R}; \quad \therefore \cdot 005 = \frac{1}{R}, \text{ or } R = 200 \text{ ohms};$$

$\therefore$ resistance required in series $x = 200 - 38 = 162\ \omega$.

Similarly for the second range—

$$\cdot 005 = \frac{4}{R}, \; R = 800; \quad \therefore x = 762.$$

(2) A second method consists in using a set of proportionally divided resistances as explained in connection with the potentiometer. (Art. 305.)

307. Method of Obtaining a Small Known P.D.

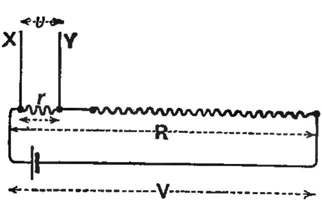


Fig. 189

It is sometimes required to apply a small known P.D. to a circuit of high resistance. This may be done as shown in fig. 189. A constant cell of known E.M.F. is required. X, Y are the terminals of the circuit to which the P.D. is

to be applied. Join a high resistance R (say 10,000 ohms) in series with the cell. Join the ends of a small portion of it, r, to X and Y. Practically no current is tapped off along XY, hence the current in r is the same as in the remainder of R. If v is the P.D. at XY, and V the P.D. at the ends of R—

$$v = Cr,\ V = CR;\ \therefore \frac{v}{V} = \frac{r}{R}.$$

Since R is a high resistance, we have V = E, and

$$v \doteq \frac{r}{R} \cdot E,$$

from which v may be calculated.

308. Conversion of Voltmeters into Ammeters.

In order to avoid sending heavy currents through the working coils of ammeters, these instruments are now usually constructed on the following principle:—The heavy current to be measured is sent through a fixed resistance of small value. Across the terminals of this a voltmeter system is joined (fig. 190). The indications of the instrument will depend on the current passing through the low resistance, and the voltmeter scale may be marked to indicate the current which passes through the low resistance. The latter should be in the form of a flat or corrugated strip so that it may remain cool during the passage of the current.

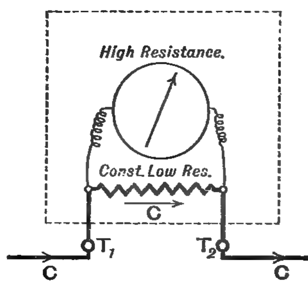


Fig. 190

309. Calibration of Ammeters and Voltmeters.

Since instruments of this class do not usually give strictly proportional indications, it is necessary to determine the value of the scale readings throughout the entire range.

Ammeters (fig. 191).—A standard resistance is arranged in

series with the ammeter under test, a variable resistance, and several accumulators. The terminals of the standard resistance are joined to the point A and to one side of the switch. The standard cell is joined to A and to the other side of the switch. The current through the potentiometer is first adjusted as explained in Art. 304. A current is then sent

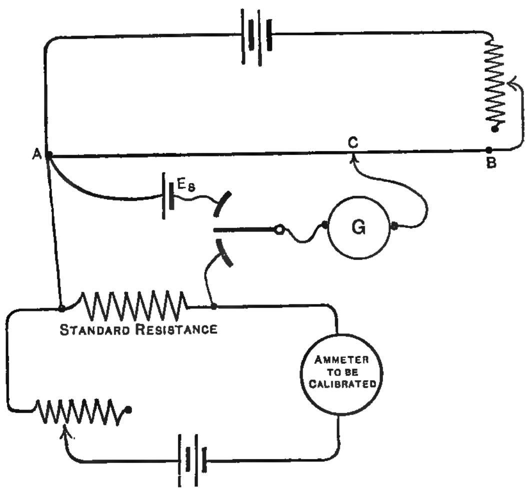


Fig. 191.—Calibration of Ammeter

through the ammeter circuit, and adjusted to produce a deflection of 1 division. The P.D. at the ends of R_s, the standard resistance, is then read off by means of the potentiometer.

Then— $$C = \frac{V}{R_S}.$$

The current through the ammeter is then increased to the next division, and the new P.D. at ends of R_s determined.

Then— $$C_1 = \frac{V_1}{R_S}.$$

This process is continued throughout the entire scale of the ammeter.

Voltmeters.—Up to 1·5 volts, the test may be made by joining the voltmeter as a shunt across the terminals of the standard resistance in the experiment just described.

For high-reading instruments the principles explained in Art. 307 may be applied (fig. 192). r_1 and $(r - r_1)$ are the small and high resistances respectively in series with a large battery of accumulators. The voltmeter under test is joined

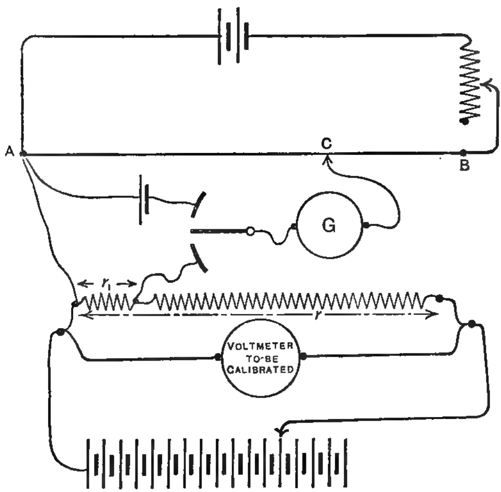


Fig. 192.—Calibration of High-Reading Voltmeter

as a shunt across r, and the potentiometer switch across r_1. The various readings on the voltmeter are obtained by varying the number of cells included. Each voltmeter reading is obtained from the corresponding potentiometer reading by multiplying by $\frac{r}{r_1}$. Thus, if $\frac{r}{r_1} = 100$, when a P.D. of 1·45 is shown on the potentiometer the voltmeter is indicating 145 volts.

310. Measurement of Very Low Resistance.

The metre-bridge cannot be satisfactorily applied to the comparison of resistances of the order of ·001 ohm. For the

measurement of very low resistances special methods have been devised depending on the potentiometer principle. The following is an exceedingly convenient apparatus for the purpose, and gives very consistent results.[1]

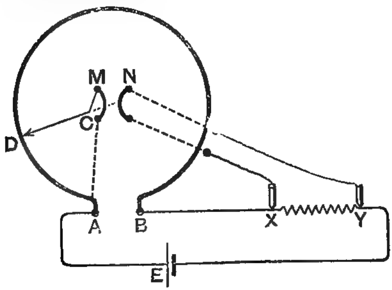


Fig. 193.—Measurement of Low Resistance

ADB (fig. 193) is a stout wire bent into a circle and adjusted to be exactly $\frac{1}{10}$th ohm. A scale divided into 1000 parts runs alongside the wire, the scale and wire are mounted on a board. An arm CD pivoted at C can be brought into contact with the wire by depressing a key at D. A small differential voltmeter is mounted at the centre of the board. The ends of one coil M are joined to A and the arm CD. The ends of the other coil are brought to two terminals, which may

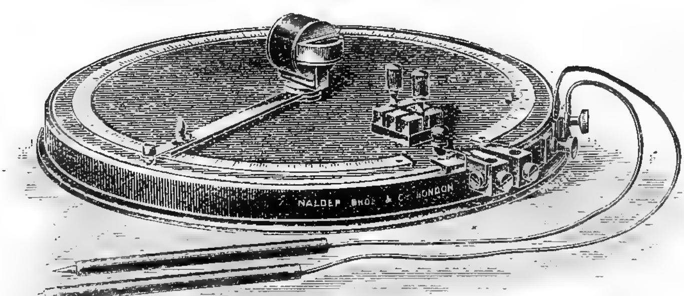


Fig. 194.—Low-Resistance Tester

be connected as a shunt across the resistance under test XY. The latter is joined in series with AB. The main circuit is EADBXYE. The voltmeter shunts XNY, AMCD. The currents in the shunt circuits flow in opposite directions round

[1] Made by Messrs. Nalder Bros. & Co.

M and N. If XY is less than a tenth of an ohm, a point of contact on AB, say D, can be found where the voltmeter is unaffected. Then the potential-difference of AD = potential-difference of XY. But since the currents are equal in AB and XY, the resistances will be equal, and will be given by the scale reading of D.

The instrument reads direct to ·0001 ohm, and shunts are provided for varying the sensitiveness. The arrangement of the parts will be perceived from fig. 194. The instrument is frequently used for the measurement of armature resistances.

STANDARD CELLS AND RESISTANCES

311. Cells in which the depolarizing action is of the replacement type (Art. 244) are especially suitable for standards of E.M.F. The following cells have been found the most reliable for this purpose.

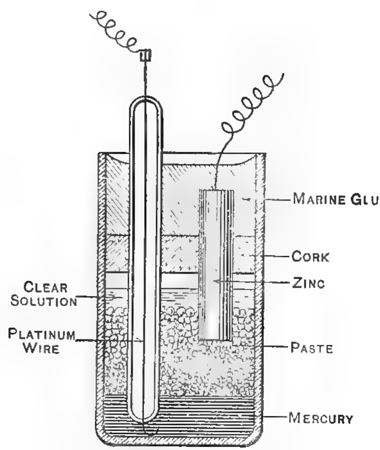


Fig. 195.—Clark Cell

1. **The Clark Cell.**—This form is adopted by the Board of Trade (fig. 195). The poles are zinc and mercury, the former being a short rod, and the latter forming a layer at the bottom of a small glass cylinder. The exciting liquid is a saturated solution of zinc sulphate, and the depolarizer the insoluble mercurous sulphate. These substances, which are prepared with great care, are mixed to form a paste just above the mercury, but a clear solution of zinc sulphate is left in the upper part of the tube. The cork carrying the zinc rod is sealed in with marine glue, and a platinum wire fused through

the bottom of a glass tube makes contact with the mercury. The tip of the platinum wire is amalgamated previously by dipping it, when red-hot, into mercury.

A Clark cell *must not be allowed to carry any appreciable current*, and should be used always in series with a high resistance (10,000 ohms). The E.M.F. diminishes with rise of temperature. The decrease is in accordance with the formula—

$$E = 1{\cdot}4345\{1 - {\cdot}00077\,(t - 15^\circ)\}.$$

2. **The Weston Cadmium Cell.**—Two test-tubes are connected by a horizontal tube. Cadmium amalgam is placed at the bottom of one tube, and mercury in the other. These are covered with a mixture of mercurous sulphate and saturated cadmium sulphate. The tubes are closed above, and a platinum wire is fused through the bottom of each tube to serve as a terminal. The E.M.F. is 1·109, and the temperature coefficient is almost negligible.

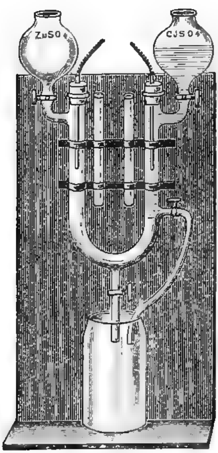


Fig. 196.—Fleming Standard Daniell

3. For ordinary purposes a Daniell cell serves as a sufficiently accurate standard. The best form is that devised by Dr. J. A. Fleming (fig. 196). The two limbs of the U-tube contain solutions of pure copper sulphate and zinc sulphate respectively. These should be of equal density (1·2), and should be poured simultaneously into the tubes, so that they do not mix. The terminals are a copper-plated copper rod, and a rod of pure zinc. The E.M.F. is 1·105 volt.

312. Measurement of E.M.F. of a Cell.

The E.M.F. of a cell may be determined with the aid of a standard resistance and current-measuring instrument, say a tangent galvanometer (or current-balance). The standard

resistance is arranged in series with the tangent galvanometer, a variable resistance, and constant battery. The high potential terminal of the standard resistance is joined through a sensitive galvanometer and high resistance to the positive pole of the cell. The negative pole is joined to the low potential terminal of the standard resistance. The current is adjusted by the variable resistance until no deflection is obtained in the sensitive galvanometer. Then if C is the current indicated by the tangent galvanometer and R the standard resistance, $E = CR$.

Composition of Electrical Alloys.

The following are the principal alloys used for resistance coils:—

Platinum-silver (largely used for resistance coils): Silver 66 per cent, platinum 34 per cent.

German-silver: Copper 4 parts, zinc 1 part, nickel 2 parts.

Platinoid: German-silver with 1 or 2 per cent of tungsten.

Manganin (valuable on account of its almost negligible temperature coefficient): Copper 84 per cent, manganese 12 per cent, nickel 4 per cent.

Phosphor Bronze and Silicon Bronze: Copper alloyed with 3 per cent tin (with traces of phosphorus or silicon from the fluxes used).

313. Standard Resistances.

Standard coils for testing purposes may be constructed of bare platinum-silver or manganin wire soldered to two stout copper terminals. The coil should be immersed in an oil bath, the temperature of which can be observed with a thermometer.

An important quantity in connection with standard resistances is the **temperature coefficient** of the material. Metals and alloys increase in resistance when the temperature rises, and the increase per degree is a nearly constant fraction of the resistance at a given temperature. This

fraction, calculated on the resistance at 0°, is the *temperature coefficient.* If R_0 is the resistance at 0°, the increase of resistance for 1° is αR_0 where α is the coefficient. The increase for $t°$ rise is therefore $\alpha R_0 t$. Hence the total resistance at $t°$ is given by—

$$\begin{aligned} R &= R_0 + \alpha R_0 t \\ &= R_0(1 + \alpha t). \end{aligned}$$

EXAMPLE.—On a winter's day when temperature = 0° C. a copper coil has a resistance of 5000 ohms. What is its resistance on a summer's day when the temperature = 25° C.? Temperature coefficient = ·004.

$$\begin{aligned} \text{Increase of resistance for } 1° \text{ rise} &= \cdot 004 \times 5000 \\ \text{,, \quad ,, \quad } 25° \text{ ,,} &= \cdot 004 \times 5000 \times 25 \\ &= 500. \\ \text{Resistance at } 25° &= 5500. \end{aligned}$$

The material for a standard resistance should have a high specific resistance and low temperature coefficient. For the former quality will allow of the use of stout wire without undue length, so diminishing the rise of temperature due to Joule heat; and the low temperature coefficient keeps the variations of resistance within narrow limits. The materials which possess these properties in a marked degree are the alloys German-silver, platinum-silver, platinoid, manganin, and constantin.

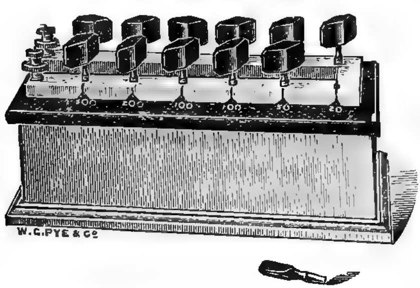


Fig. 197.—Resistance Box

Fig. 197 shows the usual form of a box of resistance coils. The cover of the box is of ebonite, and carries a number of brass blocks. Each of these ends in a rod inside the box, and the coils are joined across successive rods. The coils are

wound usually on reels, and *non-inductively*. That is, each coil is doubled on itself to form a long, narrow loop, and then wound on the reel. Thus no magnetic field is produced by the coil, and no inductive effects occur. Each coil may be short-circuited by the insertion of a plug, and the resistances are therefore brought into circuit by the *removal* of the plugs. The sets of resistances usually adopted are—

1, 2, 2, 5; 10, 20, 20, 50; etc.,

or 1, 2, 3, 4; 10, 20, 30, 40; etc.

In the form of standard resistance suitable for the potentiometric experiments four terminals are provided. Two of these are merely for the purpose of joining the resistance in circuit. The other two are the "potential" terminals, and are joined to two points on the coil carefully adjusted so that the resistance between them is exactly equal to the standard resistance.

Example.

Explain why an electromagnetic voltmeter should have as high a resistance as possible. Given a voltmeter of this kind, reading from 0 to 5 volts, with a resistance of 500 ohms; how may the same instrument be adapted to read from 0 to 50 volts with the same scale? (1904.)

(*a*) See Art. 306. The voltmeter must practically form no split in the circuit.

(*b*) Arrange a resistance of 4500 in the shunt circuit containing the voltmeter.

Let C be the current required to produce a deflection of 5 divisions. Then in the first case—

$$C = \frac{5 \text{ volts}}{500 \text{ ohms}} = \frac{1}{100} \text{ ampere.}$$

In the second case—

$$C = \frac{x \text{ volts}}{(500 + 4500) \text{ ohms}} = \frac{1}{100} \text{ ampere;}$$

$$\therefore x = 50 \text{ volts.}$$

QUESTIONS

1. Describe some practical form of standard of electromotive force, and explain carefully the method by which such a standard, in conjunction with a set of resistance coils and a galvanometer, may be utilized for current measurement. (1902.)

2. A constant cell is employed to send a steady current through a wire of uniform resistance. How would you show experimentally that the fall of potential along the wire was uniform, and how would you determine the potential difference per unit length of the wire? (1904.)

3. Explain the terms *specific resistance*, and *temperature coefficient of resistance* of a material. What material would you employ for constructing a standard resistance, and how would you wind the wire? (1906.)

4. Describe carefully, stating the precautions necessary, how you would test the accuracy of an ammeter reading to about 1·5 ampere. (1906.)

CHAPTER XXIV

ELECTROLYSIS

314. We have already mentioned the chemical action which takes place when a current passes through an electrolyte, and have referred to the use of a copper voltameter for the comparison of current strengths. Before proceeding to the general laws of electrolytic action we shall here describe two other useful forms of voltameter.

315. **The Hydrogen Voltameter.**

This consists of a graduated glass-tube having two platinum wires fused through its walls near the bottom, the wires being welded to two platinum plates. The tube, which is closed at the upper end, is filled with dilute sulphuric acid and inverted in a vessel of the same solution. When the electrodes are joined to a battery, the gases evolved collect in the upper portion of the tube. From the volume of the mixed

gases the weight[1] of hydrogen can be calculated. (Allowance may be made for the pressure due to the column of the liquid and for the temperature.) 1 c.c. of the mixed gases contains 2/3 c.c. of hydrogen. 1 c.c. of hydrogen weighs ·0000896 gram. The weight of hydrogen liberated is therefore ·0000896 × 2/3 × (volume of mixed gases).

Fig. 198 shows a form of voltameter which may be used when it is required to collect the gases separately.

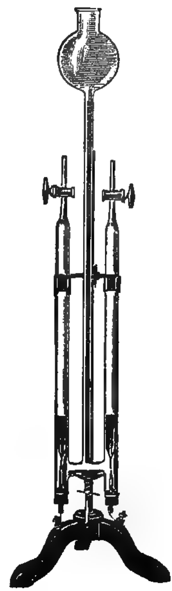


Fig. 198. — Voltameter for Decomposition of Sulphuric Acid

316. The Silver Voltameter.

The cathode may be made in the form of a platinum bowl which at the same time forms the receptacle for the liquid. The anode consists of a plate of silver suspended horizontally in the liquid. The electrolyte is a solution of silver nitrate, to which a few drops of acetic acid may be added. The anode may be wrapped in filter paper to prevent particles of disintegrated silver from falling into the bowl. The silver deposited on the bowl by electrolysis may be dissolved off by nitric acid. This form of voltameter, devised by Lord Rayleigh, is suitable for the standardization of current balances or other instruments used for absolute current measurements.

317. Faraday's Laws of Electrolysis.

I. Arrange a hydrogen voltameter in series with a copper voltameter. Allow the current to flow for, say, half an hour, and determine the weight of copper and the volume of hydrogen. Repeat the experiment with a stronger current. A greater weight of copper is obtained, and a greater volume of hydrogen. It will be found that the volume of hydrogen and the weight of copper are both increased in the same ratio. This ratio is also that of the current strengths.

[1] Strictly the *mass*, but the distinction need scarcely be attended to here.

The volume of oxygen is also increased in the same ratio, and this will be found true of any other electrolytic deposit. If the time is varied, it will be found that the weight of the ion deposited is directly proportional to the time, for a given current.

Taking the expression "amount of electrolytic action" to mean weight of substance deposited or set free at the electrodes, we may express the first law of electrolysis thus—

I. The amount of electrolytic action is directly proportional to the current strength and to the time.

If w is the weight of substance liberated,

$$w \propto C \times t \qquad (1)$$

or

$$w \propto Q.$$

We have therefore the following alternative statement—

The quantity of electrolytic action with a given substance is proportional to the total quantity of electricity which passes through the cell.

II. By arranging in series voltameters containing different electrolytes, we may compare the weights of different materials deposited by the same current in the same time (*i.e.* deposited by the same quantity of electricity). Thus with copper and hydrogen voltameters it will be found that the weight of copper is 31·8 times the *weight* of hydrogen liberated. If the oxygen is collected separately it will be found 8 times the weight of the hydrogen.

These numbers are in the proportion of the *chemical equivalents* of the substances (or radicals) forming the ions. Chemical equivalents are a series of numbers representing the relative weights of the elements (or radicals) which enter into combination with each other, hydrogen being represented by unity as a standard of comparison. The chemical equivalent may be found by dividing the atomic weight (or sum of atomic weights in the case of a radical) by the *valency*, that is by the number of atoms of hydrogen chemically equivalent to one atom of the element. Thus—

The atomic weight of oxygen is 16, valency 2, and the chemical equivalent is 8.

The atomic weight of silver is 107·9, valency 1, and chemical equivalent 107·9.

The atomic weight of copper is 63·6, valency 2 for cupric salts, chemical equivalent for cupric salts 31·8.

8 grams of oxygen combine with 1 gram of hydrogen, 107·9 grams of silver, or 31·8 grams of copper.

The second law may therefore be expressed—

II. The amount of electrolytic action for the passage of a given quantity of electricity is proportional to the chemical equivalent of the substance liberated.

It follows that if we know the weight of hydrogen set free by unit quantity of electricity, we can calculate the weight of any other substance set free. The quantity of hydrogen set free by 1 ampere in 1 second (that is, by 1 coulomb) is—

$$\cdot 00001036.$$

If k is the chemical equivalent of any ion, the weight of this ion set free by 1 coulomb is—

$$e = k \times \cdot 00001036 \qquad \text{(2)}$$

The weight of ion liberated by a unit current flowing for unit time is termed the **electrochemical equivalent** of the substance.

The following table gives the electrochemical equivalents of the more important ions along with their chemical equivalents. The electrochemical equivalents are expressed in **grams per coulomb.**

Element.	Chemical Equivalent.	Electrochemical Equivalent.
Hydrogen........	1	·00001036
Copper...........	31·5	·000328
Silver.............	107·7	·001118
Mercury..........	100·0	·001037
Iron...............	18·6	·000193
Zinc...............	32·5	·000337
Sodium..........	23·0	·000239

The weight of any ion liberated by a current of C amperes in t seconds is—

$$w = k \times \cdot 00001036 \times Ct,$$

or
$$w = eCt \qquad \text{...(3)}$$

where e is the electrochemical equivalent.

[*Note.*—If the C.G.S. unit of current is adopted instead of the ampere, the constant for hydrogen becomes ·0001036, and the other electrochemical equivalents must also be multiplied by 10.]

Examples.—1. A current flows through a hydrogen voltameter and a silver voltameter in series. If 5 cubic centimetres of hydrogen are liberated in 10 minutes, find how much silver would be deposited in 1 hour.

The weight of 1 c.c. of hydrogen is ·0000896 gram.
The weight of 5 c.c. of hydrogen is ·000448 gram.
The weight of hydrogen liberated in 1 hour $= \cdot 000448 \times 6$.
Wt. of silver liberated in 1 hour ($k = 107 \cdot 9$)
$$= \cdot 000448 \times 6 \times 107 \cdot 9 = \cdot 2904 \text{ gm.}$$

2. A current flowing through a copper voltameter deposits ·15 gram copper in 1 hour. What is the strength of the current (supposed constant)?

We have—
$$w = eCt$$

$w = \cdot 15$, $e = \cdot 000328$ gram per ampere-second, $t = 3600$.

Hence—
$$C = \frac{\cdot 15}{3600 \times \cdot 000328} = \cdot 126 \text{ ampere.}$$

318. Theory of Electrolytic Conduction.

The theory of Grotthus serves as a first approximation to an explanation of electrolytic conduction. But the results of many experiments made in recent years have led to a considerable modification of the theory. One important fact of which the Grotthusian theory gives no explanation is that electrolytes are nearly always *solutions.* Perfectly pure sulphuric acid and pure water are almost non-conductors, but dilute sulphuric acid forms a good electrolyte. Dry hydrochloric-acid gas is a non-conductor, but its solution in water

conducts readily. Pure liquids like ether, carbon-bisulphide, are good insulators.

It is now believed that the groups of atoms are separated or dissociated through the agency of the solvent—usually water. The separated groups form the ions. The ions derived from each molecule carry electric charges, and in consequence are urged from the side of high potential (anode) to the side of low potential (cathode), or vice versa, according as the charge carried is positive or negative.

Thus when sulphuric acid is mixed with water, some action of the water molecules breaks up (*i.e.* dissociates or ionizes) the molecules of sulphuric acid into the groups H_2 and (SO_4). The group H_2 forms the positive ion and carries a positive charge. The (SO_4) group forms the negative ion and carries a negative charge. When the electrolytic cell is joined in a circuit, the P.D. between anode and cathode makes the H_2 ions travel in one direction and the (SO_4) ions in the reverse direction.

The theory that the molecules of the substance dissolved are broken up into ions by the action of the solvent does not rest on electrical evidence merely. The lowering of the freezing-point produced by dissolving small quantities of substances in water, the laws of osmosis, and even the colour of the solutions all yield evidence of the same process.

It is not necessary to suppose that *all* the molecules of the dissolved substance are dissociated. Dissociation and recombination may be taking place continually, but at any instant there will always be a certain percentage of dissociated ions. The percentage depends on the strength of the solution, and it appears that in very dilute solutions the molecules are all dissociated.

Electrolytic dissociation must not be confused with ordinary chemical dissociation. Thus a solution of ammonium chloride consists of ions (NH_4) and Cl. But when the dry salt is heated it breaks up into NH_3 and HCl (chemical dissociation).

It must be noticed that if C_p is the current carried by the

positive ions, and C_n the current carried by the negative ions, the total current—

$$C = C_p + C_n.$$

This is because a current of negative electric charges in one direction produces the same effect (magnetic, heating, etc.) as a current of positive electricity in the *reverse* direction.

319. Secondary Actions.

In addition to the electrolytic action of the current there are usually secondary actions of a purely chemical nature. We shall here consider only the most important of these.

> I. **If the anode consists of the same metal as the positive ion of the electrolyte, it dissolves at the same rate as the metal is deposited on the cathode. The solution remains at constant mean strength.**

Thus, consider the electrolysis of copper sulphate with a copper anode. For every atom of copper deposited on the cathode there is a group (SO_4) liberated at the anode. The (SO_4) group cannot exist alone, and combines at once with a Cu atom from the anode. For each molecule of $Cu\,SO_4$ electrolysed a new one is formed by the secondary action, and the mean concentration of the solution remains constant.

A precisely similar process goes on when zinc sulphate is electrolysed with a zinc anode, or a silver salt with a silver anode.

The principles just mentioned are always adopted in electroplating.

> II. **If the anode consists of platinum, or other metal not readily attacked by the nascent negative ion, the solvent round the anode becomes decomposed, oxygen being evolved.**

An example occurs in the electrolysis of sulphuric acid solution. H is liberated at the cathode. The (SO_4) set free

at the anode is unable to combine with the platinum, and attacks the surrounding water molecules. Thus—

$$H_2O + (SO_4) = H_2SO_4 + O.$$

The secondary action thus forms a new molecule, which takes the place of that decomposed, and the O is given off at the anode. The net effect is the same as if the water were electrolysed. In fact, the experiment is frequently referred to as the electrolysis of *water*, but the real electrolyte is H_2SO_4.

Similarly $CuSO_4$, $ZnSO_4$, K_2SO_4, etc., will all yield oxygen when electrolysed with a platinum anode.

III. If sodium, potassium, or other alkaline metal is liberated at the cathode, it attacks the water immediately, with the formation of caustic soda, potash, etc., and liberation of hydrogen.

$$K + H.HO = KHO + H$$
$$Na + H.HO = NaHO + H.$$

Further Examples.

Common Salt (NaCl).—The primary products are Na at cathode and Cl at anode. But the secondary action (III) produces caustic soda and liberates hydrogen at the cathode, and Cl combines with the metal forming the anode, even if of platinum.

Hydrochloric Acid (HCl).—Hydrogen is evolved at cathode. Cl attacks material of the anode, forming a chloride.

In these two cases the Cl may be collected if a carbon anode is used. But since chlorine is a soluble gas, the solution must first be saturated with it.

Platinum Chloride.—Platinum is deposited as "platinum black", and the chlorine behaves as described above.

For certain substances, solvents other than water may be used.

320. Physical Character of the Deposit.

We usually assume that the ions consist of the positive and negative parts of the molecule of the electrolyte only. But it

is probable that in many cases the ion is more complex. In the electrolysis of $CuSO_4$, for example, the ions may be simply Cu and (SO_4), but they may also be these associated with a number of water molecules.

One fact in support of this is that the character of the deposit is greatly influenced by the nature of the salt, and it would seem that the fine uniform deposits, such as are required in plating, are produced by secondary actions. When silver nitrate is electrolysed, the silver is deposited in fine, rather loose, crystals. But if the electrolyte is the double cyanide of silver and potassium (plating solution), the silver is deposited in a close-grained condition, and can receive a high polish. Hittorf considers that in the latter case the double cyanide $KAg(CN)_2$ breaks up into the ions K and $Ag(CN)_2$. At the anode the secondary reaction is—

$$Ag + Ag(CN)_2 = 2AgCN.$$

At the cathode the silver is deposited by the *secondary* reaction—

$$K + KAg(CN)_2 = 2KCN + Ag.$$

There is a wide difference between this and the electrolysis of silver nitrate, the silver ions travelling with the negative current in the former case and with the positive current in the latter. The deposit from the nitrate grows by the addition of Ag atoms to the crystals already formed. But with the cyanide the deposit grows where the K ions are liberated, and these will not show any preference for the silver crystals. The result is a more uniform deposit.

The growth of crystals is beautifully illustrated in the well-known "lead-tree" experiment.

The ions in the electrolysis of potassium ferrocyanide are probably K_4, $Fe(CN)_6$; and in potassium ferricyanide K_3, $Fe(CN)_6$.

An important consideration in connection with electrolysis is the **current density**. This is the amount of current which passes through a unit area placed at right angles to the direction of flow.

The current density largely affects the character of the deposit; if too great, the deposit is rough and loose. If a dense current is passed through a copper voltameter, the anode becomes coated with "anode mud"—a brownish-black mixture of copper oxide and disintegrated copper. Very little deposit is found on the back of the plates, since the lines of flow of the current follow the paths of least resistance.[1]

In copper voltameters the density should not exceed 10 amperes per square foot of cathode.

321. **Fused Electrolytes.**

Insoluble substances (for example, silver chloride) may be electrolysed if they are maintained in a fused condition. The same method may be usefully employed where the products of electrolysis react chemically on water. The conductivity of a fused electrolyte is of the same order of magnitude as that of a dissolved one.

322. **Polarization.**

If we attempt to decompose dilute H_2SO_4 with a single Daniell cell, using a voltameter with platinum electrodes, we do not succeed. If two Daniell cells are used, electrolysis takes place, but with comparative slowness. The minimum P.D. which will produce continuous electrolysis may be ascertained by using a potentiometric[2] source of P.D. Thus it may be shown that the electrolysis cannot be maintained with less than 1·7 volt. The following experiment supplies the explanation of this:—

When the platinum plates are well covered with the gases, disconnect the battery and join the voltameter direct to a galvanometer. It will be found that a current flows for some time. The voltameter behaves like a temporary voltaic cell, the plates covered with oxygen and hydrogen forming the positive and negative poles respectively. The E.M.F. of the voltameter rapidly diminishes as the oxygen and hydrogen become recombined.

[1] The lines of *flow* of the current between two parallel plates immersed in a liquid, are similar to the lines of electric force between two parallel plates equally and oppositely charged.

[2] That is, by making use of the arrangement shown in fig. 191 and joining the electrodes of the voltameter to the ends of the standard resistance.

During electrolysis, therefore, the electrolytic cell is the seat of a **back E.M.F.**, owing to the deposits of different materials on the electrodes. The plates are then said to be **polarized.** If the P.D. applied to the cell terminals is not greater than the polarization E.M.F., electrolysis will not occur.

If the applied P.D. is less than 1·7 volt, say 1 volt, there is at first a weak current which causes the deposition of H and O on the plates to commence, but not visibly. The back E.M.F. increases with accumulation of deposit until it equals the P.D. The action then stops with the exception of an excessively weak leakage-current due to slow diffusion of the deposits from the plates. If the applied P.D. now increases to 1·5 volt, the back E.M.F. will increase to the same value, and so on, until 1·7 volt is reached. If the P.D. is 2 volts, electrolysis is maintained, the P.D. sending a current through the cell being $2 - 1{\cdot}7 = {\cdot}3$ volt.

In the case of the sulphuric-acid electrolyte the gaseous deposits are probably partly "occluded" in the metal electrodes, but similar results are obtained when the deposits are solid.

Irreversible Back E.M.F.—It must not be supposed that the back E.M.F. which exists *during* electrolysis will necessarily be equal to the E.M.F. which remains *after* the cessation of the main current. In the case of dilute sulphuric acid with platinum electrodes the latter is 1·07. Hence 1·07 is the reversible part of the polarization E.M.F. The difference is probably due to the work necessary to separate the hydrogen from the liquid (in the form of bubbles). If the plates are previously covered with a deposit of platinum-black, continuous electrolysis commences even when the applied P.D. is only 1·07 volt. In this case the finely divided platinum probably absorbs large quantities of the gas, and in some way facilitates its separation from the liquid.

Another cause of back E.M.F. is variation in the concentration of solution in different parts of the cell.

323. Measurement of Polarization E.M.F.

Owing to the rapid decrease of E.M.F. which takes place after the main current stops, it is necessary to measure the

polarization E.M.F. instantly after the cessation of the main current.

Fig. 199 shows one arrangement for doing this. An electrometer is connected to the voltameter on one side directly, and on the other side through a key. A battery is also connected to the voltameter through the same key. When the key is in the position shown in the figure, the current flows through the voltameter, and the plates become polarized. When the key is moved into the position shown by the dotted lines, the battery is disconnected, and the electrometer is joined up. The deflection produced indicates the potential-difference due to the polarization. The value of the potential-difference in volts is obtained by connecting up a standard cell to the electrometer in an independent experiment.

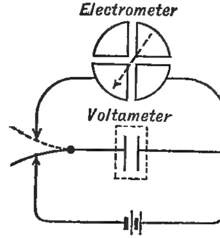


Fig. 199

In accurate determinations the key consists of the prong of a tuning-fork maintained electrically in continual vibration.

Example.—A voltameter has a resistance of 5 ohms and a polarization E.M.F. of 1·9 volt. What current will be sent through the circuit (*a*) by a cell of E.M.F. 1·5 volt and resistance 3 ohms? (*b*) by three such cells in series, connecting wires being negligible?

(*a*) Applied P.D. 1·5, back E.M.F. rises to 1·5; ∴ C = 0.
(*b*) Applied P.D. 4·5, back E.M.F. = 1·9. Total = 2·6.
Total resistance = 5 + 9 = 14.

$$C = \frac{2{\cdot}6}{14} = {\cdot}19 \text{ ampere.}$$

324. Measurement of Electrolytic Resistance.

This measurement presents special difficulties owing to the polarization of the electrodes. One of the simplest methods of overcoming the difficulty is to measure the P.D. between two points in the liquid between the electrodes by means of an electrometer. The experiment is almost identical with that described in the proof of Ohm's Law for electrolytes. The apparatus is arranged as in fig. 147. A reading of the

current is taken on the ammeter, and the corresponding reading for P.D. determined from the electrometer. The electrometer reading may be converted into volts by taking an independent reading with a cell of known E.M.F. Then if V is the P.D. and C the current, $R = V/C$.

The resistivity of the electrolyte may be determined if the distance l between the platinum loops is measured, and also the cross-sectional area of the tube.

Another method which has been frequently applied is due to Kohlrausch. A metre-bridge is used, and the tube containing the electrolyte joined in one of the gaps in the ordinary way. But instead of using a uni-direction current an alternating current is employed, polarization of the electrodes thus being prevented. A telephone is connected across the bridge in place of the ordinary galvanometer, as the latter instrument is not deflected by an alternating current. The point of balance is obtained when there is a minimum sound in the telephone. The alternating current is obtained from the secondary of a coreless induction coil, the interruptions in the primary circuit being produced by a motor break.

The following table gives the resistivities of several important electrolytes:—

Substance.	Resistivity in sq. cm. ohms per cm.
H_2SO_4 dilute (density 1·20)	1·25 at 20° C.
$CuSO_4$ solution (saturated)	25·0 „ 15° C.
$ZnSO_4$ „ „	21·0 „ 15° C.
$K_2Cr_2O_7$ „ „	29·0 „ 15° C.

325. Secondary Cells.

An electrolytic cell which continues to exert its reverse E.M.F. after the electrolysing current has ceased is termed a *secondary* cell. Sir W. Grove in 1842 constructed a battery consisting of platinum plates immersed in dilute sulphuric acid, the liquid being contained in test-tubes inverted in vessels of the same solution. When the liquid is electrolysed the oxygen and hydrogen collect in the upper parts of the tubes. When

the "charging" battery was disconnected the E.M.F. of the secondary battery so obtained was about 1·07 volt per cell. With a battery containing many of these cells Grove produced an arc light. Grove's gas battery is at present of theoretical interest merely. Secondary cells—termed also *storage cells* or *accumulators*—are now widely used, the electrodes being lead plates and the electrolyte dilute sulphuric acid. (A storage cell does not store electricity, but materials whose chemical potential energy can be converted into electrical energy.)

THE LEAD ACCUMULATOR

326. Secondary Cells of Planté and Faure.

Immerse two clean lead plates in dilute sulphuric acid and pass a current from a fairly strong battery. The hydrogen evolved at the cathode escapes without producing any effect. The oxygen combines with the anode, covering the latter with a brown deposit of lead peroxide (PbO_2). The plates become polarized, and a reverse E.M.F. is set up. Disconnect the battery and join the lead plates to a voltmeter. It will be found that the E.M.F. is nearly 2 volts. It soon falls off, however, owing to the thinness of the deposit of peroxide, which becomes used up when the cell yields a current.

By the process called "forming" the plates a deposit of sufficient thickness can be obtained to enable the cell to yield a constant current for a considerable time.

There are two principles on which plates are formed, named, after their inventors, the "Planté" and "Faure" methods.

Planté's Cell.—Gascon Planté, in 1860, constructed a cell as follows:—Two sheets of lead 60 cm. × 20 cm. and 1 mm. thick were laid one on the other with a sheet of felt between. This was then rolled into a compact cylinder and immersed in a vessel of dilute acid (fig. 200). The plates were "formed" by (1) passing the current through the cell to form peroxide deposit, (2) allowing the cell to rest on open circuit, (3) reversing the current through the cell. These

processes are repeated many times. At each reversal the peroxide becomes reduced by the action of the hydrogen, and metallic lead is formed in a "spongy" and highly active condition. When the charging process has been carried as far as possible, the hydrogen and oxygen are evolved in the proportions in which they are combined in water. The process of fully forming the plates takes several days, or even weeks.

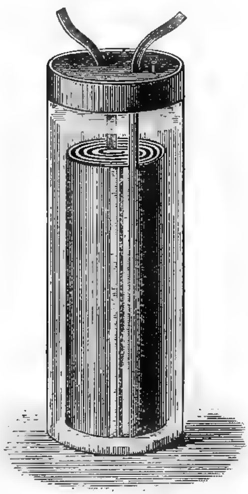


Fig. 200.—Planté Cell

Faure's Cell.—To avoid the difficulty involved in forming the plates, Faure coated both plates with a mixture of red lead (Pb_3O_4) and dilute sulphuric acid. Plates of this type are said to be "pasted". The acid acts on the red lead according to the equation—

$$Pb_3O_4 + 2H_2SO_4 = PbO_2 + 2PbSO_4 + 2H_2O,$$

so partially forming the deposit of lead peroxide.

327. Chemical Action in the Cell.

The reactions which take place in the lead accumulator have not yet been fully worked out, but much light has been thrown on the mode of action of the cell by the work of Gladstone and Tribe. The following are probably the main reactions:—

(1) Commencing with either a Planté or a Faure cell fully charged, we have peroxide on the lead plate at the anode, and spongy lead on the cathode.

(2) Now suppose the cell is allowed to rest. Since the sulphuric acid soaks through the peroxide, a *local action* takes place where the peroxide is in contact with the lead; thus—

$$Pb + PbO_2 + 2H_2SO_4 = 2PbSO_4 + 2H_2O.$$

The local action ceases as soon as the lead sulphate (which is

insoluble and a bad conductor) forms a barrier between the PbO_2 and Pb. The lead sulphate thus enables the PbO_2 to be retained in sufficient quantity to render the cell of practical value. (No local action takes place at the other plate, the spongy lead being in contact with metallic lead.)

(3) Next suppose that the cell discharges through a circuit. The acid is electrolysed; H_2 and (SO_4) are liberated at the plates, and produce—at the moment of liberation—the following secondary actions:—

(*a*) At anode: $PbO_2 + H_2SO_4 + H_2 = PbSO_4 + 2H_2O$;
(*b*) At cathode: $Pb + (SO_4)' = PbSO_4$.

Both plates become covered with a layer of $PbSO_4$, and the solution becomes *less* acid.

(4) Now let the cell be recharged, the final result of the reaction is the reverse of the last.

At anode: $PbSO_4 + (SO_4) + 2H_2O = 2H_2SO_4 + PbO_2$.
At cathode: $PbSO_4 + H_2 = H_2SO_4 + Pb$,

and the solution becomes *more* acid.

Variation in Density of the solution during the charging and discharging process is a notable feature of the cell. During the charging process the solution increases in density owing to gain of acid, and during discharge decreases in density.

It is not found advisable to allow the density to vary outside the limit 1·20 when the cell is fully charged, and 1·18 when discharged. The density is usually tested during charging with a hydrometer.

328. Construction.

The plates used in an accumulator must be of sufficient strength to retain their shape (buckling is not uncommon), and also to carry the current without heating. At the same time they must be as light as consistent with these conditions, and

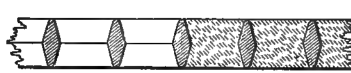


Fig. 201.—Section of Grid

must be constructed to hold the deposits firmly. The plates now used are grooved or perforated lead castings termed *grids.* The material is usually pure lead (or an alloy of lead and antimony to give the necessary stiffness).

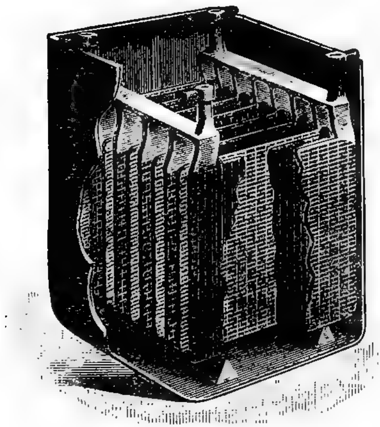


Fig. 202.—Arrangement of Plates for Heavy Currents

In some accumulators the positive plate is constructed on the Planté principle. In the Tudor accumulator the positive plate is cast in one piece, with a number of vertical ribs and horizontal ones for strength. They are formed on the Planté method described above. The negative plates are of the pasted type.

In the cells made under the E.P.S. patents, on the Faure principle, a grid of the form shown in section in fig. 201 is used. The plate is perforated with square holes tapering inwards both ways. These are filled with the red-lead paste. When the paste dries it coheres, and is prevented from falling out either way owing to the shape of the hole.

Many forms of grid are adopted by different makers. For these and other modifications the student must consult special treatises.

329. **Advantages of the Accumulator.**

(1) The E.M.F. is high, normally 2 volts, and very constant, whilst the resistance is low in all but the smallest cells and usually of the order ·01 ohm. Hence the cell can yield a current which is both *strong* and *steady.*

(2) Where lighting or dynamo installations are available

for charging purposes, secondary cells are more economical, and require less attention than primary cells.

(3) The efficiency is large enough to render the cell of practical value in lighting and power stations, where it would be impossible to use primary cells.

The energy obtained from a cell during the discharge is normally 60 per cent of the energy supplied to it during the charging process, and higher values may be obtained.

The low resistance of a secondary battery is its chief advantage for certain kinds of work.

The principal disadvantages of the cell are its initial cost, weight, and necessity of current supply for charging. There is a species of local action which results in the formation of minute crystals of lead sulphate over the surfaces of the plate (hence termed *sulphating*) when the cell is allowed to remain unused. Accumulators should be kept in continual use. Common defects are the detachment of pellets of active material, which fall to the bottom of the cell and cause short-circuiting; and buckling or warping of the plate when discharged at a high rate.

The **capacity** of a secondary cell is the total quantity of electricity it is capable of discharging through a circuit during a complete discharge.

This is usually expressed in ampere-hours (1 ampere-hour = 3600 coulombs). Theoretically a cell of 15 ampere-hours capacity would yield a current of 1 ampere for 15 hours, 5 amperes for 3 hours, and so on. But there is a certain maximum rate of discharge for each cell—depending on the size and construction—which must not be exceeded. This is determined and stated by the maker. It is evident from Faraday's laws that the capacity of the cell will depend on the amount of active material. It is found that a capacity of about 7 ampere-hours per kilogram of positive active material (PbO_2) is about the highest obtainable.

As it is inconvenient to have large plates, cells are usually made up with a number of plates in *parallel* (fig. 202). This

arrangement gives a very low resistance but does not affect the E.M.F. The outside plates are both negative, which prevents short-circuiting by contact with the cell walls. The latter are of glass, or wood lined with lead.

Short-circuiting by contact between adjacent plates is prevented by means of ebonite or glass strips.

QUESTIONS

1. Explain the application of Ohm's Law to an electrolytic cell. Describe a simple method of measuring the specific resistance of an electrolyte, and state the chief precautions necessary in the experiment. (1902.)
2. Describe some form of storage battery, and give an account of the chemical changes which go on during the charge and discharge of the cell. (1904.)
3. What is meant by polarization? Describe the changes in the current through a dilute sulphuric acid voltameter with platinum electrodes, as the E.M.F. applied to the electrodes is increased from a very small value up to 2 volts. (1903.)
4. A copper voltameter and hydrogen voltameter are arranged in series. If a current deposits 2 grams of copper in 1 hour, what volume of hydrogen will it liberate in 1 minute?
5. Describe a modern type of secondary cell, and mention some instances of its application.
6. A current is passed through a voltameter and coil of wire in series with it. If the current is altered in such a way that the heat produced in the coil is doubled, show what change will be produced in the rate at which chemical action takes place in the voltameter. (1903.)

CHAPTER XXV

THERMO-ELECTRIC CURRENTS

330. If a circuit consists entirely of metallic conductors, a current can be set up by heating or cooling some of the junctions of different metals. The effect is best shown with antimony and bismuth, but almost any pair of metals can

be used. The currents generated in this way are termed *thermo-electric* currents.

Join a bar of antimony, A, to a bar of bismuth, B (fig. 203), and complete the circuit through a *low-resistance* galvanometer. Now heat the junction gently. The galvanometer needle is deflected in a direction which shows that the current flows from B to A across the joint. A similar experiment may be made with copper and iron wires twisted together and joined direct to the galvanometer.

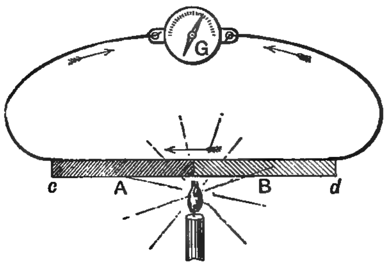


Fig. 203.—Thermo-Electric Circuit

The current is due to a very small electromotive force termed a *thermo-electromotive force.* This only exists when there is a difference of temperature between the junctions.

(As antimony and bismuth are frequently used for thermo-electric junctions, it is convenient to remember that the current flows from **A**ntimony to **B**ismuth at the **C**old junction.)

The E.M.F. in a thermo-electric circuit may be determined by the following potentiometric method, the principle being the same as that used in the case of a cell (Art. 312).

331. Measurement of Thermo-E.M.F.

Fig. 204 shows a convenient arrangement for this purpose. S is a standard resistance (a Crompton $\frac{1}{10}$ or $\frac{1}{2}$ ohm potentiometer resistance serves excellently), arranged in series with a carefully calibrated milliampere-meter. The box marked Rheo. is an ordinary box of coils used as a variable resistance, and M is an accumulator or other constant cell. The high-potential side, P_1, of the standard resistance is joined to the + terminal of the thermo-electric source, and the low-potential side, P_2, to the − terminal, through a sensitive galvanometer and key K. The current indicated by the milliampere-meter

must be adjusted by means of the variable resistance until no deflection is obtained on closing the key K. Then if V is the P.D. of the points P_1 P_2, we have—

$$V = \text{thermo-E.M.F.}$$

But if r is the value of the standard resistance in ohms, and C the current in milliamperes—

$$V = Cr \text{ millivolts.}$$

Since the thermo-E.M.F. depends on the temperature of the cold junction as well as on that of the hot one, it is necessary

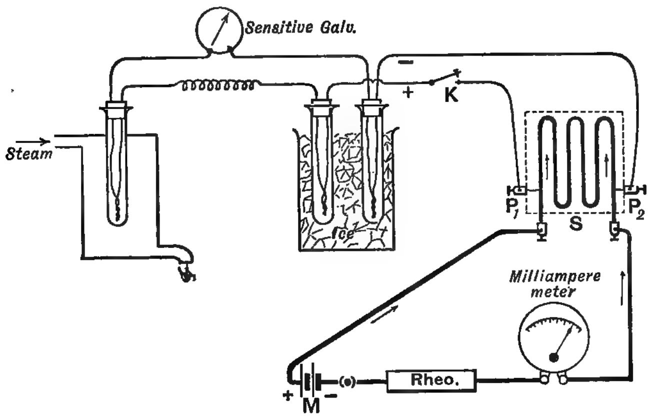


Fig. 204.—Measurement of Thermo-E.M.F.

to provide that both junctions are maintained at constant known temperatures. This is done by suspending each junction in a dry air-space surrounded by a bath at the temperature required.

EXAMPLE.—In testing a German-silver and iron junction the temperatures were 100° C. and 15° C. The standard resistance was ·5 ohm, and the current required to balance was 4·1 m.a. Find the thermo-E.M.F.

$$\begin{aligned}\text{The thermo-E.M.F. required} &= \cdot 5 \times 4\cdot 1\\ &= 2\cdot 05 \text{ millivolts}\\ &= \cdot 00205 \text{ volt.}\end{aligned}$$

332. Laws of Thermo-E.M.F.

By varying the temperatures of the junctions (in the test just described) we may prove experimentally the laws of thermo-electromotive force. The E.M.F. for a circuit of two metals is found to depend on the following conditions:—

(1) The *difference* of temperature between junctions.
(2) The *mean* temperature of the junctions.
(3) The nature of the metals.

The relation of the electromotive force to temperature may be expressed thus—

The thermo-E.M.F. is proportional to the difference in temperature of the junctions, provided the mean temperature remains constant.

Thus if the E.M.F. of a pair of metals is known for two given temperatures, it can be calculated for any other temperatures having the same mean.

EXAMPLE.—The E.M.F. of a pair of metals is 2·5 millivolts, with junctions at 0° and 100°. The mean temperature is 50°. Other temperatures having the same mean are, for example, 30° and 70°, −20° and 120°.

To find the E.M.F. for junctions at 30° and 70°, we have—

$$40 : 100 = x : 2{\cdot}5. \quad \therefore x = 1 \text{ millivolt.}$$

To find the E.M.F. for junctions at −20° and 120°—

$$140 : 100 = x : 2{\cdot}5. \quad \therefore x = 3{\cdot}5 \text{ millivolts.}$$

The thermo-electromotive force for a difference of temperature of 1° between the junctions is called the "thermo-electric power" of the two metals at the mean of the temperatures given.

In the example just quoted the thermo-electric power for a mean temperature 50° is—

$$\frac{2{\cdot}5}{100} = {\cdot}025 \text{ millivolt per degree.}$$

·025 would be the E.M.F. for junctions at 49·5° and 50·5°.

It follows from the law stated above that the E.M.F. is

equal to the total difference of temperature multiplied by the E.M.F. for 1° difference. That is—

$$\text{Thermo-E.M.F.} = \begin{pmatrix}\text{thermo-electric power}\\ \text{at mean temperature}\end{pmatrix} \times \begin{pmatrix}\text{difference of}\\ \text{temperature}\end{pmatrix}$$

333. Thermo-Electric Diagram.

Thermo-electric properties of metals are best shown graphically. In fig. 205 the thermo-electric *powers* are shown by the ordinates, and temperatures by the abscissæ. The line AD shows the increase in power of a copper-lead circuit, and EH the decrease of power of an iron-lead circuit.

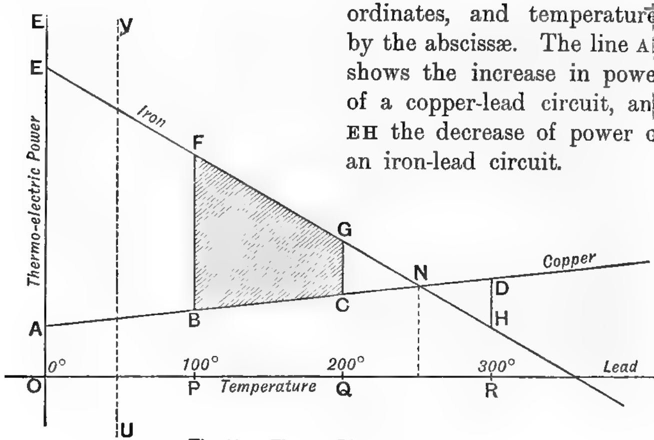


Fig. 205.—Thermo-Electric Diagram

It is found that—

(1) The thermo-electric power varies with the temperature according to a straight-line law.

(2) The thermo-electric power of two metals X/Y is equal to the algebraic sum of the powers of the pairs X/Z and Z/Y.

According to the first law we see that AD, EH are straight lines; and according to the second—

If PB is thermo-electric power of Cu/Pb at a given temperature, and PF is thermo-electric power of Fe/Pb at same temperature, then FB is thermo-electric power of Cu/Fe at same temperature.

The straight lines representing the variation of the thermo-electric powers of two metals, as in fig. 205, form the *thermo-electric diagram* for the two metals. Lead is taken as a standard of reference owing to the absence of the Thomson effect in this metal (see Art. 335), and is represented by the horizontal line. The ordinates to the different lines in the diagram show the thermo-electric powers of circuits containing the different metals and lead. The vertical distances between the lines for two metals indicate the thermo-electric power of a circuit containing these metals at the temperature shown by the abscissa. Thus the thermo-electric power of a Cu/Fe circuit at 100 is shown by FB, at 200 by CG, and so on.

Notice that the ordinates represent thermo-electric *powers*. The thermo-*electromotive force* may be represented by an *area* on the diagram.

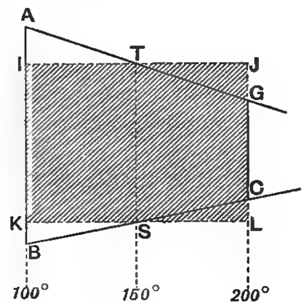


Fig. 206

Suppose that it is required to represent the E.M.F. of a Cu/Fe circuit at temperatures 100° and 200°. A portion of the previous diagram is shown in fig. 206. Bisect AG and BC. Draw IJ and KL horizontally through the points of bisection, meeting the verticals through A and G in the points I, K and J, L respectively. Then TS, IK or JL represents the thermo-electric power at the mean temperature 150°. Also KL represents the difference of the temperatures of the junctions. Hence the area IJLK represents the product

$$\mathrm{ST} \times \mathrm{KL} = \begin{pmatrix}\text{thermo-electric power}\\ \text{at mean temperature}\end{pmatrix} \times \begin{pmatrix}\text{difference of}\\ \text{temperature}\end{pmatrix}$$
$$= \text{electromotive force.}$$

By the geometry of the figure it is evident that the area IJLK is equal to the area AGCB. Hence the thermo-electromotive force for a pair of metals is represented by the area included between the ordinates for the temperatures of the junctions and the thermo-electric lines of the metals.

The direction of the current in the circuit may be represented by a point tracing out the boundary of the area in a counter-clockwise direction, the junctions being represented by the vertical lines AB, CG. Thus, for the copper-iron circuit considered, the current is from

B to C along the copper, C to G across the hot junction,
G to A along the iron, A to B across the cold junction.

334. The Neutral Point—Inversion.

The thermo-electric lines for some metals intersect. Copper and iron furnish an example, the point of intersection occurring for a temperature of 270°. The thermo-electric power vanishes at this temperature—termed the *neutral point.*

The neutral point for a pair of metals is the temperature at which their thermo-electric power becomes zero.

We must ascertain what effect this has on the total E.M.F. when one temperature is below the neutral point and the other above.

Referring to fig. 207, if N is the neutral point, CG the

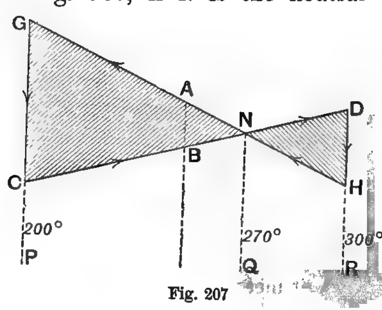


Fig. 207

thermo-electric power at the lower temperature, DH that at the upper temperature, CG and DH must be taken with opposite signs. The electric power at the mean temperature is—

$$\frac{CG + (-HD)}{2}.$$

The E.M.F. of the circuit is—

$$\frac{CG - HD}{2} \times PR$$

$$= \frac{CG}{2}(PQ + QR) - \frac{HD}{2}(PQ + QR)$$

$$= \left(\frac{CG \cdot PQ}{2} - \frac{HD \cdot QR}{2}\right) + \left(\frac{CG \cdot QR}{2} - \frac{HD \cdot PQ}{2}\right).$$

But the quantity within the second bracket vanishes, for by similar triangles

$$\frac{CG}{PQ} = \frac{HD}{QR}.$$

The quantity within the first bracket is the difference in areas of the two triangles CNG, NDH, and therefore the E.M.F. is represented by the difference in the areas of the triangles on opposite sides of the neutral point.

The direction of the current round the circuit may be indicated by a point tracing out the boundary, the outline of the larger triangle being traced out counter-clockwise.

Thus in the case shown in fig. 207 the directions are C to D along the copper, D to H (that is, copper to iron) at hot junction, H to G along the iron, G to C at cold junction.

Let us suppose that the cold junction is maintained at a constant temperature whilst the hot junction is gradually heated. The area representing the total E.M.F. is thus bounded on the left by a fixed ordinate (for the lower temperature) and on the right by an ordinate which travels further towards the right as the temperature rises. The area increases (and therefore the E.M.F.) up to the neutral point. Beyond this the area cut off by the moving ordinate must be deducted from the area up to the neutral point. The E.M.F. thus gradually decreases, and when equal areas are cut off on each side of the neutral point it vanishes.

The total E.M.F. thus reaches its maximum value in one direction when the temperature of one junction passes the neutral point, that of the other being steady. If the temperature of the hot junction rises still more, then, when the

area to the right is the larger, the direction of the E.M.F. changes. This reversal is known as *thermo-electric inversion.* Its existence may be shown experimentally as follows:—

Twist an iron wire and copper wire together and join them direct to a low-resistance galvanometer. Gradually warm the junction with a spirit lamp. The deflection increases to a maximum, then steadily decreases to zero, which it passes, thus becoming reversed in sign.

335. The Peltier Effect.

When a current is sent through a circuit by means of a battery, the junctions of the different metals in the circuit are either heated or cooled. This effect, discovered by Peltier, may be shown by the apparatus represented in fig. 208.

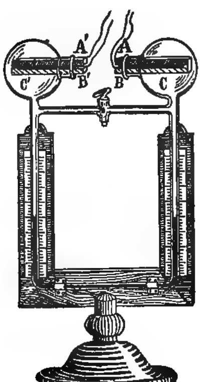


Fig. 208.—The Peltier Effect

Two bismuth-antimony couples are fitted air-tight into the bulbs of a differential air-thermometer. If the current is sent from A to B the junction is heated, and the liquid is driven down on the right. But if the current is sent from B to A the junction is *cooled*, and the liquid rises on the right.

The production of a thermo-electric current by the *application* of heat to one of the junctions was discovered by Seebeck in 1821, and is known as the Seebeck effect. The Peltier and Seebeck phenomena are closely connected; for if the current passes from a metal X to a metal Y when the junction is heated, this junction is cooled when a current is sent by a battery in the same direction.

Observe carefully the distinction between the Peltier effect and the ordinary Joule effect, considered in Chap. XXII. The former is **reversible**, absorption or evolution of heat occurring according to the direction of the current. The Joule effect is always an irreversible process.

If a thermo-junction is immersed in a calorimeter, we can determine the total heat produced by the Joule and Peltier effects together. If H is the total heat, and h the heat evolved per second by Peltier effect—

$$H = \frac{C^2R}{J} + h.$$

If we now reverse the current we have—

$$H' = \frac{C^2R}{J} - h.$$

Hence, taking the difference in the heat indicated by the calorimeter—

$$H - H' = 2h.$$
$$\therefore h = \tfrac{1}{2}(H - H').$$

In this way it has been shown that the heat produced by reversible processes is proportional to the current strength.

The Thomson Effect.—If a current is passed along a wire which varies in temperature from point to point, heat is

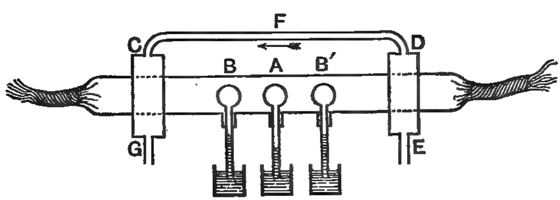


Fig. 209.—The Thomson Effect

absorbed or generated according to the direction of the current and the nature of the metal. This remarkable effect was discovered by Sir William Thomson (Lord Kelvin), in 1846, from theoretical considerations, and was subsequently confirmed by him experimentally. It is, like the Peltier phenomenon, a *reversible* process.

One method of experimentally demonstrating the existence of the Thomson effect is illustrated in fig. 209. An iron tube CD was fitted with three air-thermometers, B, A, B′. The tube was

surrounded with cooling-jackets near its ends, through which a current of cold water circulated continually. A strong current was sent through the tube, the leads being a bundle of wires at each end. The current heated the tube (Joule effect), and in consequence of the cooling arrangement there was a temperature gradient from A to C and A to D. Hence, if there were no further effects to consider, B and B′ would indicate equal temperatures. But B showed a higher temperature than B′ when the current flowed from C to D, and a lower temperature than B′ when the current was reversed, thus proving the existence of the Thomson effect.

There is no Thomson effect in lead. For this reason the line for lead in the thermo-electric diagram is taken horizontal, and is used as the main line of reference.

If a current passes along a copper wire from the warm to the cold parts the wire becomes warmed. When the current flows from cold to warm parts the copper is cooled. A similar effect is produced in all substances whose thermo-electric lines slope upwards with increasing temperature.

In iron and other metals whose thermo-electric lines slope downwards the effects are just the reverse.

336. Practical Applications.

Thermo-electric effects find their chief useful application in certain delicate instruments for detecting and measuring heat radiation. The best known of these is the *thermo-pile*. This consists of a number of delicate bars of bismuth and antimony arranged alternately as shown in fig. 210. A whole framework of such bars is arranged in cubical form and insulated by plaster of Paris or mica strips, the junctions being left exposed. When the junctions on one side are exposed to a source of radiation, an E.M.F.

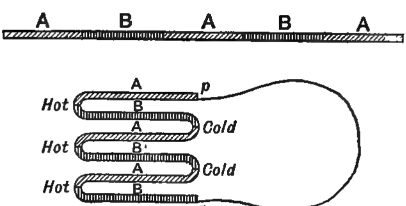


Fig. 210.—Structure of Thermo-pile

is generated by the difference of temperature so produced, and if the thermo-pile is joined to a delicate galvanometer of *low* resistance, a current will be indicated. The galvanometer deflection is proportional to the intensity of the radiation falling on the thermo-pile.

Thermo-electric junctions are also applied to the measurement of high and low temperatures, and have been recently used in the measurement of weak alternating currents.

All attempts hitherto made to apply thermo-electric E.M.F.'s to the economic production of currents have failed. Large thermo-piles, however, heated by gas flames can generate sufficient current for electro-plating.

QUESTIONS

1. Assuming the thermo-electric powers of iron and nickel with respect to lead to be $+12$ and -20 microvolts respectively, find the E.M.F. of a nickel-iron couple with junctions at 0° and 100° C. (1902.)
2. Explain what is meant by the Peltier effect, and describe an experiment by means of which it can be exhibited. (1904.)
3. Describe how you would compare a small E.M.F., say that of a thermo-couple, with the E.M.F. of a Clark cell. (1903.)
4. Under what circumstances can an electric current cool the conductor through which it passes? What will be the result if the direction of the current is reversed? (1894.)

CHAPTER XXVI

INDUCED CURRENTS

337. After the discovery of the magnetic action of a current, in 1820, many attempts were made to discover a converse action, that is, the production of an electric current by the use of a magnet. The experiments made were, however, without success, until Faraday, in 1831, at last found the key to the phenomena. He found that a current can be produced

by the action of a magnet on a coil, but only whilst the magnet is *in motion* relative to the coil. Faraday also found that an induced current is produced momentarily in a wire circuit whenever a current is started or stopped in a neighbouring circuit; but no effect is produced whilst the current is flowing steadily. These experiments of Faraday were the origin of a series of far-reaching discoveries, one outcome of which is the electrical engineering practice of the present day.

338. Induction between Circuits.

For experimental tests a pair of solenoids may be used (fig. 211). One of these, consisting of a few turns of stout wire joined in series with a battery and reversing key, is termed the *primary* coil. The other consists of a large number of turns of fine wire, and is joined in series with a sensitive reflecting galvanometer, so forming an all-wire circuit; this is termed the *secondary coil.* The primary coil should be mounted so as to slide in or out of the secondary.

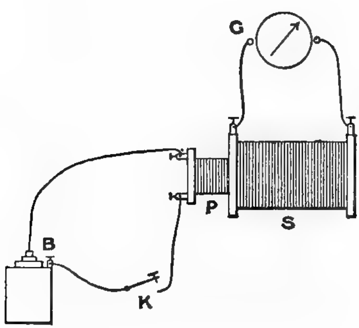


Fig. 211.—Primary and Secondary Circuits

A preliminary experiment with a cell must be made to determine which directions of the current produce deflections in the galvanometer to the right and left respectively.

The primary coil having been placed within the secondary, it will be found that there is no deflection of the galvanometer so long as the current is flowing steadily. But if the primary circuit is made or broken, or the current altered in strength, there is a momentary "kick" of the galvanometer needle. Again, if the primary current is allowed to flow steadily, and the coil is suddenly inserted into the secondary or withdrawn from it, a momentary kick of the needle occurs. The directions of the induced currents are shown in the table below, where "inverse" means that the current in the secondary is in the opposite direction to that in the primary circuit, and "direct" means that the currents agree in direction.

Primary Current.	Secondary Current.
Started, Increased, Moved towards secondary.	Inverse
Stopped, Decreased, Moved away from secondary.	Direct.

In general an induced current can be produced whatever the relative position of the coils may be, but there are particular positions for which it vanishes. If the primary is held outside the secondary, pointing at right angles to the middle of the latter, there is no induction.

If the battery is joined to the *secondary* coil, and the current is started or stopped, an induced current is produced in the primary.

339. Induction by the Motion of the Magnet.

If one pole of a bar magnet is suddenly inserted or withdrawn from the secondary coil, a kick of the galvanometer needle is produced. Here we have a current in an all-wire circuit produced without the aid of a battery. The directions are as stated below, *looking at the coil from the side where the magnet is placed.*

Magnet.	Induced Current.
N. pole approaches....	Counter-clockwise.
N. pole recedes.........	Clockwise.
S. pole approaches.....	Clockwise.
S. pole recedes..........	Counter-clockwise.

340. Relation of Induction to Magnetic Field.

When a coil is placed near a magnet or current the magnetic field of the latter surrounds the coil, and a number of the

lines of induction are *linked* with the coil for the time being.[1] If we represent the field by unit tubes of induction, these will crowd together when the field is strengthened, and separate when it is made weaker. If you think this over in connection with the experiments mentioned above, you will see that an induced current only occurs when there is a *change* in the number of tubes of induction linked with the circuit. The cause of the induced current is therefore to be sought in the variations of the magnetic field. In all cases the current is produced by a "cutting" of the lines of induction by the circuit.

341. General Rules for Direction of Induced Current.

The direction of the current induced in a closed conducting circuit can be found in several ways. The student should become familiar with both the rules given below, since, although they lead to the same result, one rule is often easier of application than the other.

> **I. Look along the lines of the inducing field so that they point away from you. Then if the number of lines linked with the circuit increases, the direction of the induced current is counter-clockwise.**

Confirm this by use of the experimental results obtained above (Arts. 338, 339). A formal proof is given in Art. 348.

The induced current is considered to have a positive direction when its own lines of force, within the coil, would have the same direction as those of the inducing field, and a negative direction when its own lines would oppose the inducing field. The law of induction may therefore be stated as follows:— An increase of magnetic flux through a circuit produces a negative induced current, and a decrease a positive one.

Strictly speaking, the above rule determines the induced

[1] It must be remembered that a line of induction is an endless curve, like the link of a chain.

E.M.F. The direction of the induced current does not always agree with that of the E.M.F., owing to the influence of self-induction (Art. 343).

Lenz's Law.—The following rule, enunciated first by Lenz, has many important applications:—

> **II. The induced current has always such a direction that its action opposes the motion which produces the induction.**

Thus, suppose the N. pole of a magnet is caused to approach a wire ring (fig. 212). According to Lenz's Law, the motion

Fig. 212

must be opposed by a repulsion. Hence the near face of the ring acts like a N. pole, and the induced current has a counter-clockwise direction, as seen from the magnet. When the magnet is withdrawn the motion is retarded by an attraction, and the induced current makes the near face of the ring of S. polarity.

The motion referred to in Lenz's Law is the *relative* motion of the conductor and field.

If the field is steady, the current is induced by the motion of the secondary across the field.

If the secondary is fixed and the primary moving, the latter carries its own lines of force with it, and causes them to move across the secondary. A similar statement applies to a moving magnet.

If both primary and secondary are fixed, the induced current is produced by a variation in the primary current. We must here suppose that the field lines retain their identity and move outwards across the field (like waves), or shrink inwards. The

following example will show the application of Lenz's Law in this case.

Let A (fig. 213) represent a primary and B a secondary coil. When the current is started in A some of the lines of induction due to this current thread through the coil B. If we look along these lines in the direction of the field, the direction of the induced current appears counter-clockwise. The field within the circle B, due to the induced current, is in a direction which opposes the inducing field.

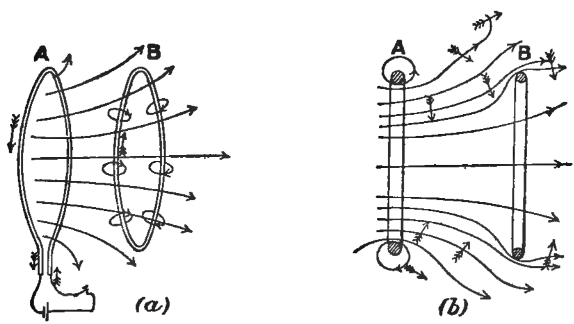


Fig. 213

The field due to the induced current may be compounded with the inducing field. The resultant is shown at (*b*). It is apparent from the form of the lines in (*b*) that *the induced current temporarily prevents the resultant field from crossing the wire* B. This effect is therefore in accordance with Lenz's Law.

When the current in A is steady the lines of induction are not distorted, as there is no induced current.

If the current in A ceases, its field vanishes at the same time, and we may consider that the lines close up and finally collapse in the wire A. During the short period of disappearance the lines which were linked with the secondary must cross the wire B. The passage is again momentarily retarded. During the time of passage there is a mutual pressure between the secondary and the field, which is transmitted through the field to the primary.

It may be stated as a general rule that when the number of lines of induction linked with a circuit tends to change, the induced current temporarily opposes the change.

Experiments illustrating Lenz's Law.—(1) Make a small flat coil and join its ends to two long flexible wires. Suspend the pendulum so formed between the poles of a powerful electromagnet. Set the pendulum swinging. If the circuit of the coil is completed by joining the suspending wires at the top, the motion is instantly checked. (2) The coil of a low-resistance D'Arsonval galvanometer will oscillate for some time if the circuit is left open. But if the galvanometer terminals are joined by a short wire, the induced current immediately checks the motion. (3) If a copper cube is suspended and set spinning between the poles of an electromagnet, it is quickly brought to rest when the current is turned on. (4) If a copper plate is moved about in the strong field of a powerful electromagnet, the retardation can be distinctly felt, an impression of dragging the copper through a soft substance, such as butter, being produced. (5) If a long compass-needle is suspended just above a horizontal copper disc, and the latter is set in rotation, the currents induced in the disc tend to stop the motion. But since the disc is caused to move, the reaction drags the magnet round in the same direction. The draughts of air produced may be kept from the needle by a glass plate placed between the disc and the needle. This phenomenon is known as Arago's Rotation. It was one of the earliest observed facts of electro-magnetic induction.

342. Use of Iron Cores.

If an iron core is placed within the primary of a pair of induction coils, the inductive effects are greatly increased. The core becomes magnetized by the field of the primary, and on account of the great permeability of iron, a large number of tubes of induction are suddenly added to the inducing field. If the iron is soft, these tubes collapse into the core when the current is stopped. Thus more powerful direct and inverse currents are produced.

Faraday's Ring.—If one portion of an anchor ring is wound with a primary coil, and another portion with a secondary joined to a galvanometer, the large flux of induction through

the ring may be demonstrated. If a current is started or stopped in the primary the galvanometer is suddenly deflected, even when the distance between primary and secondary is so great that no effect would be observed without the core.

343. **Self-Induction.**

When a circuit containing an electromagnet is broken, a bright spark is usually observed at the point of break. This may be considered as due to a *self-induced* current. Before the break takes place there is a large number of lines of induction linked with the circuit. When the current is stopped, the sudden collapse of this field produces a *direct* current in the circuit (in accordance with the rules already enunciated). The momentary direct current at break is frequently called the "extra" current. Similarly, when a current is started in a circuit, the sudden production of the lines of induction causes a momentary *inverse* current. This momentarily retards the production of the main current. Its existence may be demonstrated experimentally.

Join a large coil to a Post-office bridge and proceed as if to find its resistance. When a balance has been obtained in the ordinary way for steady currents, keep the galvanometer key down and open the battery key. There will be a sudden kick of the galvanometer needle in a direction which indicates a "direct" current from the coil. Now keeping the galvanometer key closed, establish the battery connection. There is a momentary kick of the needle in the reverse direction, showing the inverse self-induced current. These effects are obtained from the coil under test. The resistances in the P.O. box are non-inductively wound.

344. **The Ruhmkorff Induction Coil.**

This important instrument consists of a primary and a secondary coil, provided with an automatic arrangement for make and break of circuit. The main parts and connections are shown diagrammatically in figs. 214, 215.

(1) The *primary* coil consists of a comparatively few turns of thick wire, provided with a soft iron core. The latter consists of a bundle of soft iron wires. (2) The *secondary* is

wound with a very large number of turns of fine copper wire (several miles in the larger coils), the successive layers being

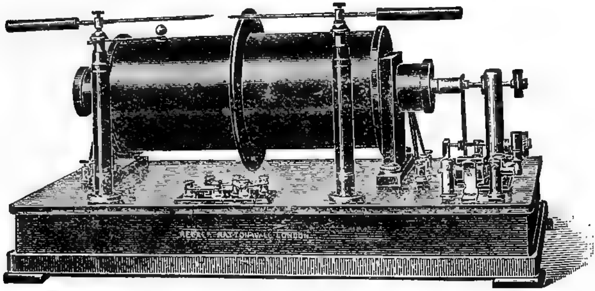


Fig. 214.—Ruhmkorff Coil

highly insulated. The resistance of the primary is usually less than 1 ohm, whilst the resistance of the secondary is several thousand ohms. One end of the primary is joined to an

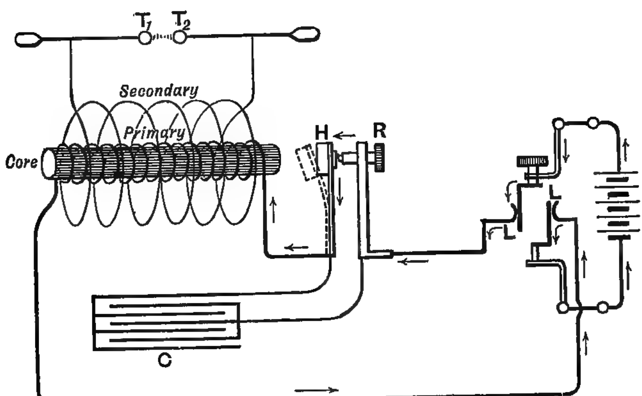


Fig. 215.—Ruhmkorff Coil—Connections

upright spring carrying an iron head H. The back of the spring has a small platinum plate which touches the screw R, also tipped with platinum. The latter is joined through a

fixed brass support to the commutator. The other side of the commutator is joined to the end of the primary.

The ends of the secondary coil are joined to two well-insulated discharging rods.

The action is as follows. The spring makes contact in the primary circuit, and the current magnetizes the iron core. The core at once attracts the iron head H. This movement, however, breaks the primary circuit; the current ceases, and the iron core loses its magnetism, so allowing the hammer head to be pulled back by the spring. This re-establishes the contact, and the movements are repeated. The spring is maintained in rapid vibration, and an intermittent current passes through the primary coil. At each make or break of contact an induced current is produced in the secondary. The discharge is similar to that from an electrostatic machine, and there is a very high P.D. between the secondary terminals.

The secondary discharge at make is much less intense than that at break of circuit. The former may be made almost negligible by the following arrangements:—(1) The spring must not vibrate too rapidly; (2) a condenser must be connected across the primary spark gap as shown in fig. 215; (3) the secondary discharge must take place across an air space or vacuum tube. Under these conditions the *self-induced* or *extra* current produced in the primary at break rushes into the condenser, so diminishing the spark at HR. The condenser then discharges round the primary (generally in an oscillatory manner), and so helps to destroy the residual magnetism of the core. The collapse of the primary field is thus much more rapid than its re-establishment; and in the presence of an air gap in the secondary circuit the E.M.F. induced in the latter case will be too weak to produce a spark. Since the induced current at make is negligible, *the secondary current consists of a succession of discharges in one direction only.*

The commutator allows a reversal of the current in the primary to be made, the secondary discharges being reversed also.

If a vacuum tube is joined across the terminals, the tube

lights up with a beautiful glow. The difference in the character of the glow at the ends may be used for determining the direction of the secondary discharge. Induction coils may be used in place of electrostatic machines for many purposes. They can be made more powerful, and are generally more convenient.

345. Induced Electromotive Force.

We have so far confined our attention to the induced *current*. The current is, however, produced by an electromotive force generated in the secondary circuits, and the laws of this induced E.M.F. must now be considered.

Calculation of Induced E.M.F.—Consider the arrangement shown in fig. 216. The rectangular frame ABCD is placed with its plane at right angles to a uniform field. The rod PQ slides along the rails AB, CD. The rod *cuts* the lines of induction, and hence we get an induced current in the circuit.

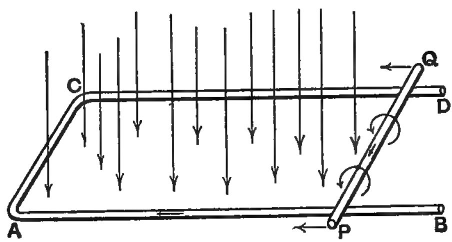


Fig. 216

(This statement is confirmed by the experiment described below.) Let the induced current be C, and let PQ be moved at such a rate that this current remains constant. Now the field exerts a force f on the rod PQ, which is given by—

$$f = \mathrm{CB}l, \text{....................(Art. 280.)}$$

where B is the induction in the field, and l the length of PQ. Hence, since this force opposes the motion, work is done against the field. The work done for a distance d is—

$$\begin{aligned} \mathrm{W} &= f \times d \\ &= \mathrm{CB}ld \\ &= \mathrm{C} \,.\, \mathrm{B} \,.\, a, \end{aligned}$$

where a is the area swept through by the bar. But Ba is the total number of lines of induction (*i.e.* the magnetic flux). Hence—

$$\text{W} = \text{C} \times (\text{number of lines cut by PQ}),$$
$$(\text{Work per second}) = \text{C} \times (\text{number of lines cut per second}).$$

Now the electromotive force is equal to the work done per second per *unit* current. Thus—

$$\text{E} = (\text{number of lines cut per sec.}) \ldots\ldots\ldots\ldots (1)$$

Experimental Test.

This result may be tested experimentally. A circular coil (shown in section in fig. 217) is joined in series with a battery and variable resistance. A copper disc (seen edgewise in the figure) is

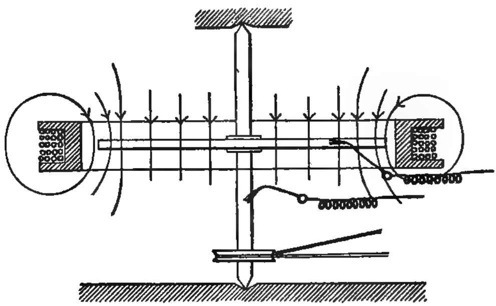


Fig. 217.—Experimental Test of Induction Laws

pivoted so that it can be rotated in the plane of the coil. The disc may be imagined divided into radial strips. All these radii cut the lines of induction at equal rates, and an equal E.M.F. is (by symmetry) generated along each radius. There is no current in the disc, but the E.M.F. produces a difference of potential between the centre and circumference. If two springs are allowed to touch the axle and circumference respectively, these will serve as high and low potential terminals (corresponding to the poles of a cell). The E.M.F. generated may therefore be measured with a potentiometer just as in the case of a cell. It is found that the E.M.F. is proportional to

(1) the speed of rotation;
(2) the current in the coil.

But since the flux-density B in the field is proportional to the current, the E.M.F. varies as the number of tubes cut per second.

If a conductor moves obliquely across a magnetic field, the formula

$$E = (\text{lines cut per second})$$

will still apply. (This may be proved by resolving the motion, field, and length of conductor in directions mutually at right angles.)

EXAMPLE.—A disc 5 cm. in radius rotates about a vertical axis at 3000 revolutions per minute. If the horizontal component of the earth's field = ·2, and the angle of dip is 60°, find the P.D. in volts between the centre and edge of the disc.

The field at right angles to the disc is the earth's vertical component V, and

$$V = H \tan \delta = {\cdot}2 \times \sqrt{3}.$$

This is the number of unit tubes or "lines" penetrating the disc per square cm. Hence—

$$\text{Total vertical flux through disc} = \pi r^2 V = \pi \times 25 \times {\cdot}2 \times \sqrt{3}.$$

Each radius cuts this number of lines per revolution. Hence the number of lines cut per *second*

$$= \left(\frac{3000}{60}\right) \times \pi \times 25 \times {\cdot}2 \times \sqrt{3}$$

$$= 1362 \text{ C.G.S. units} = {\cdot}00001362 \text{ volts.}$$

Since the circuit is open this is the P.D. required.

346. **Direction of Local Induced E.M.F.**

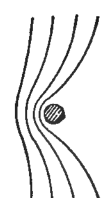


Fig. 218

A rule for finding the direction of the induced E.M.F. may be arrived at from a consideration of the case shown in fig. 216. The energy of the current is derived from the work done in moving the bar PQ. Hence *the resultant lines of induction must offer a resistance to the motion.* By the principles of electro-dynamics we see that the lines of induction will therefore be crowded together in front of the moving rod. (See fig. 218, where the rod is shown in section

and the observer is looking towards Q.) Now these resultant lines are compounded of those belonging to the vertical inducing field and the circular lines round the wire due to the current induced. Hence these component fields must agree in direction immediately in front of the moving conductor. This condition determines the direction of the current (and electromotive force). The rule for finding the direction may be stated thus—

> **Look at the wire end-on, and imagine that the field lines are displaced by its motion. Then if the displaced lines in front of the wire curve round it in the clockwise direction, the E.M.F. acts away from you along the wire.**

In fig. 218, if the field lines are directed upwards, the E.M.F. acts away from you through the page.

If the conductor is fixed and the field moving, the conductor may be imagined to move in the reverse direction to the field, since we are concerned only with relative motion.

347. Distribution of Induced Electromotive Force.

Since electromagnetic induction is produced by a relative motion of the circuit and field, the electromotive force is only generated in those parts of the circuit where this relative motion exists. Thus in the experiment illustrated in fig. 216 the field is constant and the frame is fixed. Hence no E.M.F. is generated in AB, AC, or CD. The bar PQ is the source of the whole E.M.F.

Further, it is evident that the E.M.F. is *distributed* along the bar. In a voltaic cell the E.M.F. is discontinuous, *i.e.* it is generated at the junctions of the different materials and not spread throughout the cell. In the case of induced E.M.F., each unit length of the conductor contributes its share towards the total E.M.F. of the circuit. Each centimetre of the rod PQ contributes an equal share. If we imagine the rod PQ divided into a large number of elements, the whole rod may be compared with a series battery of the same number of cells.

The potential rises from Q to P (when the conditions are as shown in the fig. 216), and Q would correspond to the negative pole, P to the positive pole of the battery.

In the case of the rotating disc described above, the potential rises from the centre to circumference, or vice versa, according to the direction of rotation, each element of the radius contributing its share to the P.D.

In some cases, however, *all* portions of the circuit are in motion relative to the field. If the circuit is fixed and the field varying, they may still be considered to be in *relative* motion. In such cases all portions of the circuit help to produce the total E.M.F., and the circuit may be compared with one consisting *entirely* of voltaic cells. The total E.M.F. in the circuit is the sum of the local E.M.F.'s of the elements. The sum must be taken algebraically, for the E.M.F.'s in some parts of the circuit may tend to produce a current in the reverse direction to the remainder. (The corresponding circuit of voltaic cells would have some of the elements arranged in opposition to the others.)

It is important to distinguish between the total E.M.F. of the whole circuit and the local E.M.F. generated in any portion of it. The *current* depends on the total E.M.F., and therefore in some parts of the circuit the local E.M.F. may act against the current.

The laws from which we determine the magnitude and direction of the local E.M.F. are given above. We now consider the total E.M.F.

348. Calculation of Total E.M.F. in a Circuit.

The convention as to signs must first be understood. If we look along the lines of force and *through* the circuit, lines pointing away from us may be called positive, those pointing towards us, negative. A clockwise current is then positive (see Art. 341). An *increase* in the number of lines passing through the circuit is either an increase in those pointing away from us or a *decrease in those pointing towards us.*

Consider the general case of a circuit and field in relative

motion. The lines of induction may be crossing the boundary of the circuit, inwards at some parts and outwards at others. From the rule for direction of local E.M.F. above, we see that the local E.M.F.'s will act in the positive direction round the circuit when positive lines move across the boundary outwards, or when negative lines move inwards. The number of lines which cross any centimetre of the wire per second is the E.M.F. generated in that centimetre. Hence, adding these effects all round the circuit, we have—

Total E.M.F. in the positive direction

$$= \begin{pmatrix}\text{decrease per second} \\ \text{of positive lines}\end{pmatrix} + \begin{pmatrix}\text{increase per second} \\ \text{of negative lines}\end{pmatrix}$$

$$= \begin{pmatrix}\text{algebraical decrease per second} \\ \text{in number of lines}\end{pmatrix}.$$

Thus, if N_1 is the number of lines linked with the circuit at any instant, and N_2 the number linked after a very short interval t, the number which becomes unlinked from the circuit per second is $(N_1 - N_2)/t$.

Thus the total E.M.F.

$$E = \frac{N_1 - N_2}{t},$$

or

$$E = -\frac{N_2 - N_1}{t} \qquad \text{(2)}$$

$\left(\text{In the notation of the calculus we have } E = -\frac{dN}{dt}.\right)$

Expressed in words, this equation reads—

The total E.M.F. generated in a complete circuit is at any instant equal to the rate of *decrease* of the number of lines of induction linked with the circuit.

We have supposed that the circuit is a single turn of wire, and linked only once with each tube of induction. In a spiral or solenoid the circuit may be linked many times with each tube of induction. When all the tubes of induction thread all the turns of the circuit, the total number of linkages may be expressed in terms of the flux and number of turns. If there are n turns of wire and a flux of P tubes, we have—

$$N = P \times n, \qquad \text{(3)}$$

where N is the number of linkages. This is sometimes called the effective flux.

Thus **linkage** (or effective flux) is the product of the number of turns in the coil and the number of unit tubes of induction linked with those turns.

The **direction of the total E.M.F.** may be found by the application of Rule I (Art. 341), (E.M.F. being substituted for current). This rule is expressed also in the formula for total E.M.F. (Equation (2).)

EXAMPLE.—The number of tubes of induction threading a coil of 500 turns is at a certain instant 6500 lines. One-thousandth of a second later the number is 6000. Find the E.M.F. induced during the short interval.

$$\text{Initial number of linkages} = 500 \times 6500.$$
$$\text{Final number of linkages} = 500 \times 6000.$$
$$N_2 - N_1 = 500 \times -500 = -250000.$$
$$E = -\frac{N_2 - N_1}{t} = \frac{250000}{\frac{1}{1000}} = 250 \times 10^6.$$

This is in C.G.S. units. In volts we have—

$$E = \frac{1}{10^8} \times 250 \times 10^6 = 2{\cdot}5 \text{ volts.}$$

Since 1 volt $= 10^8$ C.G.S. units, it is evident that Equation (2) will give the E.M.F. directly in volts if we take as our unit of linkage 10^8 C.G.S. units of linkage. The **practical unit of linkage** is therefore 10^8 maxwell-turns.

349. Currents Induced in a Rotating Coil.

Let a circular hoop of wire be mounted so that it can be rotated about an axis coinciding with its diameter. If the coil rotates in a uniform field, then the component of the field at right angles to the axis of rotation induces currents in the coil. Thus, if the coil rotates in the earth's field about a horizontal axis parallel to the meridian, the horizontal component of the field will not be linked with the coil in any position of the latter. In this case the vertical component only is effective. Fig. 219 shows six typical positions. In (1) the hoop embraces the greatest number of lines of induction, and for a small movement of the hoop this number remains practically constant. Hence for the moment there is no

induced E.M.F. In (2) the number of lines threading the hoop is rapidly diminishing. Hence looking from above the E.M.F. is clockwise. In (3) the coil is for the instant in the plane of the meridian, and there are no lines threading the coil. But in this position the number of lines threading the coil is *changing most rapidly*. Hence here we have the greatest E.M.F. In (4) the number of lines girdled by the hoop is increasing. The direction of E.M.F. as seen from above is counter-clockwise, but since we are now looking at the opposite face of the coil this is the same

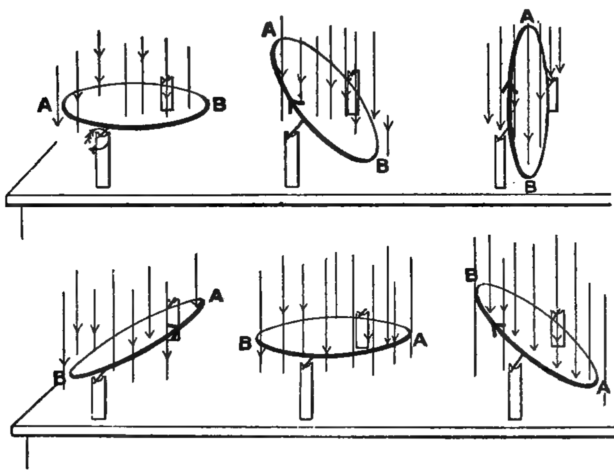


Fig. 219.—Induction in Rotating Hoop

direction as before, namely, B to A in the nearer half of the hoop. The conditions in (5) are the same as in (1), and the E.M.F. is again zero. In (6) the E.M.F. is from A to B in the nearer half, and has therefore been reversed. The changes in the remainder of the revolution are similar to those already given.

The rotating hoop is an important typical case to which it is often necessary to refer. The following points must be kept in mind:—

(i) The E.M.F. does not depend on the number of lines girdled by the hoop, but on the **rate of change** of this number.

(ii) When the greatest number of lines is encircled the E.M.F. is *zero.*

(iii) When no lines of force are encircled the E.M.F. is for the moment a *maximum.*

(iv) The E.M.F. is reversed in direction as it passes through its zero values, *i.e.* when the greatest number of lines is encircled, as at (1) and (5).

The E.M.F. induced in the coil is said to be *alternating.* Its strength increases to a maximum, then decreases to zero; becomes reversed in sign and increases to a negative maximum, and finally returns to a zero value. These changes are completed in each revolution.

The above considerations all apply to the E.M.F. The current strength will also wax and wane, but the maxima and minima of current do not in general occur at the same instants as the maxima and minima of E.M.F.

350. **Faraday's Disc.**

A copper disc is mounted on an axle so that it may be rotated (fig. 220). The portion of the disc near the circumference passes between the poles of a strong magnet, and two springs press lightly on the axle and edge of the disc respectively. These springs are connected with terminals, so that they may be joined to a sensitive galvanometer.

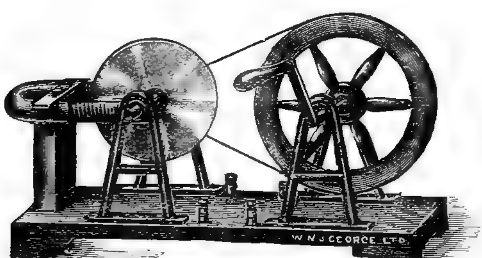


Fig. 220.—Faraday's Disc

If the disc is rotated at a constant rate a steady deflection is obtained in the galvanometer.

Each radial strip of the wheel cuts the lines of induction, and the local E.M.F.'s generated act along the radii. A current is therefore produced which leaves the disc by one contact spring and returns by the other. The strength of the current may be varied by changing the speed of rotation, or (if an electromagnet is used) by changing the strength of the inducing field.

This arrangement is interesting as forming a simple means of producing a *steady* current in one direction by electromagnetic induction. It is, in fact, a very simple dynamo.

351. Eddy Currents.

The current generated in the disc does not all flow out into the external circuit. A portion completes its circuit in the disc itself, forming what is termed an "eddy current". The lines of flow are somewhat as shown in fig. 221. According to Lenz's Law, we must have the currents flowing in such a direction that they retard the motion. Hence, when the poles and rotation are as indicated in the figure, the upper half of the front surface of the disc acts like a S. pole, and the lower half like a N. pole. The currents form a clockwise whirl in the upper half and a counterclockwise one in the lower half.

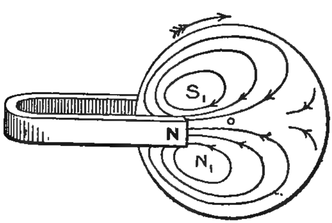


Fig. 221.—Eddy Currents

In the experiment with Arago's disc (Art. 341) the motion of the needle is due to the field set up by the eddy currents in the disc.

Eddy Currents in Electromagnets.—When a mass of iron becomes magnetized the flux of induction set up in it generates eddy currents momentarily in the iron. If the magnetization is continually varying, the surging of these currents is kept up. Since the iron offers resistance, energy is dissipated in the form of Joule heat. In order to prevent the consequent waste

of power, the cores of induction coils, armatures, transformers, and other arrangements subject to varying magnetization are *laminated*, *i.e.* built up of plates or strips of iron carefully insulated from each other.

A useful application of eddy currents is found in the arrangements used for making ammeters, etc., dead-beat. A strip of metal, usually aluminium, is attached to the moving part of the instrument, and moves between the jaws of a permanent magnet. The eddy currents, temporarily induced in the strip or "damper", check the motion, and so help to bring the pointer quickly to rest.

Another application occurs in the governor of some types of electricity meters.

It is important to notice that **the retarding force in Faraday's disc and similar arrangements is proportional to the velocity of rotation.**

Let B be the impressed field, C the induced current, and f the retarding force. Then—

$$f \propto C \,.\, B.$$

But C varies as the induced E.M.F. (which in turn is proportional to the rate of rotation), and as the inducing field B. Thus, if n = no. of revs. per sec.,

$$C \propto B \,.\, n$$
$$\therefore\ f \propto B^2 n.$$

Hence, if the field is constant,

$$f \propto n.$$

i.e. the retarding force is proportional to the velocity.

352. Magnetic Induction.

In dealing with the forces exerted on current-conductors, and with induced currents, we have referred to the lines of induction, rather than to lines of force. We must now explain the reason for this distinction.

So long as the medium in which the field exists is air, and the quantities are expressed in electromagnetic measure, the

distinction is unnecessary, for the equations hold when magnetic force is substituted for induction.

But in the experiment with Faraday's Ring we find that the iron core enormously increases the inductive discharge in the secondary. This is equally true when the primary is uniformly wound over the whole ring, and since in the latter case there is no free magnetism, the increase of inductive effect cannot be ascribed to a change in the magnetic force.

A similar effect is produced by other magnetic metals, and if experimental arrangements are made to ensure accuracy, it is found that almost any substance will influence the inductive discharge in some degree.

Further, if the quantities are expressed in electrostatic measure, the value obtained for the induced electromotive force is only $1/(9 \times 10^{20})$ of that calculated from the rate of cutting of lines of force.

We are thus led to regard induced electromotive force as depending on a measure of the field distinct from magnetic force. This we term *magnetic induction*. Taking as a guide the experimental relations obtained in air, we have the following definition:—"The measure of the magnetic induction at any point is the electromotive intensity induced at that point in a conductor which moves with unit velocity relative to the field in such a direction that the intensity is a maximum."

The induction, the intensity, and the direction of motion are mutually at right angles when the intensity is a maximum.

When the velocity v is not at right angles to the induction B, the electromotive intensity F acts at right angles to the plane containing the directions of B and v. If θ is the angle between these directions,

$$F = vB \sin \theta.$$

(Electromotive intensity in a distributed source is the E.M.F. generated *per unit length.*)

353. Induction in Insulators.

It is only in conductors that continuous currents can be produced by electromagnetic induction. If a mass of dielectric

is displaced relative to the magnetic field, an E.M.F. is induced in it as in the case of a conductor. But the current produced by this E.M.F. can only be the momentary current which exists during the polarization of the dielectric (Art. 215). Thus in the experiment of the rotating copper disc, with a given speed and field, the E.M.F. generated produces a certain displacement of electricity from the centre to the edge of the disc, and consequently a potential-difference between the centre and the edge. The E.M.F. generated (with the same speed and field) if the disc were, say, ebonite, would produce a potential-difference between the centre and edge of the disc, which is only $\left(1 - \frac{1}{K}\right)$ times that which occurs with a conductor. This relation, which was deduced from theoretical considerations by Dr. Larmor, has recently been confirmed experimentally by Dr. H. A. Wilson, the test being made with an ebonite cylinder rotating in the field of a concentric solenoid.[1]

QUESTIONS

1. A bar magnet is suspended on a stirrup by a string and oscillates in a horizontal plane. How are the oscillations affected (if at all) when a thick non-magnetic metal plate is placed horizontally beneath the needle so as to be close to without touching it? (1896.)

2. A rectangular wire frame is laid on a table so that two of its edges are parallel to the magnetic meridian. The frame is then suddenly turned over through 180°, about one of these edges as axis. Find the direction of the total E.M.F. induced in the frame at several typical stages of the motion.

3. Describe Barlow's wheel, and explain its action (1) when used as a dynamo, (2) when used as a motor. (1901.)

4. A light metal ring is suspended over the end of a solenoid; if a large current is suddenly sent through the solenoid, show that the ring will be repelled. (1903.)

5. Describe the Ruhmkorff induction coil. What is the object of the condenser? How must the secondary coil be wound to avoid as far as possible the tendency to sparking between neighbouring turns of wire?

[1] *Proc. Roy. Soc.*, vol. lxxiii.

6. A copper ball, 3 cm. in diameter, is allowed to fall freely through a distance of 500 metres. Find the magnitude of the E.M.F. induced, at the moment before the ball strikes the ground, in the E.-W. horizontal diameter—given that the earth's total magnetic force = ·4 C.G.S., the dip = 60°, and $g = 980$.

CHAPTER XXVII

ELECTROMOTIVE FORCE AND POTENTIAL-DIFFERENCE

354. The distinction between electromotive force and potential-difference has already been dealt with, and for the reasons explained we restrict the term electromotive force to primary causes of finite electric displacement, of which the chief are the following:—

(1) Volta-E.M.F.—arising from chemical action or diffusion in voltaic cells, and polarized electrolytic cells.
(2) Thermo-E.M.F.—due to junctions of different metals or unequally-heated metals.
(3) Induced E.M.F.—due to a cutting of lines of magnetic induction.

In the present chapter we consider more fully the relation between the P.D. at the terminals of the generator and the E.M.F.

355. Potential-Difference on Open Circuit.

The electromotive force of the generator acts whether the circuit is open or closed. In the former case, as we have seen—

P.D. of terminals = E.M.F. generated.........(1)

If we suppose that the seat of the whole electromotive force generated by a cell is where the zinc touches the acid, the potential diagram for open circuit is as shown in fig. 222. The potential of the zinc is shown by the line AB. In the

layer of acid touching the zinc there is a sudden rise of potential BC, due to the E.M.F. The potential through the acid is uniform. The terminal P.D. is shown by the difference in level of the points A and D, and this is equal to the E.M.F. BC.

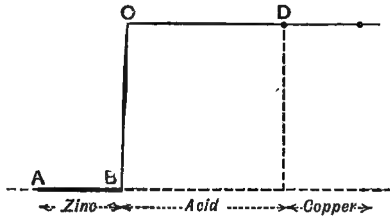


Fig. 222.—Potential Diagram for Simple Cell

In the actual cell the distribution of potential is more complicated than this. The seat of the E.M.F. has been much discussed, but it is probable that it originates chiefly in the places where the plates touch the liquids, and where different liquids come into contact, as at a porous pot. The total force is the algebraic sum of these local E.M.F.'s. The internal distribution of potential on open circuit is somewhat as shown in fig. 223. In the metals, and solutions of uniform density and nature, there is no E.M.F., and the potential lines for these are horizontal. Since we are usually concerned with actions outside the cell, we may suppose, for most purposes, that the local E.M.F.'s are all transferred to the contact of the negative pole with the exciting liquid. Thus, referring to fig. 223, instead of dealing with three separate electromotive forces, BC, DE, FG, we take the algebraic sum of these and suppose this to exist at BC, as shown in fig. 222.

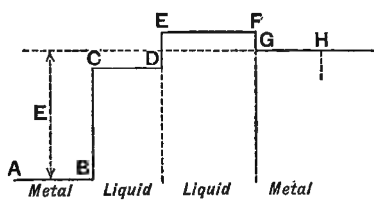


Fig. 223.—Potential Diagram for Compound Cell

The distribution of potential described above occurs not only when the circuit is actually broken, but whenever the conditions allow no current through the cell (as in balancing the E.M.F. of a cell with a potentiometer).

356. Potential-Difference on Closed Circuit.

If the circuit be closed there is a gradual fall of potential in the direction of the current all round the circuit. The diagram then takes the form shown in fig. 224 (supposing as above that the whole E.M.F. is at the zinc-acid layer). The sudden rise of potential in the zinc-acid layer is the same as before,

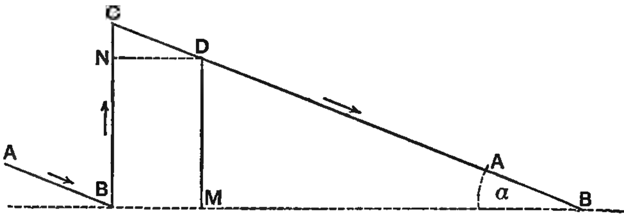


Fig. 224.—Diagram for Closed Circuit

but there is now a drop of potential through the cell in the direction of the current. Let the resistances of the various sections of the circuit be represented by the horizontal distances in the corresponding parts of the diagram.

We have then—

E.M.F. of the cell	$= E$	$= BC$
P.D. of the poles	$= V$	$= MD$
Fall of potential inside the cell	$= v$	$= CN$
Total resistance of the circuit	$= R$	$= BB'$
Internal resistance of the cell	$= b$	$= BM$
External resistance	$= r$	$= MB'.$

We may write down expressions for the current thus—

$$C = \frac{E}{R} = \frac{BC}{BB'}$$
$$= \frac{V}{r} = \frac{MD}{MB'}$$
$$= \frac{v}{b} = \frac{CN}{BM}.$$

These expressions are equal to the tangent of inclination of the lines CB′, DB′, CD respectively. Hence, since the current has the same strength at all parts of the circuit, the line CDB′ is straight.

The strength of the current is represented by the "steepness" of the line CB′ or by tan α.

From the similar triangles DMB′, CBB′ we have—

$$\frac{\mathrm{MD}}{\mathrm{BC}} = \frac{\mathrm{MB'}}{\mathrm{BB'}},$$

that is—

$$\frac{\mathrm{V}}{\mathrm{E}} = \frac{r}{\mathrm{R}} \qquad \text{(2)}$$

The diagram refers to the case where there is only one generator or battery in the circuit. The important relation stated in (2) may be expressed in words thus—

> **When there is only one generator in a circuit, the P.D. of the terminals is the same fraction of the E.M.F. that the external resistance is of the total resistance.**

Example.—A cell has E.M.F. 2 volts and resistance 5 ohms. What is the P.D. at its poles (1) on open circuit? (2) when the poles are joined through a resistance of 15 ohms?

(1) V = E = 2 volts.

(2) The total resistance is 20 ohms. The external resistance is 15/20, or 3/4 of the total resistance. Hence—

$$\mathrm{V} = \frac{3}{4} \times \mathrm{E} = 1{\cdot}5 \text{ volt.}$$

The following particular cases should be specially noted:—

(*a*) If the external resistance is very great, the P.D. is practically equal to the E.M.F.

(*b*) If the external resistance is very small, the P.D. becomes almost zero, and the whole E.M.F. is expended within the cell.

(*c*) If the cell has a negligible resistance (*e.g.* an accumulator), the P.D. can be considered equal to the E.M.F. (except on excessive short-circuit).

357. Universal Battery.

When several circuits are joined in parallel across the terminals of a battery of *very low internal resistance*, any of the circuits may be opened or closed without causing appreciable change in the currents in the remaining circuits.

Let a battery of accumulators be joined to several circuits of equal resistance r, as shown in fig. 225. If one key only (A) is down, a current V/r flows through A. If all the keys are down, a current V_1/r flows through *each* circuit, V_1 being the new potential-difference at the battery poles. But owing to the low resistance of the cells there is practically *no drop of potential in the battery* in either case. Therefore V_1 and V are both equal to the full electromotive force of the battery, and each circuit gets as much current when all the other circuits are working as it does when it is working alone. The current through the battery is proportional to the number of circuits closed. A battery used in this way to supply several circuits is said to be connected on the "universal" system.

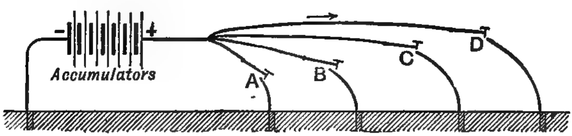


Fig. 225.—Universal Battery System

In the figure the circuits are shown completed through the earth—an arrangement adopted in telegraphy. If the number of circuits is too great, the external resistance may become so low that there is an appreciable drop of potential at the poles of the battery, and consequently in the current received by each circuit.

EXAMPLE.—A battery of 20 cells, each internal resistance ·01 ohm, is connected on universal principle. How many circuits each of 100 ohms resistance may be joined up so that the current received by each circuit may not vary more than 1 per cent?

Let E = E.M.F. of the battery.

$$\text{Resistance of battery} = 20 \times {\cdot}01 = {\cdot}2\ \omega.$$

When one circuit is in use, external resistance = 100.

$$V = \frac{100}{100{\cdot}2} \text{ of E, by equation (2).}$$

When n circuits are in use, external resistance $= \frac{100}{n}$. Hence—

$$V_1 = \frac{100/n}{100/n + {\cdot}2} = \frac{100}{100 + {\cdot}2 \times n}.$$

The current in each circuit depends on the potential-difference which exists for the time being at the terminals of the battery. Hence V_1 must not be less than 99/100ths of V.

Taking the limit allowed—

$$\frac{V_1}{V} = \frac{99}{100}.$$

Thus $$\frac{100{\cdot}2}{100 + {\cdot}2n} = \frac{99}{100}.$$

$$\therefore n = 6{\cdot}06.$$

Thus the greatest number of circuits allowable = 6.

Similar principles apply to the connection of lamps in electric lighting. The mains are of low resistance, and may be considered merely as extensions of the generator poles. The lamps are of high resistance, and are joined in parallel across the mains.

358. Measurement of Battery Resistance.

Equation (2) can be applied in the determination of the resistance of a battery or cell.

Join the cell to an electrometer, and read the potential-difference in terms of the electrometer deflection. The cell is here on *open* circuit. Let deflection = θ_1.

Next, join the cell through a resistance box so that a *closed* circuit is formed, but at the same time leave the poles connected to the electrometer. Diminish the resistance until the electrometer deflection is considerably less than its previous value (say about one-half). Let new deflection = θ_2.

Then $$\frac{\theta_2}{\theta_1} = \frac{V}{E};$$

But by (2) $$\frac{V}{E} = \frac{r}{r+b}; \quad \therefore \frac{\theta_2}{\theta_1} = \frac{r}{r+b}.$$

Thus $$\frac{\theta_1 - \theta_2}{\theta_2} = \frac{b}{r},$$

and $$b = \frac{\theta_1 - \theta_2}{\theta_2} \times r.$$

Since many cells polarize, it is necessary to repeat the first observation immediately after the second, and in the calculation take the value of θ_1 so found.

In place of the electrometer, any instrument for measuring potential-difference may be substituted, *e.g.* a potentiometer, a condenser and ballistic galvanometer, or if the cell resistance is not great, a high-resistance galvanometer or voltmeter.

359. General Relation of P.D. to E.M.F.

As already mentioned, Equation (2) is only applicable when the generator considered is the only one in the circuit. In other cases the P.D. may be found by considering the effects of E.M.F. and current separately.

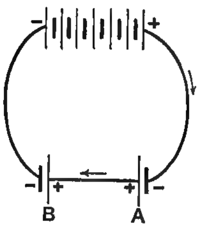


Fig. 226

Suppose we have a battery of six cells joined in circuit with two others, as shown in fig. 226. The cell A assists the current, and B opposes it. (Since B now exerts a "back" E.M.F., the ordinary action of the cell is reversed. Thus if B is a Daniell cell, copper is *dissolved* from the positive pole and zinc is *deposited* on the zinc plate.)

The P.D. at the terminals of A or B may be resolved into two parts—

(1) A rise of potential from the negative to the positive pole of the cell, due to the E.M.F.;

(2) A fall of potential through the cell, due to the current.[1]

The first component = E. The second is the product of the current strength and resistance of the cell, and takes place in the direction of the current.

Thus in the cell A, where the E.M.F. is forward, the *rise* of potential in the direction of the current is given by—

$$V = E - Cb \qquad (3)$$

[1] We usually say that the P.D. produces the current. But since the P.D. in a simple conductor ceases at the same time as the current, it is sometimes convenient to consider that the P.D. is *due to* the current.

In the cell B, where the E.M.F. is backward, the *fall* of potential in the direction of the current is given by—

$$V = E + Cb \text{.........................} (4)$$

Each equation gives the excess of potential at the positive pole over that at the negative pole. If in the former equation V should work out as a negative quantity, the potential of the positive pole is lower than that of the negative pole.

EXAMPLES.—1. A cell of E.M.F. 2 volts and resistance 3 ω is arranged in opposition to a battery of five similar cells. Find the P.D. at the terminals of the former cell.

$$\text{Current} = \frac{\text{total E.M.F.}}{\text{total resistance}} = \frac{10-2}{18} = \frac{4}{9} \text{ amp.}$$

Fall from positive pole to negative due to the current $= Cb = 4/9 \times 3 = 1\frac{1}{3}$ volt.

Fall from positive pole to negative due to E.M.F. = 2 volts.

Therefore the total potential-difference $= 2 + 1\frac{1}{3} = 3\frac{1}{3}$, the positive pole being at a higher potential than the negative.

2. A strong battery sends a current of 5 amperes through a cell of resistance 3 ohms and E.M.F. 1·5, the direction of the current through the cell being from the negative to the positive pole. Find the P.D.

Drop from negative to positive pole *due to the current* $= 5 \times 3 = 15$ volts.

Rise from negative to positive pole due to the E.M.F. = 1·5 volt.

Therefore the total fall of potential $= 15 - 1{\cdot}5 = 13{\cdot}5$ volts.

The positive pole of the cell is at a potential 13·5 volts *lower* than the negative pole.

360. Back E.M.F. in Voltameters and Motors.

The polarization in an electrolytic cell always produces a back E.M.F., and the P.D. is therefore calculated from (4) above.

Similarly, the motion of the armature of a motor across the field lines generates an electromotive force opposed to the current. The magnitude of the counter electromotive force in a motor depends on the rate at which the armature windings cut the field lines. This variation of back E.M.F. with speed may easily be illustrated experimentally.

Join a small motor to a source of constant potential (accumulator or lighting mains according to the voltage for which the motor is wound). Join a glow lamp in the armature circuit. Allow the motor to run light, and notice that the lamp glows very dimly. Now prevent the armature from rotating, and notice that the lamp at once glows brightly. In the former case the rapid motion of the armature generates a back E.M.F. e, and since V is constant, the current is weak, for if r is the armature resistance,

$$V = Cr + e.$$

But when the rotation is prevented $e = 0$, and the current is therefore stronger.

361. Distributed E.M.F.

Where the E.M.F. is concentrated in a film or layer, of molecular dimensions and of no appreciable resistance, as in a voltaic cell, the change of potential is *sudden* and always equal to the E.M.F. whether a current is flowing or not. A similar discontinuity occurs at the junctions in a thermo circuit (Peltier effect). But when the E.M.F. is *distributed* as in a dynamo armature or other source of induced E.M.F. there are no sudden changes of potential.

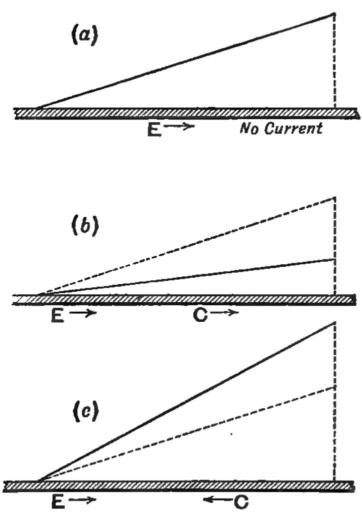


Fig. 227.—Distributed Electromotive Force

On open circuit the potential rises gradually in the direction of the electromotive force, every element of the wire adding its share to the total.

On closed circuit the drop of potential due to the current

in the wire must be compounded with that due to the electromotive force. The potential curve rises—

(1) less steeply if the E.M.F. acts in a "forward" direction;
(2) more steeply if the E.M.F. acts in a "backward" direction.

In fig. 227 (*a*) (*b*) (*c*) the rise of potential due to E.M.F. is shown by a dotted line, and the actual rise of potential by a continuous line, the space between the two representing the effect of the current.

These diagrams would apply to the wire PQ in the experiment shown in fig. 216—(*a*) if the circuit is broken at some point; (*b*) when the circuit is closed, as shown in fig. 216; (*c*) to the case where a current is sent from P to Q by a battery placed, say, between A and C.

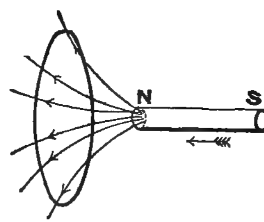


Fig. 228.—Symmetrical Distribution

If the E.M.F. is uniformly distributed along a uniform wire, and the current is such that the drop due to it is equal to the rise due to E.M.F., there is no actual P.D., and we have the unusual case of *a current flowing along a conductor which is everywhere at the same potential.*

An example occurs when a cylindrical magnet with a symmetrical field is moved towards a coaxial circle of wire (fig. 228). At any instant the E.M.F. in every centimetre of the wire is the same, and this E.M.F. alone urges the current. The energy absorbed from the field is converted directly into Joule heat at the points of absorption.

Another example is possible in connection with the Thomson effect. For a certain temperature-gradient, strength, and direction of current, the E.M.F. generated in each portion of the wire may be equal to the drop due to the current, and the whole wire remains at one potential. In this case there is an

electric field, but the lines of force end on the wire at right angles.

In dealing with the quantitative laws of distributed electromotive force it is frequently simpler to express the results in terms of *electromotive intensity, i.e.* the electromotive force generated per *unit length* of the conductor.

362. Conducting Net-works—Kirchhoff's Laws.

Cases sometimes arise where the wires, etc., are joined to form a net-work of conductors. We may require the total effective resistance of the net or the current through some branch. (The Wheatstone bridge is the most frequent example.) The solution of these problems is effected by the following laws due to Kirchhoff:—

(1) **The algebraic sum of the currents flowing towards any junction is zero.**

(2) **The algebraic sum of the E.M.F.'s taken round the complete boundary of a mesh in one direction is equal to the algebraic sum of the decrements of potential due to the currents in the boundary.**

The first law is simply a mathematical way of stating that the quantity of electricity reaching any junction per second is equal to the quantity which leaves it per second.

The second law is a consequence of the fact that the potential can only have one value at each point, so that in going round a mesh the total rise of potential due to E.M.F. is equal to the total fall produced by the currents.

The laws may be stated algebraically thus—

$$c_1 + c_2 + c_3 + \dots + c_m = 0.$$
$$(C_1 r_1 + C_2 r_2 + C_3 r_3 + \dots + C_n r_n) = (E_1 + E_2 + \dots + E_n).$$

The application of these equations to any but the simplest examples gives rise to simultaneous equations which are tedious or difficult to solve. The following device, due to Maxwell, simplifies the calculation considerably.

363. Maxwell's Rule.

"Let the boundary of each mesh be traversed by an imaginary current which flows in *one* direction round the whole boundary. Apply Kirchhoff's second law to these imaginary currents and find their algebraical values. Then the actual current in any wire is the sum of the imaginary currents."

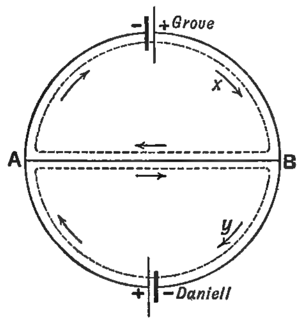


Fig. 229.—Maxwell's Rule

EXAMPLE 1.—A Grove cell, E.M.F. 2 and resistance 3, and a Daniell cell, E.M.F. 1, resistance 5, are joined to a resistance A B of 10 ohms as shown in fig. 229. Find the currents in the conductors.

Suppose that a current x amperes flows in the upper mesh, and a current y amperes flows in the lower mesh, both being clockwise.

Then the current from A to B $= y - x$, or from B to A $= x - y$.

For the upper mesh, *taken clockwise*, we have—

$$\text{Rise due to E.M.F.} = 2 \text{ volts.}$$
$$\text{Fall due to current} = (3 \times x) + (x - y)10.$$

$\therefore$ by Kirchhoff's second law—

$$\begin{aligned} 2 &= 3x + 10x - 10y \\ &= 13x - 10y \ldots\ldots\ldots\ldots\text{(i)} \end{aligned}$$

For the lower mesh, *taken clockwise*, we have—

$$\begin{aligned} \text{Rise due to E.M.F.} &= 1 \text{ volt.} \\ \text{Fall due to current} &= (y - x)10 + 5y. \\ \therefore 1 &= 10y - 10x + 5y \\ &= -10x + 15y \ldots\ldots\ldots\text{(ii)} \end{aligned}$$

Solving the simultaneous Equations (i) and (ii), we have—

$$x = \frac{8}{19}; \quad y = \frac{33}{95}.$$

Since these are positive, the assumed directions of x and y are the actual directions of current through the cells.

The actual current in AB

$$= y - x = \frac{33}{95} - \frac{8}{19} = -\frac{7}{95}.$$

Since this is negative, the actual current is from B to A and $= 7/95$ ampere.

The P.D. at the points AB is $10 \times \frac{7}{95} = \frac{14}{19}$ volt.

EXAMPLE 2.—The sides AB, BC, CD, DA of a Wheatstone parallelogram have resistances 2, 3, 4, 6 respectively. The bridge connecting B, D has resistance 10 ohms. What is the joint resistance between A and C?

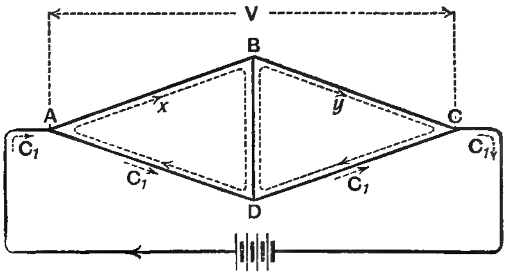


Fig. 230.—Wheatstone Net

Let V be the P.D. of the points A and C (fig. 230), and let C be the current entering at A and leaving at C. Let a clockwise current x pass round ABD, and a similar current y round BCD. Then resistance required $=$ V/C.

Since there are no E.M.F.'s in either mesh, we have—

$$\text{V} = x \times 2 + y \times 3 \text{ (path ABC)} \ldots\ldots\ldots\text{(i)}$$
$$\text{V} = (\text{C} - x)6 + (\text{C} - y)4 \text{ (path ADC)} \ldots\ldots\ldots\text{(ii)}$$
$$\text{V} = 2x + 10(x - y) + (\text{C} - y)4 \text{ (path ABDC)} \ldots\ldots\ldots\text{(iii)}$$

We have to eliminate x and y between these equations and obtain the ratio V/C.

Solving for y from these equations, we have—

$$-110y = 15\text{V} - 120\text{C} \text{ from (ii) and (iii)}$$
$$110y = 88\text{V} - 220\text{C} \text{ from (i) and (ii).}$$
$$\therefore 103\text{V} = 340\text{C}$$
$$\frac{\text{V}}{\text{C}} = \frac{340}{103}$$
$$\text{R} = 3{\cdot}3 \text{ ohms.}$$

Example.

The electrodes of a quadrant electrometer are joined to the terminals of a battery of five cells in series. In what ratio will the deflection of the needle be altered if the electrodes are joined to the terminals of a battery of three cells in series similarly arranged, the cells being all alike and the connecting wires thick. (1899.)

Let e be the E.M.F. and b the resistance of each cell.

(*a*) In the first case there is no current, and the P.D. = the E.M.F.

$$\therefore \text{ Electrometer indicates a P.D. } = 5e.$$

(*b*) In this case a circuit is completed and a current flows. The three cells are arranged in opposition to the other five, if like poles of the two batteries are joined to the same quadrant. We must therefore determine the P.D. at the terminals of the battery of five cells, or that of three cells. Taking the latter, we have—

$$\begin{aligned} \text{Drop from pos. to neg. pole due to current} &= C \times 3b. \\ \text{Drop due to E.M.F.} &= 3e. \\ \text{Total drop} &= 3e + 3Cb. \\ \text{Now} \qquad C = \frac{5e - 3e}{8b} &= \frac{1}{4}\,\frac{e}{b}. \\ \therefore \text{ Potential-difference} &= 3e + \frac{3e}{4b}\,.\,b. \\ &= 3\tfrac{3}{4}e. \end{aligned}$$

Comparing this with the previous case, we have the ratio required

$$= 5e : 3\tfrac{3}{4}e,$$
$$\text{or} \qquad 4 : 3.$$

Similarly we may show that if the cells are in conjunction the second P.D. is zero.

QUESTIONS

1. The terminals of a battery, of E.M.F. 4 volts and resistance 3 ohms, are connected by a wire of resistance 9 ohms. By how much is their difference of potential altered thereby? (1900.)

2. What are the conditions that a current may flow through a cell, (*a*) with the poles at the same potential, (*b*) with the positive pole at a lower potential than the negative pole? Give the potential diagrams for the two cases, supposing the E.M.F. to be located entirely in the zinc-acid surface.

3. A motor takes a current of 15 amperes when the P.D. at its terminals is 110 volts. If the internal resistance is 1·4 ohm, what is the reverse electromotive force developed?

4. In a Wheatstone net ABCD, the resistances of the wires AB, BC, CD, DA are 2, 5, 3, and 4 ohms respectively. BD contains a battery of 2 ohms resistance and 2 volts E.M.F. Find the current through the galvanometer if the latter has a resistance of 10 ohms and is joined across AC.

5. Four points, A, B, C, D, are connected together as follows:— A to B, B to C, C to D, D to A, each by a wire of 1 ohm resistance; A to C, B to D, each by a cell of 1 volt E.M.F. and 2 ohms resistance. Determine the current flowing through each of the cells. (1900.)

6. Explain how it is possible for a current to flow in a wire which is everywhere at the same potential.

CHAPTER XXVIII

ENERGY—POWER—EFFICIENCY

364. Forms of Energy.

When a body possesses the power of doing work it is said to possess energy, and the various conditions in which a body possesses this power are termed forms of energy. With many of these the student is already familiar. Thus a flying bullet, the steam pent up in a boiler, a raised weight, a clock-spring when wound up all possess energy.

It is convenient to divide the forms of energy into two classes—

> **(1) Kinetic energy — due to inertia and the actual motion of a body or its parts.**
>
> **(2) Potential energy — due to forces exerted between different bodies, or different parts of the same body.**

A flying bullet or a rotating wheel possesses kinetic energy. If m is the mass of the bullet, and v its velocity, the energy

can be shown $= mv^2/2$. In the case of a rotating body the energy is $= K\omega^2/2$, where K is the moment of inertia and ω the angular velocity.

A wound-up spring possesses potential energy owing to the elastic forces which the different parts of the spring exert on one another. When the relative position of these parts alters, the forces do work.

In these examples the existence of the motion or force to which the energy is due is obvious. But there are other examples where the energy can only be classed as kinetic or potential when some theory of the cause of the energy has been adopted.

Heat.

The phenomena of heat are now universally regarded as due to a rapid motion of the molecules of a body, the velocity increasing with rise of temperature. Hence, since the amount of work capable of being derived from a hot body depends on the quantity of heat, we regard heat as *molecular kinetic energy*.

Chemical Affinity.

During chemical reactions heat may be evolved or absorbed. The reactions are attributed to forces between the atoms of reacting substances. Hence the substances possess, before the reaction, mutual *potential energy* of an atomic kind. If the potential energy of the products of the change is less than the mutual potential energy of the factors, the difference appears as heat evolved.

Electric Field.

The energy of an electric field is usually regarded as *potential energy* owing to the analogy between electric induction and elastic displacement. The potential energy per unit volume of the field is $F^2K/8\pi$.

Magnetic Field.

The energy of the field of a magnet or current is regarded as due to some kind of motion in the ether, and is therefore termed *kinetic*. (But in speaking of the energy due to two magnets, two currents, or a magnet and a current, as due to the mutual forces in

play between them, the term mutual *potential energy* is frequently employed.) The energy per unit volume of the magnetic field is $H^2\mu/8\pi$.

It is possible that a complete knowledge of the mechanical structure of the ether might lead us to classify all energy as kinetic.

Most physical processes involve a transformation of energy from one form into another. The transformation is subject to the two following laws, which may be taken as the main axioms of physical science:—

(1) **The Law of Conservation of Energy.**—This states that "when one form of energy is converted into another, the quantity which disappears in one form is exactly equal to the quantity which reappears in the other forms".

This is equivalent to saying that "energy is never annihilated or created".

(2) **The Law of Degradation of Energy.**—It is not equally easy to derive mechanical force from the different forms of energy. In other words the *availability* of the energy for doing work varies. In the case of a body in rapid motion, *e.g.* a fly-wheel, the energy is highly available. But the same quantity of energy existing in the form of heat in a body at low temperature would be far less available.

The law of degradation of energy states that "when any transformation of energy takes place some of the energy becomes less available, or degraded".

Friction is the most common cause of degradation of energy. Other causes are conduction and radiation of heat.

365. Transformation of Energy in a Circuit.

Consider a simple circuit consisting of a wire connecting the poles of a voltaic cell. We know that heat is produced in the wire at the rate C^2R/J calories per second. The energy necessary for this is derived from the surrounding electromagnetic field. But since the field remains of constant strength, it must be replenished at the same rate. Hence energy flows into the magnetic field from the cell at the rate C^2R per second.

The source of this is the chemical potential energy of the materials in the cell.

The energy derived from the source may be expressed in terms of the E.M.F. By Ohm's Law—

$$E = CR, \text{ or } EC = C^2R.$$

$$\left.\begin{array}{l}\text{Therefore energy transformed}\\ \quad\text{into the electromagnetic}\\ \quad\text{form per second}\end{array}\right\} = \text{E.M.F.} \times \text{current.}^1$$

In the ordinary chemical experiment of dissolving zinc in sulphuric acid the chemical energy is transformed *directly* into heat. In a simple voltaic circuit it is transformed into heat, but *indirectly*, since, through the agency of the magnetic field, the energy is transferred to the wires before it is converted into heat.

In the case of a dynamo the energy transformed into the electromagnetic form by the E.M.F. generated is derived from the mechanical energy of the moving parts.

If the circuit contains a motor, or polarized voltameter, part of the energy absorbed from the field is converted into kinetic energy or chemical potential energy instead of into Joule heat. In these cases there is a "back" E.M.F., the value of which is given by the equation—

$$\left.\begin{array}{l}\text{Energy transformed } \textit{from} \text{ the}\\ \quad\text{electromagnetic form per}\\ \quad\text{second by a reverse electro-}\\ \quad\text{motive force}\end{array}\right\} = \text{Back E.M.F.} \times \text{current.}$$

366. **Work Done by Electric Forces.**

We have seen that when the current flows from a point A in a circuit to a point B, the electric forces do an amount of work per second = VC. If there is a source of back E.M.F. between A and B, the value of V is given by Equation (4), Art. 359—

$$V = Cr + E'.$$

$$\text{Hence} \quad VC = C^2r + E'C.$$

[1] In some modes of treating the subject this is taken as a definition of E.M.F., transformation by reversible processes being considered.

Here the work done by the electric force is expended in two ways—

(1) in heating the wires or other conductors (Joule heat);
(2) in doing work against a reverse electromotive force.

When we are dealing with steady currents in one direction only (*i.e.* continuous currents), the chief sources of back electromotive force met with are: (*a*) motors; (*b*) polarized voltameters. The energy expended against the back E.M.F. of a voltameter is stored chiefly in the form of chemical potential energy. Some energy may, however, be required to do the mechanical work of separating the products of decomposition from the electrodes, and the back E.M.F. due to this is irreversible.

EXAMPLES.—1. The terminals of a motor are joined to a 110-volt constant-pressure main. If the resistance of the motor (series) is 1 ohm, and the current taken 5 amperes, find the back E.M.F. and disposal of energy.

$$\text{Here}\quad V = Cr + E'.$$
$$110 = 5 \times 1 + E'.$$
$$E' = 105 \text{ volts.}$$

The disposal of energy is shown by—

$$VC = C^2r + E'C.$$
or
$$110 \times 5 = 25 \times 1 + 105 \times 5.$$

Hence, energy supplied per second = 550 joules.
Energy converted into heat per second = 25 joules.
Energy converted into mech. work per second = 525 joules.

2. A current of 3 amperes does work in an electrolytic cell at the rate of 25 joules per second. If the polarization E.M.F. is 2 volts, find the resistance of the voltameter.

$$VC = C^2r + E'C.$$
$$25 = 9 \times r + 2 \times 3.$$
$$\therefore r = 2\tfrac{1}{9} \text{ ohms.}$$

In the most general case, where there are sources of forward E.M.F. as well as sources of back E.M.F., we have, if E =

direct, and E′ = back E.M.F., the *fall* of potential in the direction of the current given by—

$$V = Cr + E' - E.$$

Therefore $$VC + EC = C^2r + E'C.$$

The right-hand side gives the energy withdrawn from the field by the part of the circuit considered, and the left-hand side shows that the work done by the electric forces is helped by the supply of energy sent into the field by the direct E.M.F.

367. Power (or Activity).

It will be noticed that in all the above cases we deal with the energy transformed, or the work done, *per second*. In most calculations relating to transformation of energy in a circuit it is convenient to deal with the *rate* of doing work rather than with the total work done, and the term "power" or "activity" must be frequently used.

Power (or activity) is the rate of doing work.

The C.G.S. unit of activity is 1 erg per second. **The practical unit** adopted in electricity is a rate of 1 joule per second, and is termed a **watt.**

Thus, if W joules of work are done in *t* seconds, the activity A in watts is—

$$A = \frac{W}{t}.$$

Thus $$\text{watts} = \frac{\text{joules}}{\text{seconds}},$$

or, $$\text{joules} = \text{watts} \times \text{seconds}.$$

An activity of 1000 watts is termed a *kilowatt.*

In engineering the unit of power or activity is that of a "theoretical" horse, and is defined as a rate of 33,000 ft.-lb. per minute.

To express a horse-power in watts we have 1 foot = 30·48 cm.; 1 lb. = 453·6 grams; and 1 gram = 981 dynes.

$$\begin{aligned} \therefore\ 1 \text{ lb. wt.} &= 453{\cdot}6 \times 981 = 444981 \text{ dynes.} \\ 1 \text{ ft.-lb.} &= 444981 \times 30{\cdot}48 = 13{,}560{,}000 \text{ ergs.} \\ 1 \text{ H.P.} &= 33000 \text{ ft.-lb. per minute.} \\ &= 550 \text{ ft.-lb. per second.} \\ &= 550 \times 13{\cdot}56 \times 10^6 \text{ ergs per second.} \\ &= 746 \text{ joules per second.} \end{aligned}$$

$$\text{Thus}\quad 1 \text{ horse-power} = 746 \text{ watts}$$
$$= \tfrac{3}{4} \text{ kilowatt nearly.}$$

Activity in Electric Circuits.—The work done per second by electric forces maintaining a current C is given by—

$$\frac{W}{t} = V \times C \quad \ldots\ldots\ldots\ldots\ldots\ldots (\text{Art. } 289)$$

Denoting the activity by A, we may write—

$$A = V \times C,$$

or in practical units—

$$\text{Watts} = \text{volts} \times \text{amperes.}$$

The distinction between this equation, which gives *rate* of doing work, and that for *total* work done, namely—

$$\text{Joules} = \text{volts} \times \text{coulombs,}$$

must be carefully observed.

368. Wattmeters.

A *wattmeter* is an instrument designed to indicate directly the activity of any part of a circuit. It may be described as an ammeter and voltmeter combined, since its reading at any instant indicates the *product* of the current existing at that instant into the potential-difference. In fact, if the current is steady an ammeter and voltmeter may be used in place of a wattmeter, but it is convenient to have a single instrument. Most wattmeters depend on electrodynamic force, and the general principle of their action will be understood from

the following description of two forms suitable for laboratory use.

Siemens's Electrodynamometer.

This instrument, shown diagrammatically in fig. 231, consists of the following parts:—

(1) A fixed coil of stout wire brought to two terminals on the base.

(2) A suspended coil of a large number of turns of fine wire brought to separate terminals on the base.

(3) A fine spring (helical) attached to the suspended coil and to a torsion head.

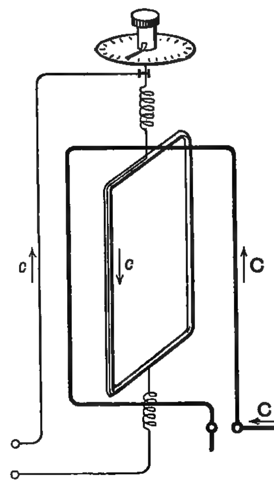


Fig. 231.—Principle of the Wattmeter

The spring serves the double purpose of supplying the controlling torque and carrying the current from one terminal to the upper end of the suspended coil. A light flexible wire conveys the current from the lower end of the suspended coil to the other terminal.

The suspended coil rests normally with its plane at right angles to that of the fixed coil, this position being indicated by a pointer attached to the suspended coil. There are also two stops which limit the play of the pointer and the amount of deflection.

Let us suppose that it is required to determine the power supplied to a glow lamp. We join the stout coil *in series* with the lamp, and the terminals of the fine wire coil to the lamp terminals A, B, through an additional high resistance. The fine wire coils are therefore joined as a *shunt* across the lamp,

on the voltmeter principle (Art. 306). Then the P.D. of A, B causes a current c to flow through the fine wire coil, and

$$c \propto \text{V}.$$

Now the current c is so minute that the main current, C, which flows through the thick wire coils, is equal to that in the lamp. But the power A expended in the lamp = CV.

Therefore— $$\text{A} \propto \text{C}c,$$

i.e. the product of the currents in the two coils is proportional to the activity of the portion of the circuit joined up. But the electrodynamic force between the two coils will tend to deflect them so that their planes coincide, with a torque proportional to Cc (Art. 287). This torque may be counteracted by a rotation of the torsion head, so that the opposing twist in the spring maintains the suspended coil in its normal position. If θ is the angular rotation of the torsion head necessary, and G the torque,

$$\theta \propto \text{G}.$$

But G is proportional to Cc, which in turn is proportional to A.

Therefore— $$\theta \propto \text{A}.$$

Thus the number of watts is directly proportional to the rotation of the torsion head necessary to restore the suspended coil to its initial position.

The thick wire or current coils (*i.e.* the "ammeter" portion of the instrument) must be joined in series with the part of the circuit to be tested, so that they carry the whole current.

The fine wire coils must be connected as a **shunt** across the part of the circuit under test. Since this shunt must have a high resistance (see Voltmeter Principle, Art. 306), it is usually necessary to include a separate high resistance in series with the fine wire coils.

Kelvin Watt Balance.

The current balance may be arranged as a wattmeter. In the composite balance already described the fixed coils formed

of thick wire rope are joined in series with the main circuit. The terminals of the movable coils are joined through a separate high resistance, as a shunt across the part of the circuit to be tested. The current in the movable coils is therefore proportional to the P.D. The attractive force between the wire rope and the movable coils is proportional to C*c*, and therefore proportional to CV, or the number of watts. Hence, when the constant of the instrument has been determined, the number of watts may be obtained by reference to the scale of *equal* divisions engraved on the suspended frame.

When the composite balance is used as above, the fixed fine wire coils are cut out of circuit by means of a switch.

This form of wattmeter is suitable for high activities—hekto- or kilowatts.

369. Efficiency.

The law of degradation of energy leads to many results of practical importance. The energy put into a machine is partly dissipated in useless frictional heating of the bearings, etc., and, in the case of electrical machines, in heating the conductors (joule heat) and the iron of electromagnets (hysteresis). Hence the power derived from the machine is always less than the power put into it.

The efficiency of a machine is the ratio of the useful power derived from the machine to the total power supplied to it.

Thus, more briefly expressed—

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}.$$

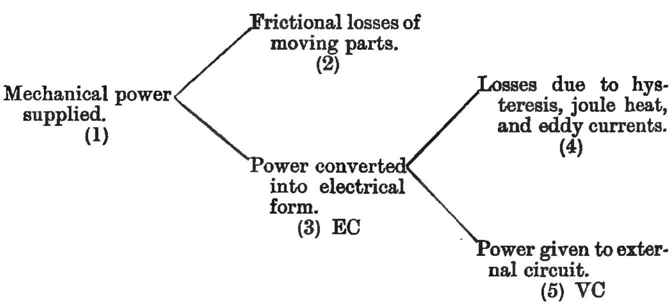
The numerical value of the efficiency depends on the points at which we choose to consider the energy supplied and taken out. Suppose that a dynamo is driven by an engine and used to supply a set of lamps. We might consider the relation of the "light" energy emitted per second to the heat supplied to the boiler per second; or the relation of the energy supplied

by the engine to the dynamo, to the electrical energy given by the latter to the lamps; or again, to bring the points of input and output nearer together, we may take the relation of the electrical energy emitted by the armature to the mechanical energy transformed by it. It is evident, therefore, that in stating efficiency the points of input or output must be stated or implied.

370. Application to Electromagnetic Machines.

The electromagnetic machines which are used for the conversion of energy to or from the electrical form, namely, dynamos and motors, furnish instructive examples of the principles we have enunciated.[1]

Efficiency of a Dynamo.—The energy expended in driving the armature is partly used in producing current, and partly against the friction of bearings, etc. The former part is not entirely given to the external circuit, on account of heating and inductive effects. The main steps in the conversion of energy may be represented diagrammatically thus—



The ratio of (3) to (1) is called the mechanical efficiency since its value depends indirectly on (2).

The ratio of (5) to (3) is called the electrical efficiency since its value depends indirectly on (4).

[1] For the main principles of construction of these machines the student should consult an introductory work on electrical engineering.

The ratio of (5) to (1) is called the total efficiency (sometimes the "commercial efficiency").

Example.—A series dynamo has a total resistance of 1 ohm. The terminal P.D. is 200 volts and current 30 amperes at a certain speed. Find the efficiency under these conditions, if the power supplied to the machine is 8 kilowatts.

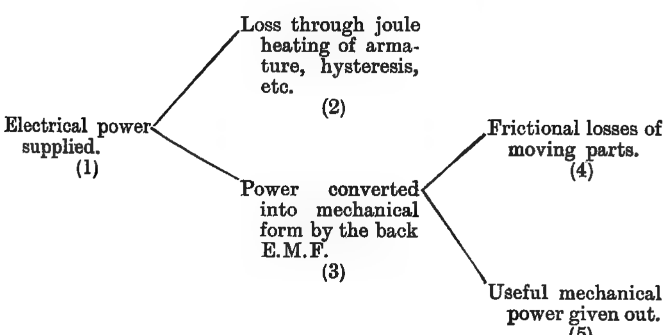
(5) Power given to external circuit $= VC = 200 \times 30 = 6$ kw.
(4) Internal electrical losses, supposing these are entirely due to joule heating of armature (neglecting hysteresis, etc.), $= C^2R = 900 \times 1 = \cdot 9$ kw.
$\therefore$ Electrical power generated $= 6{\cdot}9$ kw.
(1) Power supplied $= 8$ kw.

Thus— Electrical efficiency $= \dfrac{6{\cdot}0}{6{\cdot}9} = 87$ per cent.

Mechanical efficiency $= \dfrac{6{\cdot}9}{8{\cdot}0} = 86$ „

Total efficiency $= \dfrac{6{\cdot}0}{8{\cdot}0} = 75$ „

Efficiency of a Motor.—The action of a motor may be considered as the reverse of that of a dynamo—electrical energy is supplied and mechanical energy given out. The conversion of energy may be represented thus—



The ratio of (3) to (1) is called the electrical efficiency because the losses are electrical.

The ratio of (5) to (3) is called the mechanical efficiency because the losses are mechanical.

The ratio of (5) to (1) is the total or commercial efficiency.

EXAMPLE.—Current is supplied to a series motor at a P.D. of 110 volts. If the resistance of armature and field-magnet coils is ·3 ohm, find the efficiency when the current taken is 10 amperes, assuming all losses due to joule heat.

(1) Power supplied electrically $= CV = 1100$ watts.
(2) Power dissipated $= C^2R = 100 \times \cdot 3 = 30$ watts.
(3) Power converted into mechanical effect $= 1070$ watts.

$$\text{Electrical efficiency} = \frac{1070}{1100} = 97{\cdot}2 \text{ per cent.}$$

371. Efficiency of Secondary Cells.

The efficiency of a cell is measured with reference to *total* energy—not power. During the charging process energy is stored up as chemical potential energy in the deposit of lead peroxide. When the cell is used to produce a current, the energy so stored is reconverted into current energy. The efficiency of the cell is the ratio of the total electrical *energy* so recovered to the electrical energy supplied by the charging current. If, during the charging, a quantity of electricity $= Q$ coulombs is passed with a P.D. $= V$ volts, and during discharge a quantity $= Q'$ is supplied by the cell at P.D. $= V'$,

$$\text{Energy efficiency} = \frac{V'Q'}{VQ}.$$

This is not a constant; it varies with the rate of discharge, which must therefore be mentioned in stating the efficiency. With moderately low rates of discharge the energy efficiency amounts to 60–70 per cent with the present form of cells.

The ratio of the maximum number of ampere-hours which the cell will yield (without injury) to the normal number of ampere-hours required for complete charging is the "current efficiency". (This is more strictly called the quantity efficiency.) For moderately low rates of discharge this may amount to 80–90 per cent.

372. Joulemeters.

A *joulemeter* is an instrument which registers the *total* energy expended in a portion of a circuit during a given period, and therefore differs from a wattmeter, which simply indicates the *rate* of working at each instant.

A common form of joulemeter now in use is the motor type—originally suggested by Profs. Ayrton and Perry. The

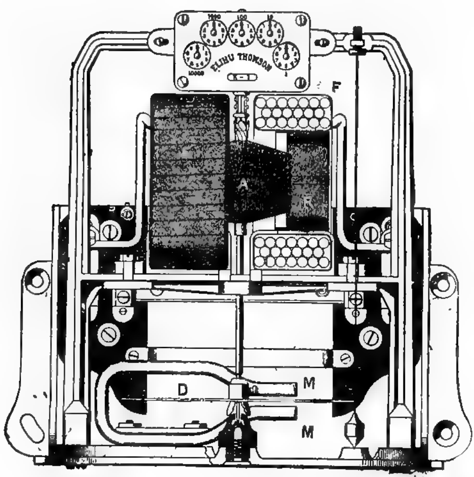


Fig. 232. --Thomson Joulemeter—Part Section

instrument as designed by Prof. Elihu Thomson is shown in fig. 232. The motor is constructed of coils without iron cores. The armature A consists of fine wire coils, and the field coils F are of stout wire. These correspond to the suspended and fixed coils of a Siemens dynamometer. The armature is wound so as to give a uniform torque. The commutator (seen just below the dial in the figure) consists of eight delicate rods, and the "brushes" are two silver strips bearing lightly, and edgewise, on the commutator rods. The spindle is mounted, like the arbor of a watch, on jewelled bearings, thus reducing friction to a minimum. The armature

spindle is geared to the indicator so that the number of revolutions may be (proportionally) indicated. The fixed coils are joined in series in the circuit, and the armature, along with a high resistance, as a shunt across the portion of the circuit under test.

The *total work done* on the circuit is recorded in terms of the *number of revolutions*, the proportionality between these two quantities being obtained as follows:—In the first place, we have

$$\text{number of revolutions} \propto \text{angular velocity} \times \text{time}.$$

Also, $$\text{work done} \propto \text{activity} \times \text{time}.$$

Hence, if we arrange so that velocity of rotation is proportional to activity, number of revolutions will be proportional to work done.

Now the spindle of the armature is fitted with a copper disc D, which rotates between the poles M of a permanent magnet. Eddy currents are thus set up in the disc, and retard the motion with a force proportional to the velocity (Art. 351). When the current is put on, the speed at first increases, but soon acquires a steady value, namely when the retarding force becomes equal to the driving force (or strictly the torque). Hence if f represents the driving force—

$$\text{Speed} \propto f.$$

Now the thick wire and fine wire coils are joined to the circuit under test, as in a wattmeter.

$$\therefore f \propto \text{CV};$$
$$\therefore \text{speed} \propto \text{CV}.$$

Hence number of revolutions varies as work done.

The dial of the instrument is usually graduated in Board of Trade Units of Work (or Energy).

(The Board of Trade unit of work = 3,600,000 joules. It may also be defined as 1000 watt-hours.)

In another type the time factor is introduced by means of a clock. The pendulum of a clock carries the shunt coil at its lower end, which passes just above the other coil at each

swing. The clock gains or loses at a *rate* depending on the force between the coils, and therefore on the activity. The gain or loss in a given period is proportional to the total work done in the circuit under test, and can therefore be used as a measure of this work.

373. Electricity Meters.

If a current is supplied to a circuit at a *constant potential-difference*, the total work done (QV) is proportional to the total quantity of electricity which has passed. Hence an instrument for measuring the quantity of electricity also serves for measuring the quantity of work, and is termed an "electricity meter", a "coulombmeter", or "constant potential joulemeter".

If the current were constant, Q could be determined by multiplying the readings of an ammeter and a chronometer. But since the current varies, an integrating instrument is required.

The main principles applied in the construction of electricity meters are those of (1) the electric motor, (2) the voltameter.

1. *Motor Form.*—This only differs from the joulemeter described above in that a permanent magnet replaces the fixed coils. The speed is then proportional to the current in the armature; and the number of revolutions to the quantity which has passed. The armature (suitably shunted) is therefore placed in series with the circuit.

The dial—if marked in Board of Trade units or other units of *energy*—will only be correct for a circuit of given potential.

2. *Electrolytic Form.*—The weight of a metal deposited from solution by electrolysis is proportional to the quantity of electricity which passes. This fact is utilized in the construction of electrolytic meters. Figs. 233, 234[1] show an interesting modern form—the Wright Electrolytic Meter.

The electrolyte, a solution of mercurous nitrate, is contained in a vertical cylinder with an expansion at the top. (In the figures the liquid is only shaded in the expansion.) The

[1] From *Modern Electric Practice.*

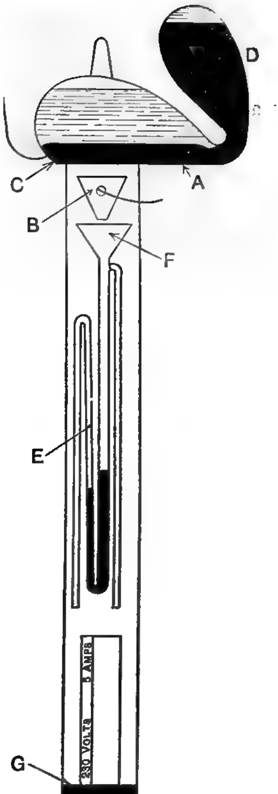


Fig. 233.—Wright Electrolytic Meter—Side view

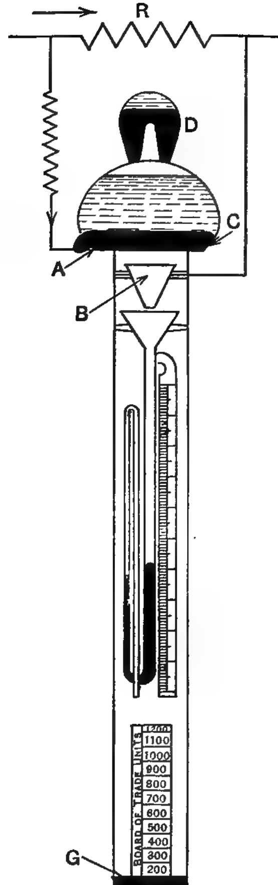


Fig. 234.—Wright Electrolytic Meter—Front view

anode A consists of a *ring*-shaped layer of mercury. The current rises from all parts of this ring and curls over through a central hole (not visible in the figures) to the cathode B, which is of platinum. Mercury is dissolved at the anode and

deposited in the form of drops in the funnel-shaped cathode. These drops run down and fall into a bent tube E placed below B. When the tube is full, the mercury siphons over so that the tube E automatically empties itself into the wider tube G. The latter, besides containing the mercurous nitrate, serves as a measuring vessel. It is graduated in Board of Trade units usually up to 1000. By the use of a shunt R only a fraction of the main current is allowed to traverse the electrolytic cell. The vessel is hermetically sealed, a small space being left at the top for expansion. When the mercury in A has all been transferred by electrolytic action, the instrument may be reset by inverting the whole tube so that the mercury runs back into D.

Example.

An E.M.F. of 3 volts is required to force a current of 1 ampere through a voltameter containing acidulated water. If the work required to separate 1 gram of H is 142,000 watt-seconds, and the e.c. equivalent of H is ·00001035, find the resistance of the voltameter. (1904.)

Work required to liberate ·00001035 gram H

$$= 142000 \times \cdot 00001035 \text{ watt-seconds or joules}$$
$$= 1{\cdot}47 \text{ nearly.}$$

Current required to liberate ·00001035 gm. per second

$$= 1 \text{ ampere.}$$

$$\text{Back E.M.F.} = \frac{\text{energy absorbed per second}}{\text{current}} = \frac{1{\cdot}47}{1}$$
$$= 1{\cdot}47 \text{ volts.}$$

In the case of a back E.M.F. we have—

$$V = E + CR;$$
$$\therefore 3 = 1{\cdot}47 + 1 \times R.$$
$$R = 1{\cdot}53 \text{ ohms.}$$

QUESTIONS

1. A dynamo feeds 1000 sixteen-candle glow-lamps. What current must the dynamo supply if the difference of potential at its terminals is 200 volts and each lamp absorbs 3·6 watts per candle? (1903.)

2. Enumerate the principal sources of waste of power in an electric motor.

Current is supplied to a series motor at 100 volts, the resistance of the circuit being ·5 ohm. Determine the power expended in turning the armature when the current is 10 amperes. Determine also the current when the power thus expended is a maximum. Compare the values of the electric efficiency in the two cases.

(1905.)

3. Show that the electrical efficiency of a motor increases with the speed at which the armature runs; also that for a constant applied potential-difference the mechanical power obtained is a maximum when the efficiency is ·5.

4. A series dynamo has total resistance 1·5 ohms, and maintains 110 volts at its terminals when the current is 9 amperes. Find its electrical efficiency.

Also find its efficiency when it is used as a motor, the speed and current being the same.

CHAPTER XXIX

MAGNETIC POTENTIAL—INDUCTANCE

374. The general forms assumed by the lines of force due to currents in circuits of various shapes have already been studied (Ch. XX), but an expression for the strength of the magnetic field has only been obtained in one case, namely, the field on the axis of a circular coil. We shall now consider a more general method of dealing with the field depending primarily on the idea of potential.

375. **Magnetic Potential.**

Just as in dealing with the problems of electrostatics, we find that calculations are simplified by the use of the quantity termed electric potential, so we find that the introduction of a similar quantity facilitates the treatment of the properties of magnetic fields.

The amount of work done by the magnetic forces when a unit N. pole is transferred from

any point to an infinite distance is termed the magnetic potential at the point.

The potential due to a quantity of magnetism m collected at a point may be calculated on exactly the same lines as given in Arts. 142, 143 for electric potential. In fact, if we substitute m, m', for e, e', the calculation there given holds all through, and we have—

Magnetic potential at distance d from a quantity of magnetism m

$$= \frac{m}{d} \quad \text{.............................(1)}$$

$$= \text{U, say.}$$

The magnetic potential due to a very short magnet, or that in the remote field of an ordinary magnet, may be calculated as follows.

Let NS be the magnet, P the point, comparatively distant. Let θ = angle PSN, l = length of the magnet, and m = strength of poles, SP = d. Then—

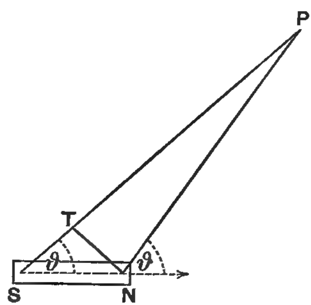


Fig. 235.

$$\text{Potential at P due to N. pole} = \frac{m}{\text{NP}}.$$

$$\text{,,} \quad \text{,,} \quad \text{S.} \quad \text{,,} \quad = \frac{-m}{\text{SP}}.$$

$$\therefore \text{ resultant potential} = m\left(\frac{1}{\text{NP}} - \frac{1}{\text{SP}}\right)$$

$$= m\left(\frac{\text{SP} - \text{NP}}{\text{SP . NP}}\right).$$

Draw NT at right angles to SP. Then taking account of the great distance of P, we have approximately—

$$\text{SP} - \text{NP} = \text{ST} = l \cos\theta.$$

$$\text{SP . NP} = \text{SP (SP} - \text{ST)} \doteq \text{SP}^2 \text{ approximately,}$$

since ST is negligible in comparison with SP.

$$\therefore \ U = m\frac{l \cos \theta}{d^2}$$

$$= \frac{M \cos \theta}{d^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In the limit, when l is indefinitely small in comparison with d, the equations become exact, and this expression gives the magnetic potential due to an indefinitely short magnet.

376. **Work Done in a Closed Path.**

The magnetic potential due to a magnet, *i.e.* due to a distribution of free magnetism, has only one value at a given point. The proof of this is identical with the corresponding proposition in Electricity (see Art. 142). The work done in taking a unit magnetic pole from a point A to a point B in a field due to magnets only is independent of the path chosen between the two points.

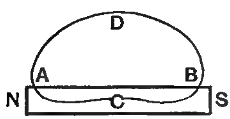


Fig. 236.

It follows from this that the work done against the polar forces of a magnet, in taking unit N. pole round any completely closed path, is zero.

Let ACBD (fig. 236) be any closed curve in the field. Then if W units of work are done *by* the magnetic forces in taking a pole from A to B *via* C, the same amount of work is done if the journey from A to B is traversed *via* D. Hence, if we follow the path ACBDA so that a closed curve is traced out, W units are done *by* the field along ACB, and W units are done *against* the field along BDA. Hence the algebraical sum of the amounts of work done against the field is zero.

If the path lies entirely outside the magnet, the same result is obtained. It must be remembered that lines of force are considered (not lines of induction). The above proposition should be studied in connection with fig. 10.

377. Magnetic Shell.

A *magnetic shell* may be imagined as a thin sheet of steel magnetized in a direction perpendicular to its surface. We say "imagined", because it is not possible to obtain magnetic shells practically, owing to the powerful demagnetizing force within the magnetized substance. The conception of a magnetic shell is, however, of great theoretical importance. It is necessary to mention at the outset that in all theoretical investigations we suppose the thickness of the shell indefinitely small.

Let the surface density of free magnetism be uniform over each face of the shell and $= \sigma$. If a small area a be marked out on the N. face of the shell there will be a corresponding area on the S. face, and these two areas may be regarded as the ends of a short cylindrical magnet having its axis perpendicular to the face of the shell. The poles of this little magnet are each numerically $= a\sigma$, and its moment $= a\sigma t$, where t is the thickness of the shell. Hence, if the area a is unity, the moment is σt.

This product is termed the *strength* of the shell.

> **The strength of a magnetic shell is its magnetic moment per unit area.**

The strength of a shell is denoted by ϕ. Thus—

$$\phi = \sigma \times t \qquad (3)$$

Note.—When we imagine the thickness of the steel to decrease indefinitely, we must imagine the surface density of free magnetism to increase indefinitely, and so that the product σt is constant.

378. Potential due to the Shell.

Let an area a be marked out as above. This is the end of a little magnet which forms part of the shell, and whose moment

$$= \sigma a t$$
$$= a\phi.$$

The axis is normal to the shell surface. Take a point P at a distance, and draw lines from all parts of P to the boundary of the area, so forming a narrow oblique cone. If θ is the inclination of the axis of the cone to the axis of the magnet, and d the distance of P from a,

$$\text{Potential at P due to the element} = \frac{\text{M} \cos \theta}{d^2},$$

$$= \frac{a \cos \theta \, . \, \phi}{d^2},$$

since $\text{M} = a\phi$.

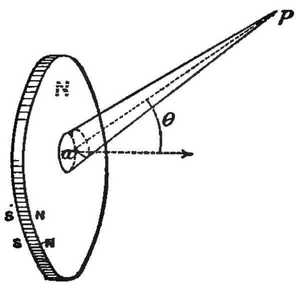


Fig. 237

Now $a \cos \theta$ is the area of the right section of the cone close to the shell, and $a \cos \theta / d^2$ is the solid angle subtended by the area a at P. Hence, calling this solid angle ω_1, we have—

Potential at P due to elemental magnet $= \omega_1 \times \phi$.

In a similar manner, if we mark off areas a_1, a_2, etc., the potentials due to these are $\phi\omega_1$, $\phi\omega_2$, etc., where ω_1, ω_2 are the corresponding solid angles at P. Hence, if the whole surface of the shell is divided up in this way, we have—

$$\begin{aligned} \text{Total potential at P} &= \phi(\omega_1 + \omega_2 + \omega_3 + \ldots\ldots) \\ &= \phi\omega \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4)} \end{aligned}$$

where ω = solid angle subtended at P by the boundary of the shell.

In the above we have not introduced any assumption as to the shape of the shell. Hence—

"The magnetic potential due to a shell depends on the strength of the shell and on the boundary, *but is independent of the shape of the shell surface.*"

When the lines drawn from P within the solid angle ω meet the S. surface of the shell first, ϕ must be reckoned negative.

If we choose two points, one close to the surface on one side and the other close to the surface on the other side, there is a finite difference of potential between these points, although they are indefinitely close together. Thus in fig. 238 the solid angle subtended by the boundary at A, a point in contact with the S. face of the shell, is ω, and the potential at A is $-\phi \times \omega$. The solid angle CBD is also $= \omega$ (approximately, and in the limit absolutely). Hence the solid angle at B *facing the shell* is $(4\pi - \omega)$, and the potential at B is $(+\phi) \times (4\pi - \omega)$.

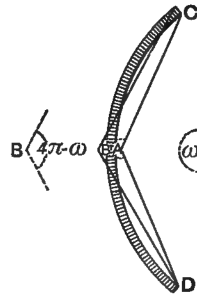


Fig. 238.

Thus the difference of magnetic potentials at A and B

$$\begin{aligned} &= \phi(4\pi - \omega) - (-\phi\omega) \\ &= 4\pi\phi. \end{aligned}$$

"The difference in magnetic potentials at two points separated only by the thickness of the shell is 4π times the strength of the shell."

379. Ampère's Theorem.

Experiment shows that the distribution of magnetic force near a circuit is in perfect agreement with the following theorem:—

The field due to a current is identical in form with the external field of a magnetic shell whose boundary coincides with the circuit.

This is known as *Ampère's Theorem.*

The general method of experiment is to test with a magnetometer the distribution of magnetic force along certain lines for which the force due to the shell can be calculated. The calculated force and the observed force may then be compared. The experimental results show that the actual force is accurately proportional to that obtained by calculation.

The importance of magnetic shells in electrical theory lies in the coincidence of their external fields with current fields, and we may express the strength of a current in terms of that of an equivalent shell. If the field due to a shell is calculated, and if that due to a current with the same boundary is calculated from Ampère's Law (Art. 268), the results become identical if we make—

$$\phi = C.$$

Hence we have the following relation:—

"The electromagnetic measure of a current is equal to the strength of a shell which produces externally the same field as the current when its boundary coincides with the circuit."

Observe very carefully that the field *inside* the material of the shell does not agree with that of the current. The magnetic force inside the shell is a powerful demagnetizing field[1] in the reverse direction to the field just outside the shell. The form of the shell can, however, always be chosen so that the point under consideration is outside. In all cases we must suppose that the medium surrounding the shell is air, whatever the medium surrounding the current may be.

380. Work Done in a Closed Path.

A very important result is obtained from considering the amount of work done in taking a unit pole round a completely closed path in the field of a current.

The path chosen may or may not be *linked* with the circuit.

(1) **The path is not linked with the circuit.**—Imagine an equivalent magnetic shell substituted for the current. Then at *all* points of the path the field of the current is identical with that of the shell, and since the latter is a magnet, by the result of Art. 376 above the total work vanishes.

(2) **The path is linked with the circuit.**—Compare the fields of force due to the current and to the shell respec-

[1] The mathematical student will perceive that in the limit the external field is the finite difference of two infinite quantities. The infinite flux of magnetization breaks up into an infinite demagnetizing field and a finite external field.

tively. These are identical outside the shell, but within the substance of the latter the fields would, if superposed, be in opposite directions. Hence, since the work done against the field of the shell would by the result of Art. 376 vanish for a complete path traversing the substance of the shell, it will not do so for the current's field.

This is also obvious from the fact that the lines of force in the current field are closed curves.

The work done in the closed path may be found by choosing first a path which is *not quite* closed, but ends at points just on each side of the surface of the shell. Since the fields agree at all points of this path, we have by the result of Art. 378—

$$\begin{aligned}\text{Work per unit pole} &= 4\pi\phi \\ &= 4\pi C \quad \ldots\ldots\ldots\ldots\ldots (5)\end{aligned}$$

But the path is all but completely closed, and in the case of the current (since the field is finite at all points) only an infinitesimal amount of work would be done in the completion. Thus we obtain the following important result:—

The work done in taking a unit positive pole round a closed path linked once with the circuit is 4π times the current strength.

It follows from this relation that the magnetic potential at any point in the field of a *current* may have any number of values. If U is one of these values, then the others are given by the expression $U \pm n \,.\, 4\pi C$, where n is an integer depending on the number of times the path chosen is linked with the circuit. (Contrast this with the case of a magnet where the potential is a single-valued function.)

The circuital relation expressed by (5) is believed to be universally true, and we must now regard it as expressing, in the most general form, the definition of current. Thus—

"The current through any area is in electromagnetic measure $1/4\pi$ times the work done in taking a unit N. pole once round the boundary of that area".

This applies to dielectric currents as well as to ordinary conduction currents.

If the work is that done *against* the magnetic field, it is positive when the path is traced out in the opposite direction to the lines of force, and vice versa.

The term **magnetomotive force** is frequently applied to denote the work done in taking a unit pole round a path linked with the circuit.

$$\text{Magnetomotive Force} = 4\pi n\text{C} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

where n is the number of times the path is linked with the circuit. The abbreviation M.M.F. is used for this quantity.

381. Application to Straight Wire and Solenoid.

By making use of facts concerning magnetic fields which may be proved experimentally, and applying the equation for magnetomotive force (6), we may derive expressions for the magnetic force in a simple manner.

1. *Field due to a Current in a Straight Wire.*—Experiment shows that the field in this case consists of circular lines of force concentric with the wire. Let r be the radius of one of these lines, and F the magnetic force at any point of the line. The work done in taking a unit pole once round the circle is

$$\text{F} \times (\text{circumference}),$$

or

$$\text{F} \times 2\pi r.$$

Now this work $= 4\pi\text{C}$.

Hence—

$$\text{F} = \frac{4\pi\text{C}}{2\pi r} = \frac{2\text{C}}{r}.$$

Thus the magnetic force due to an indefinitely long straight wire is inversely proportional to the distance of the point from the wire.

2. *Field due to a Current in a Solenoid.*—Experiment shows that the field outside a long closely-wound solenoid is negligible; also that the field inside is uniform.

Let l be the length of the solenoid, n the number of turns per centimetre, C the current strength, F the field inside.

The work done in taking unit pole round a path which is threaded through the solenoid is practically all done in that part of the field which lies inside the solenoid. Thus—

$$\text{Work done per unit pole} = \mathrm{F} \times l.$$

Since the total number of turns is nl, the path is linked nl times with the circuit, and

$$\text{Work done per unit pole} = nl \times 4\pi\mathrm{C}.$$

Thus—
$$\mathrm{F} = \frac{nl \times 4\pi\mathrm{C}}{l}$$
$$= 4\pi n\mathrm{C}.$$

The field due to a solenoid carrying a given current depends only on the *closeness* of the windings, and is independent of the diameter of the coil.

3. *Field Inside a Conductor.*—The theorem of M.M.F. enables us to calculate the field *inside* a wire or rod of circular cross-section. By symmetry the lines of force are circular. Also the laws of conduction show that the current is uniformly distributed through the whole cross-section, *i.e.* the current density is constant.

Let R be the radius of the circular cross-section, r the radius of a line of force within the wire, F the magnetic force on this line, C the total current. Then work done in taking unit pole round the line of force

$$= \mathrm{F} \times 2\pi r.$$

The current encircled by this line is proportional to the area, and is equal to

$$\frac{\pi r^2}{\pi \mathrm{R}^2} \times \mathrm{C}.$$

Therefore the work done on unit pole

$$= 4\pi \cdot \frac{r^2\mathrm{C}}{\mathrm{R}^2}.$$

$$\therefore \mathrm{F} = \frac{4\pi r^2\mathrm{C}}{2\pi \mathrm{R}^2 r} = \frac{2r\mathrm{C}}{\mathrm{R}^2}.$$

Thus the force inside is directly proportional to r, and vanishes at the centre of the wire.

382. Comparison of Methods of Calculation.

In Chapter XX we have obtained expressions for the force on the axis and at the centre of a circular coil by considering the total force as made up of components due to each *element* of the wire. In the present chapter we have seen that a different method of treatment is possible. Current may be defined as the strength of an equivalent shell, and the force due to a given circuit can be calculated on the basis of this definition. The latter method of treatment is in one respect preferable, since currents are only known to flow in *complete* circuits. Heaviside has shown how, by "rational" current elements each of which resembles a little bar magnet and its field of induction, the former method becomes logically permissible. The formulæ for all the circuits considered may be worked out on either method.

A third method is that based on the equation for magnetomotive force, and is applicable when the force is known to be uniform along certain lines, as in the examples of Art. 381.

The current in the formulæ given is expressed in C.G.S. units. If it is expressed in amperes we must divide by 10. The various formulæ are collected below.

Position of Point.	Current in C.G.S. Units.	Current in Amperes.
Centre of circular coil...	$F = \dfrac{2\pi nC}{r}$	$F = \dfrac{2\pi nC}{10 \times r}$
On axis of circular coil..	$F = \dfrac{2\pi r^2 nC}{(r^2 + d^2)^{\frac{3}{2}}}$	$F = \dfrac{2\pi r^2 nC}{10(r^2 + d^2)^{\frac{3}{2}}}$
At distance r from long straight wire..........	$F = \dfrac{2C}{r}$	$F = \dfrac{2C}{10 \times r}$
Inside a long solenoid...	$F = 4\pi nC$	$F = \dfrac{4\pi nC}{10}$

EXAMPLE.—A solenoid 20 cm. long, wound with 500 turns, carries a current of 3 amperes. Find the field-strength.

$$F = \frac{4\pi n C}{10}.$$

$$n = \frac{500}{20} = 25; \quad C = 3.$$

Hence
$$F = \frac{4 \times 3{\cdot}142 \times 25 \times 3}{10}$$
$$= 94{\cdot}26 \text{ dynes per unit pole.}$$

383. Mechanical Action on a Circuit.

The mechanical force exerted on a circuit placed in a magnetic field may be calculated by the method of magnetic shells. We shall here give one example.

To find the couple exerted on a plane coil placed in a uniform magnetic field:—Let the plane of the coil make an angle θ with the direction of the field. Replace the current by an equivalent plane magnetic shell. The little magnets of which we may suppose the shell composed are set with their axes normal to the plane, and therefore at $(90° - \theta)$ with the field-direction. The couple acting on each component magnet of moment M is—

$$MH \sin (90 - \theta).$$

Hence the couple per unit area of the shell is—

$$\phi \,.\, H \sin (90 - \theta).$$

If a is the area of the circuit, the couple (torque) is given by—

$$G = \phi a H \sin (90 - \theta).$$

But $\phi = C$, where C is in C.G.S. units. Hence—

$$G = C \,.\, a \,.\, H \,.\, \cos \theta.$$

Notice that the torque is proportional to the area of the coil. If there are n turns, we have—

$$G = nCaH \cos \theta.$$

INDUCTANCE

384. *Linkage.*—Every line of magnetic induction due to the current is a closed curve linked one or more times with the circuit. Suppose the field divided into a number of narrow tubes of induction. The following definition may then be applied:—

"The product of the flux of induction in a tube into the number of times the tube is linked with the circuit is the *linkage* of the tube".

The sum of the products so obtained when all the tubes are taken into account is the total linkage (N).

If the total flux through a coil is P unit tubes and the number of turns in the coil n, then, if practically all the tubes are linked with all the turns, the linkage

$$N = P \times n.$$

Inductance.—When the tubes of induction considered are those due to the current only, or to magnetization called up in iron (or other magnetically soft material), their number will depend on the strength of the current. The following definition then applies:—

The inductance of a circuit is the ratio of the total dependent linkage to the current strength.

To distinguish this from mutual inductance (defined later) the term *self-inductance* may be employed. The symbol is usually L. Hence by definition—

$$L = \frac{N}{C}.$$

It must be noticed that this ratio has a definite physical meaning only when N is *dependent* on C. Hence the magnetic flux due to permanent magnets or hysteretic effects is not to be included in N.

Analogy in Electrostatics.—"Inductance" plays the same part in electromagnetics that capacity does in electrostatics. The equations

$$L = \frac{N}{C} \text{ and } S = \frac{Q}{V}$$

are closely analogous. The number of linkages N corresponds to the charge or number of tubes of electric induction, Q. The current strength C corresponds to the potential-difference V: since, except for the merely numerical multiplier $4\pi n$, the current strength is the magnetomotive force, *i.e.* a difference of magnetic potential. Thus in words—

$$\text{Inductance} = \frac{\text{Linkage of magnetic induction}}{\text{Difference of magnetic potential} \div 4\pi n}.$$

$$\text{Capacity} = \frac{\text{Electric induction}}{\text{Difference of electric potential}}.$$

Hence inductance may be regarded as the "capacity" of the circuit for magnetic linkage. A large hoop of wire has greater inductance than a small hoop; a coil of 100 turns a greater inductance than a similar one of 50 turns.

Field free from Iron.—If the field contains no iron the inductance is constant. This may be proved as follows:—

If the shape of the circuit remains constant, the shape of the lines of force is unaltered by any variation in current strength; the field may be strengthened or weakened at any point, but without change of direction. If there is no iron or other magnetic substance, the direction of the induction and intensity of induction are the same as the direction and intensity of the magnetic force.

Now imagine the whole field divided into unit tubes of induction. The density of the flux at any point will be proportional to the magnetic force and therefore to the current. Hence, if we double the current strength, we double the density of the tubes at every point in the field, and therefore double the total number. Each original tube becomes split into two unit tubes. Similarly, if the current is increased x times, the number of tubes is increased x times. Thus—

$$N \propto C.$$

Therefore $$\frac{N}{C} = \text{a constant},$$

or $$L = \text{a constant}.$$

Field containing Iron.—If the field contains a magnetically soft material, the inductance may be calculated if the induction corresponding to the magnetic force produced by the given current is considered. Since the permeability is not constant except for very weak fields, L is here variable, and is a function of C.

385. Inductance of a Solenoid.

Let l be the length of a closely wound long solenoid, and a the sectional area. Let n be the number of turns *per cm.*, C the current, and μ the permeability of the core (supposed to fill the coil space) under the existing conditions.

Magnetic force inside the coil	=	$4\pi n\mathrm{C}$.
Magnetic induction inside the coil	=	$4\pi n\mathrm{C}\mu$.
Total flux of induction	=	$4\pi n\mathrm{C}\mu a$.
Total linkage (number of turns = ln)	=	$4\pi n\mathrm{C}\mu a l n$.

$$\text{Thus} \quad \mathrm{N} = 4\pi n^2 l\mu a\mathrm{C}.$$

$$\therefore \mathrm{L} = \frac{\mathrm{N}}{\mathrm{C}} = 4\pi n^2 l\mu a.$$

If the coil has a circular section, radius r,

$$\mathrm{L} = 4\pi^2 n^2 r^2 l\mu.$$

The same formula applies to a thin anchor ring.

386. Units of Linkage and Inductance.

The C.G.S. unit of linkage is produced when 1 tube is linked once with the circuit.

The practical unit of linkage is virtually 10^8 C.G.S. units, although this has not yet received a special name. In calculations the linkage is usually first expressed in C.G.S. units, and the result then divided by 10^8.

The C.G.S. unit of inductance is 1 C.G.S. unit of linkage per C.G.S. unit of current.

The practical unit of inductance is one practical unit of linkage per ampere, and is termed a **henry.**

$$\text{Thus} \quad \mathrm{L\ (henrys)} = \frac{\mathrm{N\ (prac.\ units)}}{\mathrm{C\ (amperes)}}.$$

The henry is equivalent to 10^9 C.G.S. units of inductance. For—

$$1 \text{ henry} = \frac{1 \text{ prac. unit of linkage}}{1 \text{ ampere}} = \frac{10^8 \text{ C.G.S.}}{(\frac{1}{10}) \text{ C.G.S.}} = 10^9.$$

Thus the inductance of a solenoid is—

$$\frac{4\pi n^2 l \mu a}{10^9} \text{ henrys.}$$

Example.—Find the inductance in henrys of an empty solenoid having 500 turns, length 100 cm., and cross-sectional area of space 4 sq. cm. Also find the linkage if the current is 1·5 ampere.

Magnetic force in the solenoid $H = 4\pi . \left(\frac{500}{100}\right) . C$
$= 20\pi C.$

Since $\mu = 1$, the induction $B = 20\pi C.$

Total flux (B × a), $P = 80\pi C.$

Linkage (P × n), $N = 40{,}000\pi C$
$= \cdot 001257C$ prac. unit.

$\therefore L = \frac{\cdot 001257C}{C}$
$= \cdot 001257$ henry.

Linkage for 1·5 ampere $= \cdot 001257 \times 1\cdot 5$
$= \cdot 001885$ prac. unit,
or 188500 C.G.S. units.

387. Mutual Inductance.

When two circuits are placed in neighbouring positions, many of the lines of induction due to a current in one circuit are linked with the other. It can be shown that if a current C in one circuit produces N linkages with the other, then the same current in the second circuit will produce the same number of linkages with the first. Hence the ratio N/C is called the *mutual* inductance of the two circuits.

"The mutual inductance of two circuits is the ratio of the linkage with one circuit to the current which, flowing in the other circuit, produces that linkage." $M = N/C$.

The units are the same as for self-inductance.

Mutual Inductance of a Primary and Secondary.—Let a secondary coil of s turns be wound over a solenoid which has

n turns *per cm.* Let l be the length of the latter, a its cross-sectional area. Then for a current C in the primary solenoid the flux of induction is—

$$4\pi n C\mu a.$$

Therefore the linkage with the secondary is—

$$4\pi n C\mu a s,$$

and the coefficient of mutual inductance

$$M = 4\pi n s\mu a \text{ C.G.S. units.}$$

388. **Examples.**

Find an expression of the strength of the magnetic field near the middle of a long solenoid. Indicate by a diagram the manner in which the field varies near the ends, and find approximately the value of the field just inside the end. (1903.)

(*a*) See Art. 381.

(*b*) See diagram of lines of force, Art. 267.

(*c*) Take the case of a very long solenoid. The force at its centre is the resultant of the forces due to each half, and these act in the same direction. Hence the force due to each half is—

$$\frac{1}{2} \times \frac{4\pi n C}{10} \text{ (where C is in amperes)} = \frac{2\pi n C}{10}.$$

If now the whole solenoid is divided at the middle and the parts separated, the components just considered will be the magnetic force at the ends of the two new solenoids. Thus—

The force at the end of a long solenoid is one-half of the force at the centre, and equals $\frac{2\pi n C}{10}$.

EXERCISES

1. Prove that the magnetic field near a long straight conductor varies inversely as the distance from the conductor, and describe an experiment to test the truth of your deduction. (1904.)

2. A circular hoop of radius 10 cm. carries a current of 5 amperes and is placed in a magnetic field of ·2 C.G.S. unit. Find an expression for the work done in rotating the hoop through a right angle, about one of its diameters as axis, taking the initial position of the plane of the hoop at right angles to the field.

3. Find an expression for the magnetic force due to a current sheet, at a point just outside the sheet and some distance from its edge.

CHAPTER XXX

TRANSIENT DISCHARGE

389. The discharge of a conductor or Leyden jar is a current which lasts for a very short period, *i.e.* a transient current. Another example of a transient current is the discharge in the secondary circuit of an induction coil when a current is started

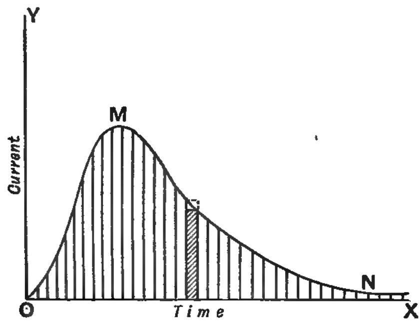


Fig. 239

or stopped in the primary. Fig. 239 shows the rise and fall of current strength during the period of discharge. The current rises rapidly to a maximum, and then falls somewhat less rapidly to zero. The whole time of discharge is usually a very small fraction of a second. The *rate* of rise to the maximum value (OM) depends on the inductance of the circuit.

390. Quantity of Electricity.

The total quantity of electricity discharged round the circuit may be represented graphically. Let the whole area OMN be divided by vertical lines into narrow strips. Each strip is practically a rectangle whose height represents the current existing at the moment and base a short interval during which the current does not appreciably alter. Hence the *area* of the rectangle represents the product (current) × (interval of time); that is, the quantity of electricity discharged during

the interval. The total quantity of electricity discharged is therefore represented by the whole area between the curve OMN and the horizontal axis OX.

The quantity discharged is given by the following equations:—

Condenser Discharge.—

$$Q = SV \qquad (1)$$

If S is in farads, V in volts, Q is in coulombs.

If S is in microfarads, V in volts, Q is in microcoulombs.

Inductive Discharge.—It may be shown that[1]—

$$Q = \frac{N}{R}, \qquad (2)$$

where N is the total *change* of linkage, and R the resistance of the circuit in which the discharge takes place. If N is in practical units of linkage, R in ohms, Q is in coulombs.

If the discharge is due to self-induction—

$$N = LC. \qquad (3)$$

If due to mutual induction—

$$N = MC \qquad (4)$$

391. Discharge in a Split Circuit.

If a part of the discharge path consists of two branches in parallel, one of which contains an electromagnet and the other does not, then the ordinary laws of the division of *current* no longer apply. For, owing to the large inductance of the electromagnet branch, the rise of current is retarded in this branch, although the resistance may be less.

But it may be shown[2] that the *quantities* of electricity discharged in the two branches follow the ordinary law of division of currents. Thus—

"When a transient discharge traverses two portions of a circuit in parallel, the quantity of electricity discharged through each branch is inversely proportional to the resistance of that branch".

[1] See Appendix (p. 588). [2] *Ibid.* (p. 589).

EXAMPLE.—A condenser charged to potential-difference 2 volts discharges through two wires resistances 5000 and 3000 ohms respectively. The quantity sent through the latter wire is ·25 microcoulomb; what is the capacity of the condenser?

Let Q_1, Q_2 be the quantities discharged along the wires respectively.

Then
$$\frac{Q_1}{Q_2} = \frac{r_2}{r_1} = \frac{3000}{5000};$$
$$\frac{Q_1}{\cdot 25} = \frac{3}{5}; \quad Q_1 = \cdot 15.$$

Charge in condenser $= \cdot 25 + \cdot 15 = \cdot 40$ m.c.
Capacity $= Q/V = \cdot 4 \div 2 = \cdot 2$ microfarad.

392. The Ballistic Galvanometer.

The principles involved in the construction of a ballistic galvanometer will be better understood if we first consider

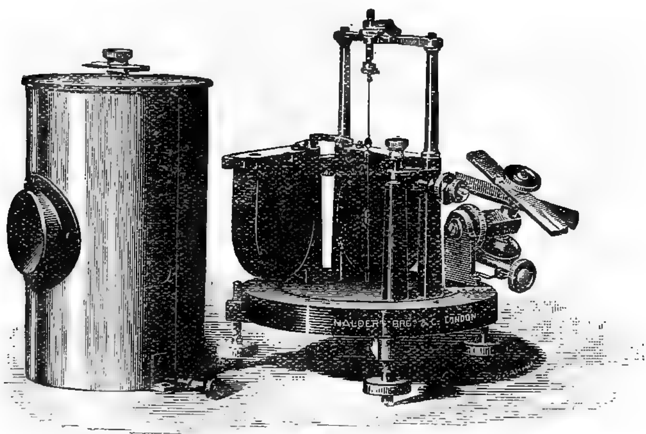


Fig 240.—Ballistic Galvanometer

a mechanical analogy. It we tie a string to the bob of a pendulum, and pull the string in a horizontal direction, the pendulum is deflected from the vertical. So long as the pull is steady the angle of deflection remains constant. But if, instead of exerting a steady pull with a string, we give the

bob a sudden blow with a hammer, the pendulum swings to one side and then oscillates about its mean position. The oscillations continue until the pendulum has been brought to rest by the frictional resistances of the air and supports. Now it is shown in mechanics that the extent of the first swing of the pendulum depends on the *impulse*. During the very short time of impact the force exerted by the hammer rises and falls, somewhat as shown by the curve in fig. 239. The total impulse is represented by the area of the curve when the ordinates represent force and the abscissæ times.

The steady deflection of the pendulum is analogous to the steady deflection of a galvanometer needle produced by a constant current. The deflection produced in the pendulum by the blow of a hammer resembles the deflection produced in a galvanometer needle by a transient discharge. A pendulum arranged to measure impulses is called a *ballistic* pendulum, and a galvanometer specially constructed for the accurate measurement of transient discharges is termed ballistic.

The measurement is made in terms of the angle of **first swing** of the needle. In order that this may indicate the **quantity** of electricity discharged, the following conditions must be observed.

(1) **Frictional resistances must be reduced to a minimum.**

(2) **The needle and its attachments must be so heavy that it does not move away from its normal position during the short period of the discharge.**

(3) **The galvanometer should be sensitive.**

One form of ballistic galvanometer (made by Messrs. Nalder Bros.) is shown in fig. 240. The first condition is satisfied by making all parts of the needle system of circular section and the mirror exceedingly small. The magnets are bell-shaped, the bell being split to form a kind of horse-shoe magnet. There are three of these magnets, two being inside the coil and the third, with polarity reversed, outside. The coil is

wound on ebonite, and no metal parts are allowed near the needle, so avoiding eddy currents and consequent damping. The suspension is a long silk thread.

The second condition is satisfied, owing to the weight of the needle system, which is further increased by a small disc screwed to the lower end of the rod. The time of swing is thus increased, and may be 5 to 10 seconds, according to the field-strength.

The third condition is attained by using a large number of turns of fine wire. The resistance is necessarily high. In a good ballistic galvanometer it is 5000 to 10,000 ohms. The controlling force in the needle instrument just described is provided by two controlling magnets. These are of different strengths for convenience of adjustment. The galvanometer is highly insulated, the base and terminals being mounted on ebonite.

When every precaution has been taken there is always a considerable frictional resistance to the motion of the needle system. The damping may be judged by the rate of decrease in the extent of swing of the needle. The amplitudes decrease in geometric progression, so that if the extent of the second swing is 90 per cent of the first, the extent of the third will be 90 per cent of the second. The theory of damping shows that the observed first swing should be increased in the ratio $1 : (1 + \frac{1}{2}\lambda)$, where λ is the logarithm of the ratio of successive swings—termed the *logarithmic decrement.* Thus, if θ' is the angle of first swing observed, the swing θ, if there had been no damping, would have been $\theta'(1 + \frac{1}{2}\lambda)$.

This expression must be used in place of θ in the equations which follow.

Constants of Ballistic Galvanometer.—It can be shown that the quantity of electricity carried through the galvanometer by a transient discharge is given by—

$$Q = \frac{t}{2\pi} \cdot \frac{H}{z} \cdot 2 \sin \tfrac{1}{2}\theta, \qquad (5)$$

or approximately

$$Q = \frac{t}{2\pi} \cdot \frac{H}{z} \cdot \theta \qquad (6)$$

t is the period of a complete oscillation, θ the angle of first swing, H the strength of the controlling field, and z the coil constant. (See Art. 271.)

If the instrument is used as an ordinary current-measuring galvanometer, the tangent law applies to the small deflections obtained, and we may write—

$$C = \frac{H}{z} \tan \delta = \frac{H}{z} \cdot \delta \text{ (ctically)}, \ldots\ldots\ldots (7)$$

where C is the current and δ the steady deflection produced (compare formula for tangent galvanometer). Thus—

$$\frac{H}{z} = \frac{C}{\delta}.$$

The factor H/z may be termed the current per unit angular deflection.

In the formula (6), if t is in seconds, H/z in *amperes*, θ in radians, Q is in *coulombs*. If H/z is in *microamperes*, Q is in *microcoulombs*.

The **absolute constant** of the instrument is the expression—

$$\frac{t}{2\pi} \cdot \frac{H}{z} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

In absolute measurements this constant must be determined, and in the application of Eqn. (6) θ must be in circular measure, and it must be remembered that angular deflection of the ray of light reaching the scale is twice that of the mirror (or needle) itself.

The **working constant** is the quantity of electricity required to produce a deflection of 1 scale division. If k is the working constant and d the number of scale divisions through which the spot is deflected by a charge Q, then—

$$Q = kd \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

It is evident from Eqn. (6) that the working constant is equal to the absolute constant multiplied by the circular measure of the angular deflection of the needle corresponding to one scale division. k, of course, varies with the position of

the controlling magnets. It is proportional to the reciprocal of the time of swing; for $k \propto t\mathrm{H}$ and $t = 2\pi\sqrt{\frac{\mathrm{K}}{\mathrm{MH}}}$. Most measurements with a ballistic galvanometer involve only a comparison of deflections. It is then unnecessary that the constant be known.

Determination of the Constant.—The absolute constant may be measured as follows:—

(1) The terminals of a cell of known E.M.F. E are joined through a high resistance and low resistance in series. The ends of the low resistance are then joined to the galvanometer. In this way a small known potential-difference is applied to the galvanometer. The arrangement of resistances is shown in fig. 189. The points X, Y are joined to the galvanometer and the deflection is noted (δ).

Then Potential-difference of XY $= \frac{r\mathrm{E}}{\mathrm{R}}$;

Resistance of galvanometer $= \mathrm{G}$ (say);

Current through galvanometer $= \frac{r\mathrm{E}}{\mathrm{RG}}$.

$$\therefore \frac{r\mathrm{E}}{\mathrm{RG}} = \frac{\mathrm{H}}{z} \cdot \delta \qquad (10)$$

H/z may therefore be calculated.

(2) The time of swing of the galvanometer needle is observed. The constant is then calculated from $\frac{t}{2\pi} \times \frac{\mathrm{H}}{z}$.

There are other methods of finding the constant, using the laws of induction. (Art. 399.)

393. Use of a Condenser with Ballistic Galvanometer.

A condenser can be used in connection with a ballistic galvanometer so that the two instruments together form a **potentiometer.**

A standard method of making the connections is shown in fig. 241. By means of the key the condenser plates can be placed in connection with the battery—the "charge" position, or joined together through the galvanometer coil—the "discharge" position.

It must be noticed that in the former case there is no current (in the ordinary sense of the term)—merely a momentary flow which charges the condenser plates + and − to the same potentials as the battery poles.

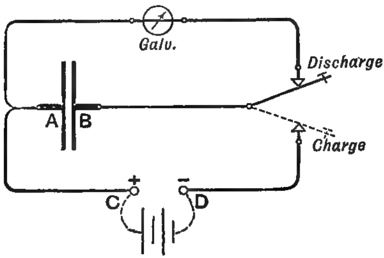


Fig. 241

The potential-difference equals that of the battery. When the key passes into the discharge position, the battery is disconnected, the galvanometer is joined up, and the condenser discharges itself through the latter.

Let V be the potential-difference of the battery terminals C, D, Q the charge which the condenser acquires, S the capacity of condenser, then—

$$Q = SV.$$

If the same condenser is retained and the battery power varied—

$$Q \propto V.$$

But $Q \propto \theta$. $\quad\therefore\ V \propto \theta$ ....(11)

Thus with a given condenser the first throws of the spot of light on the scale are proportional to the P.D. applied to the points C, D. The arrangement, therefore, forms a potentiometer, and may be used in place of the instrument described in Chap. XXIII.

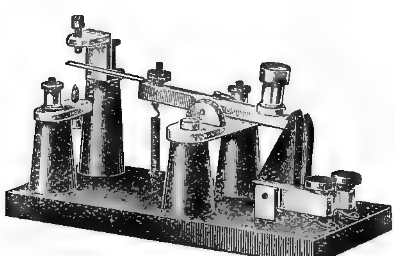


Fig. 242.—Kempe Key

Kempe Key.—A well-known type of key for use with a condenser and ballistic galvanometer is shown in fig. 242. The key is actuated by a spring. It is first

pressed down into the "charge" position, and held by one of the clutches seen on the right of the figure. On releasing the clutch the key springs into the "discharge" position. For special insulation tests an intermediate position is provided where both battery and galvanometer are disconnected. The contacts are (preferably) rubbing contacts faced with platinum.

394. Measurements with B.G. and Condenser.

(1) *Comparison of E.M.F.'s.*—Take the throw θ_1 with one of the given cell between terminals C, D (fig. 239). Retain same condenser, and take the throw θ_2 with the second cell between C and D. Then—

$$\frac{E_1}{E_2} = \frac{\theta_1}{\theta_2} \qquad\qquad (12)$$

(2) *Comparison of Capacities.*—Retain one cell throughout, and connect the condensers to be compared, in turn, between A and B. Then—

$$\frac{\theta_1}{\theta_2} = \frac{Q_1}{Q_2}, \quad Q_1 = S_1 V, \quad Q_2 = S_2 V.$$

$$\therefore \frac{S_1}{S_2} = \frac{\theta_1}{\theta_2} \qquad\qquad (13)$$

Note.—The instrument should always be adjusted so that the axis of the needle is parallel with the face of the coils.

Suspended Coil Ballistic Galvanometer.—For comparative measurements with a condenser, suspended coil galvanometers may be used. The coil must be wound on an ebonite or other non-conducting former to avoid damping through eddy currents.

Such galvanometers can be used for absolute measurements in place of the needle form, provided the circuit is open or of very high resistance.

395. Measurement of Capacity.

Since, by definition, capacity is the ratio Q/V, the quantities Q and V may be determined independently and S then calculated. But practically Q and the galvanometer constant are

determined with the same cell in order to avoid the determination of the electromotive force of the cell.

Let a constant cell E.M.F. E volts be joined to the points C, D (fig. 241) and the throw θ taken. Then—

$$Q = (\text{absolute constant}) \times \theta.$$

Next use the *same cell* to determine the constant of the galvanometer, as explained above—Equation (10). We have—

$$(\text{Constant}) = \frac{t}{2\pi} \cdot \frac{rE}{RG \,.\, \delta}.$$

$$\therefore \frac{Q}{V} = \frac{t}{2\pi} \cdot \frac{rE}{RG\delta} \cdot \frac{\theta}{V}.$$

Since the potential-difference (V) of condenser is equal to E,

$$S = \frac{t}{2\pi} \cdot \frac{r}{RG} \cdot \frac{\theta}{\delta} \quad \text{......................(14)}$$

If r, R, G are in ohms and t in seconds, S is in farads.

Determination of Working Constant.—A standard cell, together with a condenser, may be used to determine the working constant. Let the condenser be joined up as in fig. 241, the cell being at CD. If the throw is now taken, we have—

$$Q = kd.$$

But $Q = SV$, and $V = E$.

$$\therefore k = \frac{SE}{d} \quad \text{......................(15)}$$

396. Measurement of Inductance.

From Equations (2), (3), and (4) we see that the quantity of electricity discharged by a momentary inductive effect is—

$$Q = \frac{MC}{R} \text{ for mutual induction ....(16)}$$

$$Q = \frac{LC}{R} \text{ for self-induction ....(17)}$$

A determination of Q, therefore, enables us to determine the inductance M or L. As in the determination of capacity,

methods are adopted to avoid the direct measurement of the galvanometer constant and E.M.F. of the battery.

1. Mutual Inductance.

We may take as an example a pair of induction coils. Join the primary in series with a battery and key (fig. 243 (*a*)). Connect the secondary coil to the galvanometer. When contact is made in the primary circuit, there is a throw of the galvanometer needle due to the inductive discharge, and an equal opposite throw occurs at break. Let θ_1 be the inductive throw. Then $Q_1R = MC$ (R = total resistance of secondary circuit).

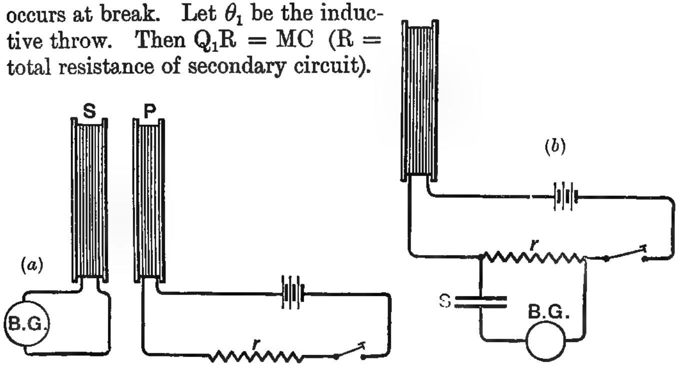


Fig. 243.—Measurement of Mutual Inductance

Next disconnect the galvanometer from the secondary and join it "through" a condenser to the terminals of a known resistance r in the primary circuit (fig. 243(*b*)). Let θ_2 be the throw observed on making or breaking contact in the primary. Then θ_2 represents the quantity of electricity required to charge the condenser to the same potential-difference as the ends of resistance r. This potential-difference $= Cr$ where C is current in the primary. Hence—

$$Q_2 = VS = CrS.$$

Combining this with the result of the first part of the experiment, we have—

$$\frac{Q_1}{Q_2} = \frac{MC}{R \,.\, CrS} = \frac{M}{R \,.\, r \,.\, S}.$$

But $$\frac{Q_1}{Q_2} = \frac{\theta_1}{\theta_2}.$$

$$\therefore\ M = RrS \,.\, \frac{\theta_1}{\theta_2}.$$

The inductance is determined in terms of resistance and capacity. If R, r are in ohms, S in *farads*, then M is in henrys.

2. Self-Inductance.

The following method is due to Lord Rayleigh. The coil under test (say an electromagnet coil) is included in one arm of a Wheatstone bridge (Post-Office pattern), and a ballistic galvanometer is used on the bridge (fig. 244).

(*a*) A balance is first obtained for steady currents in the ordinary way adopted in the measurement of resistance. The battery key

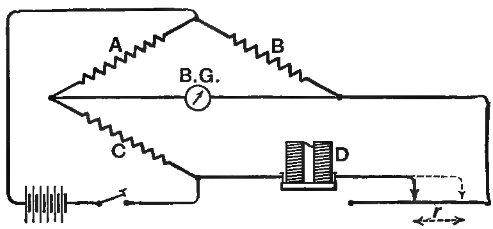


Fig. 244.—Measurement of Self-Inductance

must be put down before the galvanometer key. The balance must be made very exact. One method of doing this is to use a piece of bare wire with a sliding contact in the arm containing the unknown resistance.

(*b*) When an exact balance has been obtained the galvanometer key must be held down and the contact made or broken with the battery key. There is now a sudden throw of the galvanometer at each make or break, due to the self-induced discharge. Let this throw be θ.

(*c*) The resistance in the arm containing the coil under test is now *slightly* altered by moving the position of the sliding contact. This upsets the balance for steady currents, and if the battery key is put down first, then the galvanometer key, the *steady* deflection δ, may be read off.

Theory.—In the operation (*b*) a quantity of electricity Q is discharged through the net-work from the inductance coil. If C is the current in the latter, and R the total effective resistance of the net-work,

$$Q = \frac{LC}{R}.$$

A certain fraction of this passes through the galvanometer. If p is the fraction, the discharge through the galvanometer is $\frac{pLC}{R}$.

In the operation (c) we may imagine that, *superposed* on the distribution of currents which occurs when the balance is exact, there is a system of weak currents arising from a potential-difference due to the small change r in the resistance of the arm D. The former system produces no current through the galvanometer. The additional potential-difference Cr produces a weak current of total strength $\frac{Cr}{R}$, and a fraction of this passes through the galvanometer, namely, $\frac{pCr}{R}$. (The quantity discharged through each branch of the net-work in (b) is the same fraction of the total quantity discharged as the steady current traversing that branch in (c) is of the total steady current (Art. 391). Hence the *same* fraction p may now be used for the current through the galvanometer as was used for the quantity discharged through the galvanometer.)

Thus by Equation (6) we have—

$$\frac{pLC}{R} = \frac{t}{2\pi} \cdot \frac{H}{z} \cdot \theta.$$

Also by Equation (7)—

$$\frac{pCr}{R} = \frac{H}{z} \cdot \delta.$$

$$\therefore \frac{L}{r} = \frac{t}{2\pi} \cdot \frac{\theta}{\delta},$$

or

$$L = \frac{t}{2\pi} \cdot r \cdot \frac{\theta}{\delta}.$$

The self-inductance is determined in terms of *time* and *resistance*. With t in seconds and r in ohms, L is in henrys.

In very accurate determinations we must write $2 \sin \frac{1}{2}\theta$ $(1 + \lambda/2)$ for θ, and $\tan \delta$ for δ.

The inductance of a secondary of an induction coil may amount to 1000 henrys. That of a small electromagnet (say of a bell) is about 1 henry. The practical measurement of

inductance requires a choice of suitable resistances and various precautions to ensure success. For these and other methods of finding inductance the student is referred to laboratory manuals.

397. Inductive Methods of Testing Iron.

The above principles are applied in testing the flux of induction produced in iron by a given magnetizing force. The experimental details are described in Art. 74. If the magnetization I of a given specimen of iron is measured (say for a magnetizing force which produces saturation) by the magnetometric method (Art. 58), and if the induction B is measured (for the same specimen and same force) by the method of Art. 74, it will be found that the induction due to the magnetization is $4\pi I$. Thus the relation—

$$B = H + 4\pi I,$$

is established experimentally.

398. Examples of Inductive Discharge.

Equations (1) and (2) are of primary importance. Examples of condenser calculations have already been given, and we shall now give some examples of calculations of induced discharge in which the equation $Q = \frac{N}{R}$ is applied.

1. A wire hoop of one turn stands with its plane vertical and magnetic east and west. It is suddenly turned through 180° about a vertical diameter. Determine the quantity of electricity discharged, given radius = 50 cm., horizontal component of field = ·18 C.G.S., resistance of hoop = ·1 ohm.

When the hoop is in the initial position—

$$\begin{aligned} &\text{Induction} &&= \text{magnetic field-strength} = \cdot 18. \\ &\text{Flux} &&= \cdot 18 \times \text{area} = \cdot 18 \times \pi \times 50^2. \\ &\text{Linkage} &&= \cdot 18 \times \pi \times 50^2 \text{ C.G.S. units.} \end{aligned}$$

In the first half of the rotation the linkage is reduced to zero. In the second half the lines are reintroduced through the hoop in the reverse direction (as regards the *hoop*). Hence the total change of linkage is—

$$2 \times \cdot 18 \times \pi \times 50^2 \text{ C.G.S.}$$
$$= \cdot 00002827 \text{ prac. unit.}$$
$$Q = \frac{\cdot 00002827}{\cdot 1} = \cdot 0002827 \text{ coulomb,}$$
$$\text{or} \quad Q = 282 \cdot 7 \text{ microcoulombs.}$$

2. A ballistic galvanometer resistance 10,000 ohms and working constant ·001 microcoulomb per scale division. It is joined to a coil of wire of 500 turns, resistance 75 ohms, and area 450 sq. cm. When the coil is turned through 180° about a diameter lying at right angles to the field-direction, commencing with the plane perpendicular to the field, the throw is 60 scale divisions. Determine the field-strength.

$$Q = 60 \times \cdot 001 \text{ microcoulomb}$$
$$= \cdot 00000006 \text{ coulomb.}$$
$$N = QR = \cdot 0006045 \text{ prac. unit}$$
$$= 60450 \text{ C.G.S.} = \textit{change} \text{ of linkage.}$$

For the same reason as in Example 1, this is twice the linkage in the initial position. Hence linkage in initial position

$$= 30225 \text{ C.G.S.}$$
$$\therefore \text{flux} = \frac{30225}{500} = 60 \cdot 45.$$
$$\therefore \text{induction} = \frac{60 \cdot 45}{450} = \cdot 134.$$

Since the permeability of air is unity, the induction is equal to the field-strength.

$$\therefore \text{field-strength} = \cdot 134 \text{ dyne per unit pole.}$$

3. A long solenoid of 8 turns per cm. is surrounded at one part with a secondary coil of 300 turns and resistance 50 ohms. The solenoid has an area of 3 sq. cm. If the secondary is joined to a galvanometer of 5000 ohms resistance, find the discharge through the galvanometer when a current of 5 amperes is started in the solenoid.

$$\text{Field-strength in solenoid} = \frac{4\pi n C}{10}$$
$$= 1 \cdot 257 \times 8 \times 5 = 50 \cdot 28.$$
$$\text{Flux through secondary} = 50 \cdot 28 \times 3.$$
$$\text{Linkage with secondary} = 50 \cdot 28 \times 3 \times 300$$
$$= \cdot 0004525 \text{ prac. unit.}$$
$$\therefore Q = \frac{N}{R} = \frac{\cdot 0004525}{5050} = \cdot 089 \times 10^{-6} \text{ coulomb}$$
$$= \cdot 089 \text{ microcoulomb.}$$

Observe carefully that the quantity of electricity discharged through a circuit by an induced E.M.F. depends on the resistance of the circuit. Hence different amounts of discharge can be obtained from the same coil with galvanometers having different resistances.

399. The Earth-Inductor.

This instrument (also called *Delezenne's Circle*) consists of a coil of several hundred turns of fine wire wound on a frame which can be rotated about an axis in its own plane. The axes are supported by a second frame which can rotate about an axis at right angles to the former axis. The axis of rotation of the coil can thus be placed at any angle of inclination. The instrument is shown in fig. 245. Its chief use is to determine the constant of a ballistic galvanometer (the strength of the earth's field being previously ascertained).

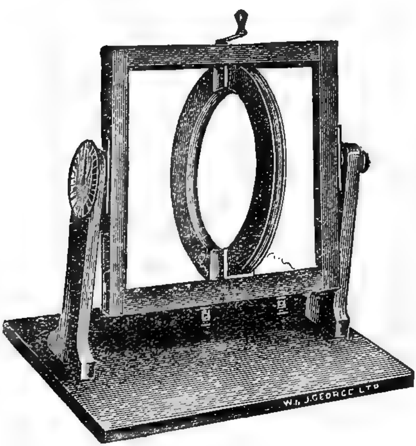


Fig. 245.—Earth-Inductor

The coil is placed with its plane E.–W. (magnetic) and axis of rotation vertical. It is joined directly to the galvanometer and then turned suddenly through 180°. The throw, d divisions, is noted.

Let a = area of coil,
n = the number of turns,
H = horizontal component of the field,
R = total resistance of the galvanometer and inductor.

Change of linkage during a revolution through 180°

$$= 2\,\mathrm{H}an.$$

Linkage change in practical units $= \dfrac{2\,\mathrm{H}.a.n}{10^8} = \mathrm{N}.$

Quantity discharged through galvanometer $= \dfrac{\mathrm{N}}{\mathrm{R}}.$

Constant = Quantity per scale division.

$$k = \frac{\mathrm{Q}}{d} = \frac{2\,\mathrm{H}an}{\mathrm{R}d} \times \frac{1}{10^8}.$$

If k is determined by other methods, the same experiment serves to determine H.

400. Examples.

1. A coil of wire of inductive area 15,000 square centimetres is connected with a galvanometer. The coil lies flat on a table, and when it is turned over the galvanometer is momentarily deflected. Discharging a condenser of 1 microfarad capacity charged to 1·5 volt through the galvanometer produces the same kick as with the coil. If the vertical component of the earth's field is ·4 C.G.S. unit, what is the resistance of the coil and galvanometer together with the connecting leads? (1904.)

From the condenser experiment we have—

$$\mathrm{Q}_1 = \mathrm{SV}$$
$$= \frac{1{\cdot}5}{10^6} \text{ coulomb.}$$

From the inductive experiment—

$$\mathrm{Q} = \frac{\mathrm{N}}{\mathrm{R}}.$$

But since the throws are equal,

$$\mathrm{Q} = \frac{1{\cdot}5}{10^6} \text{ coulomb.}$$

To find N we have—

$$\mathrm{B} = {\cdot}4 \text{ C.G.S.},$$
$$a = 15000 \text{ sq. cm.}$$
$$\therefore \mathrm{N} = 2 \times {\cdot}4 \times 15000 = 12000 \text{ C.G.S.}$$
$$= \frac{12}{10^5} \text{ practical unit of linkage.}$$
$$\therefore \mathrm{R} = \frac{\mathrm{N}}{\mathrm{Q}} = \frac{12}{10^5} \times \frac{1}{1{\cdot}5} \times 10^6$$
$$= 80 \text{ ohms.}$$

2. How would you determine the intensity of a *very strong* magnetic field such as exists between the poles of an electro-magnet?

A small coil (say 1 cm. diameter) of fine wire is mounted so that it can be rotated between the poles. The coil is joined in series with a ballistic galvanometer of known working constant. The coil is first placed with its plane at right angles to the field and then turned suddenly through 180°.

Then if H is the field strength to be measured, we find, using the same symbols and reasoning as in the case of the earth-inductor, that—

$$H = \frac{k\delta R}{2an} \times 10^8.$$

The constant k is determined by the use of a condenser and standard cell.

QUESTIONS

1. Describe some method of comparing the capacities of two condensers.

2. Having given a condenser, Kempe key, and potentiometer, how would you proceed to calibrate the scale of a ballistic galvanometer?

3. With the aid of a ballistic galvanometer and coil of wire, how may the distribution of leakage lines from the sides of a permanent magnet be determined?

4. Distinguish between a ballistic and a dead-beat galvanometer. Describe some form of suspended coil galvanometer, stating the conditions under which it is (1) dead-beat or (2) ballistic.

(1905.)

CHAPTER XXXI

CONDUCTION IN GASES

401. The study of the phenomena accompanying the conduction and discharge of electricity in gases has in recent years led to remarkable developments in the theory of electricity, and has considerably modified our views of the constitution of matter. The subject is now a very extensive

one, and we must here confine ourselves to a brief *résumé* of the main phenomena and results.

402. Ionization of Air.

Dry air carefully freed from dust is an almost perfect insulator, as is shown by the fact that a good electroscope will retain a charge for days. The air can, however, be rendered conducting in many ways.

A lighted match, if held a short distance above a charged electroscope, will cause the leaves to collapse rapidly. Flames, hot gases from flames, gases from regions traversed by electric sparks or arcs, produce a similar effect. Again, X-rays and the radiation from certain active materials such as uranium, if allowed to fall on an electroscope, will be found to dissipate the charge. The same property is possessed by ultra-violet light. If a clean zinc rod is connected to an electroscope and charged negatively, a beam of ultra-violet light will at once discharge it.

In dealing with *electrolytic* conduction, we have seen that the current is carried by definite portions of the molecule which are termed positive or negative *ions*, according to their charge. Each ion is loaded with a charge e if monovalent, $2e$ if divalent, and so on. The ion may be hampered, and its mass virtually increased, by the attachment of whole molecules of the solvent. Modern research has shown that the conductivity imparted to the air by the agencies mentioned above can be most rationally explained if we suppose that the electricity conducted through the gas is carried by particles positively or negatively charged. These are again called *ions*, but it by no means follows that the "particles" which carry the charges in a gas are identical with those which perform the same office in an electrolyte.

In electrolysis the ratio of the charge to the mass of the ion which carries it is, for hydrogen, about 10^4. It has been shown that in conduction through an ionized gas at low pressure, the ratio is nearly $7{\cdot}7 \times 10^6$, and that the charge is the same as in electrolysis. It follows that the mass of the "particle"

which forms the negative ion in a rarefied gas is about $\frac{1}{800}$ of the mass of the hydrogen atom. These extremely small negative ions are termed corpuscles by Prof. J. J. Thomson. It thus becomes necessary to recognize the existence of particles smaller than the hydrogen atom.

In air at ordinary pressure the negative ions are much heavier; the corpuscle becomes loaded up.

The positive ions, even in gases at low pressure, have a mass of the same order of magnitude as that of the hydrogen atom.

403. **Relation of Current to Potential-Difference.**

Ohm's Law is not in general obeyed by a conducting gas. If two metal plates A and B are joined to a battery and electrometer as shown in fig. 246, the conductivity of the gas between them can be studied. The space between the plates being subjected to ionizing influence, the quantity of electricity transferred from A to B per second can be ascertained from the rate at which the needle of the electrometer is deflected.

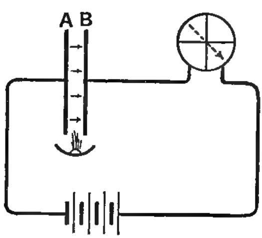


Fig. 246

By varying the battery power it is found that Ohm's Law is obeyed only when the potential-difference is comparatively low. When the battery power is increased beyond a certain point it begins to be less effective in increasing the current, and beyond a certain voltage there is no further increase in the current. This "saturation" value of the current continues until a luminous discharge occurs.

We have so far supposed the gas to be rendered conducting by some external agency. In spark or other luminous discharges we have another class of phenomena in which the gas is ionized by the powerful electric field between the discharge terminals.

404. Spark Discharges.

Gases have far less dielectric strength than solids or liquids. The potential-difference required to produce a spark, say 2 mm. long, in air is very much smaller than that required to pierce glass of the same thickness in a Leyden jar. The potential-difference required to produce a spark has been investigated by Baille, Liebig, and others. It is found that, except in the case of aluminium and magnesium, the material of the electrodes has no influence on the sparkling potential. This depends on—

(i) the length of the gap;
(ii) the pressure of the gas;
(iii) the shape of the electrodes;
(iv) the nature of the gas.

In air at normal pressure, with large spherical electrodes, and for sparks above $\frac{1}{5}$ cm. in length, the potential-difference required is given fairly accurately by Chrystal's formula for Baille's results—

$$V = 5 + 99 \cdot 6d,$$

where V is in *electrostatic* units and d in centimetres. This refers to a gap in which the air is quite "fresh". If, however, a spark has just passed, it is easier to pass a second spark—an effect which lasts several minutes.

Character of the Spark.—Short sparks take the form of a straight line of light. Longer ones consist of a short, straight line from the positive terminal, which then branches along numerous irregular paths to the negative terminal. The luminosity of small sparks may be due to particles of incandescent metal which have been detached from the terminal. In other cases the gas itself is rendered luminous.

The capacity of the terminals has a remarkable influence on the character of the spark. When the jars are attached to a Wimshurst machine the discharges become longer, brighter, and less frequent. The large quantity of electricity stored in the jars is capable of maintaining a definite spark over a longer ionized path without increase of potential-difference.

The appearance known as a brush discharge is well seen when an earth-connected plate is held a short distance from the small positive terminal of a Wimshurst machine.

405. **Discharge in a Rarefied Gas.**

Some of the most beautiful effects of electric discharge are produced in vessels which have been partially exhausted. The *high potential* discharge from an induction coil or Wimshurst is required. Platinum terminals are fused through the glass to form the anode and cathode. Commencing at the cathode end, there is first a faint luminosity covering the cathode. Beyond this there is a dark region known as the Crookes space. The length of the Crookes space depends on the degree of exhaustion, and its boundary on the side towards the anode is fairly sharp. Next follows a bluish glow called the negative glow. Beyond this there is a region known as the Faraday dark space, the extent of which varies greatly in different cases. The remainder of the tube is filled with a long, luminous—usually crimson—spindle, extending up to the anode, and known as the positive column.

If the tube is lengthened (even to several metres) the positive column lengthens accordingly, but the Crookes space and negative glow remain unchanged. Between suitable limits of pressure the positive column assumes a striated character. At a pressure of about $\frac{1}{2}$ mm. of mercury the striations are sharply defined, and are often seen in rapid oscillation. Under greater pressures they become broader and less distinct, and finally disappear. The presence of certain vapours (*e.g.* alcohol, turpentine) facilitates the formation of striæ.

The positive column can be deflected by a magnet, like a flexible conductor.

406. **Discharge in High Vacua.**

By the use of improved forms of vacuum pump Sir W. Crookes succeeded in obtaining very high degrees of exhaustion, and discovered a new series of phenomena. As the exhaustion increases the positive column shrinks, and finally

disappears. At the same time the Crookes dark space increases until it fills the whole tube. The only direct traces of the path of the discharge are now certain very faint bluish rays; but the existence of the discharge is rendered abundantly evident by the bright phosphorescence set up in the glass of the tube. The distribution of this phosphorescence (which is greenish-yellow for soda glass) shows that it is due to rays of some kind projected in straight lines from the cathode. These **cathode rays** possess some remarkable properties:

(1) The rays are stopped by glass or other solid obstacle, a shadow of the object being formed according to the straight-line law of projection.
(2) They can be **deflected by a magnet**—a property which shows that they carry a current of electricity. The direction of deflection shows that the charge projected from the cathode is negative.
(3) They can be brought to a "focus" by a concave cathode.
(4) A piece of platinum situated at the focus becomes white-hot.
(5) If a delicate paddle-wheel is supported on glass rails within the tube, and placed in the path of the cathode discharge, it is made to revolve.

The above properties are independent of the position of the anode. The effects are best observed when the exhaustion is carried to about $\frac{1}{1000000}$ of an atmosphere.

The above phenomena support the view, now generally accepted, that—

The cathode discharge consists of a stream of negatively electrified particles projected at high velocity from the cathode surface.

The ratio of the charge to the mass of these particles can be observed by deflecting the rays by magnetic and electric fields and observing the deflection produced. The result of these measurements identifies them with the corpuscles or

negative ions produced by external ionizing agencies. A corpuscle carries a natural unit of negative electricity—an "electron". Evidence has been obtained that the inertia of the corpuscle is equivalent to the electromagnetic inertia of the electron. This indicates that the corpuscle and electron are identical, and has given rise to an "electrical theory of matter". According to this theory, an atom is an assemblage of electrons held together by the attraction of a "sphere" of positive electricity in which they are situated. The nature of positive electricity is the most obscure point in the whole theory.

407. X-Rays.

When the stream of corpuscles projected from the cathode is interrupted by any obstacle within the tube, a radiation of

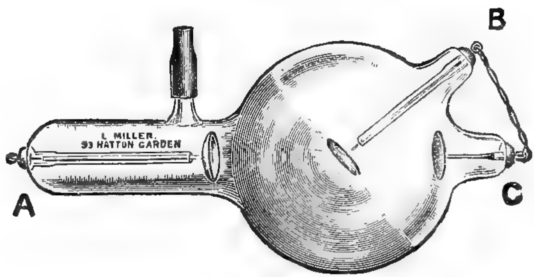


Fig. 247.—Focus Tube

different character is sent out from the surface of the obstruction. This radiation was discovered by Prof. Röntgen of Munich, in 1895, through the fogging of a photographic plate which had been left in its case near a vacuum tube at work. It was soon found that the rays will penetrate wood, leather, skin, metals, etc. The transmitting power of these substances is inversely proportional to their density, so that a heavy metal like lead is "opaque" to X-rays, whilst a light metal like aluminium is "transparent". The remarkable photographs of the bones of the human body obtained with Röntgen rays

are attributable to the difference in density between bone and flesh.

The usual form of vacuum tube used to produce X-rays is shown in fig. 247, this form being known as the "focus" tube. The cathode terminal is shown at A and the anode terminals at B and C. The cathode is of aluminium, and is concave. It is connected by an aluminium rod with the platinum terminal fused through the glass.

The anode is a disc of aluminium joined to C. In the centre of the bulb there is a "target". This is a small disc of platinum which also serves as a second anode (the use of two anodes diminishes the resistance of the tube). When the target is bombarded by the stream of cathode rays it sends out the X-radiation in all directions from its surface. These falling on the glass render it phosphorescent. All parts of the glass in front of the plane of the target emit a greenish-yellow glow, the boundary line being quite definite. (The phosphorescent area is shown shaded in the diagram.)

The following properties distinguish Röntgen rays from the cathode rays which produce them, and from light:—

(1) They are not influenced by a magnet, and therefore do not constitute an electric current.

(2) They possess great power of penetrating material bodies, whereas cathode rays are stopped by all but very thin layers of light materials.

(3) They cannot be refracted, and do not yield interference phenomena like ordinary light.

It was at first found difficult to arrive at a satisfactory explanation of these phenomena. The Röntgen radiation is now supposed to consist of disturbances in the ether, like ordinary light. But whereas light-waves are perfectly regular and periodic, the X-ray disturbance consists of pulsations of an irregular character and of duration much shorter than the period of even ultra-violet light waves. The pulses are pro-

duced by the sudden stoppage of the corpuscles at the surface of the target or anticathode.

A Wimshurst machine may be used to work an X-ray tube, but an induction coil is preferable. The coil should be capable of giving at least a 3-inch spark in air at ordinary pressure. For eye observation the shadows cast by the radiation on a fluorescent screen are utilized. This screen consists of a card coated with yellow barium platinocyanide or other fluorescent material. When exposed to the rays it glows with a bright green colour, and the shadow of an object opaque to the rays can be formed on it. Photographs are obtained by enclosing a plate in a light-tight envelope, and placing this so that the object to be photographed is between the plate and the X-ray tube.

408. **Discharge from Points.**

The discharge from a sharply-pointed conductor is due to the ionization of the air by the strong electric field round the point. A small blue glow exists at the tip of the point, and this is the only luminous portion of the discharge. To produce ionization a certain minimum potential is required which depends on—

(1) the sharpness of the point;
(2) the pressure of the gas;
(3) the sign of the electrification, being less for negative than positive charges;
(4) the nature of the gas.

The *electric wind* is caused by the stream of ions communicating its momentum to the air in the neighbourhood of the point. For a given current strength and air pressure the reaction is less for negative ions than for positive ones, as may be expected from the smaller mass of the former.

409. **Radio-Activity.**

M. Henri Becquerel found, in 1896, that compounds of uranium emit rays which affect a photographic plate. These

rays, termed Becquerel rays, penetrate opaque materials and ionize gases.

M. and Mme. Curie, in 1900, found that some specimens of pitchblende from Bohemia were more active in emitting these rays than uranium itself, and subsequently compounds of three new elements—actinium, polonium, and radium—were separated, all of which are active. The most highly radio-active material is radium. This substance closely resembles barium in its chemical properties. It is only present in minute quantities in pitchblende, some tons of this mineral yielding less than a gram of radium compound.

Prof. Rutherford has shown that the Becquerel rays are of a composite character. Three different kinds of rays can be distinguished—

(1) The α-rays. These have very little penetrating power, being capable of passing through only a very thin leaf of aluminium. The ionizing effects are, however, due almost entirely to the α rays. They carry a positive charge, and are deflected by a magnet.

(2) The β-rays. These have been identified with the cathode discharge. They carry a negative charge, and are deflected in the opposite direction to the α-rays.

(3) The γ-rays. These are unaffected by a magnet, and resemble the X-rays in their properties.

Radium also gives off a gas which is radio-active.

For details of results and experiments see J. J. Thomson's *Conduction of Electricity through Gases*, and Rutherford's *Radio-activity*.

CHAPTER XXXII

ELECTRICAL UNITS

410. Fundamental Equations.

I. The definitions of all electrical quantities are based primarily on one or other of the two following relations:—

$$f = \frac{mm'}{d^2} \qquad (1)$$

$$f = \frac{ee'}{d^2} \qquad (2)$$

These expressions each contain an electrical or magnetic quantity of *one kind* only. They express the experimental laws of magnetic and electric force between magnetic poles or electric charges *in vacuo*; and they also serve to define the primary measures of magnetic pole and electric charge.

In the ordinary system of electrical definitions these are the only equations which involve an electrical or magnetic quantity of one kind. The equation for the mechanical action between two current circuits may be considered as a third example, but in the established treatment of the subject, currents are represented by a distribution of magnetism (a magnetic shell) and the equation reduces to a particular case of (1).

II. The following relations each involve *two* quantities of an electrical or magnetic nature:—

Magnetic—

Magnetic force	=	mechanical force per unit pole	$H = \frac{f}{m}$.
,, moment	=	pole-strength × length	$M = m \times l$.
Magnetic potential or M.M.F.	=	work per unit pole = field-strength × distance	$U = H \times d$.
Current (electromagnetic definition)	=	magnetomotive force per turn ÷ 4π	$i = \frac{U}{4\pi n}$.
Magnetization	=	magnetic moment per unit volume	$I = \frac{M}{v}$.
Surface density	=	quantity of magnetism per unit area	$\sigma = \frac{m}{a}$.

Magnetic induction	= {equivalent magnetization × 4π}	$B = 4\pi\frac{m}{a}$.
Magnetic flux	= induction × area	$P = B \times a$.
" linkage	= flux × number of turns	$N = P \times n$.

Electrical—

Electric force	= {mechanical force per unit charge}	$F = \frac{f}{e}$.
Electric potential or E.M.F.	= {work per unit charge = field-strength × distance}	$V = F \times d$.
Surface density	= charge per unit area	$\sigma = \frac{e}{a}$.
Electric induction	= {surface density of equivalent charge}	$D = \frac{e}{a}$.
Total induction	= induction × area	$Q = D \times a$.
Conduction current	= {charge transferred per unit time}	$C = \frac{Q}{t}$.
Dielectric current	= {increase of total induction per unit time}	$\gamma = \frac{Q}{t}$.

It will be noticed that some quantities occur more than once on the list, owing to different modes of definition. Thus total electric induction is mathematically equivalent to charge, magnetization to surface density of free magnetism. The different meanings of "current" are especially important, and are again referred to below.

III. The following equations express the definitions of the quantities given in the left-hand members and involve *three* quantities of an electrical or magnetic nature:—

(1) Equations referring to properties of fields.

Magnetic—

$$\text{Inductance} = \frac{\text{Linkage}}{\text{Current}} \quad L = \frac{N}{i}.$$

Electrical—

$$\text{Capacity} = \frac{\text{Total elec. induc.}}{\text{Potential-diff.}} \quad S = \frac{Q}{V}.$$

(2) Equations referring to electrical or magnetic circuits.

Magnetic—

$$\text{Reluctance} = \frac{\text{Mag. mot. force}}{\text{Flux}} \quad Z = \frac{U}{P}.$$

Electrical—

$$\text{Resistance} = \frac{\text{Elec. mot. force}}{\text{Conduction current}} \quad R = \frac{E}{C}.$$

(3) Equations referring to properties of materials.

Magnetic—

$$\text{Susceptibility} = \frac{\text{Magnetization}}{\text{Mag. force}} \quad \kappa = \frac{I}{H}.$$

$$\text{Permeability} = \frac{\text{Mag. induction}}{\text{Mag. force}} \quad \mu = \frac{B}{H}.$$

Electrical—

$$\text{Permittivity} = \frac{\text{Elec. induction}}{\text{Elec. force}} \quad \omega = \frac{D}{F}.$$

$$\text{Resistivity} = \frac{\text{Electric force}}{\text{Current density}} \quad \rho = \frac{F}{C/a}.$$

Specific inductive capacity as ordinarily defined is the ratio of two capacities (Art. 175), or, what is equivalent to this, the ratio of two permittivities. In electrostatic measure, permittivity and inductive capacity are numerically equal.

It must here be mentioned that some of the above quantities may be defined in other ways. Resistance and E.M.F. may be defined on energy principles, electric and magnetic induction from the circuital laws. The definitions given above are equivalent to those of the more elaborate treatment, which must be left for a later stage of reading.

411. Two Meanings of "Current".

It will be noticed that "current" occurs in the lists of both electrical and magnetic quantities. The significance of the term is, however, different in the two cases.

According to the *electrical* definition the term "current" has its original meaning of "flow". The equations $C = Q/t$ or $\gamma = Q/t$ refer to the flow of electricity in a conducting circuit, or to the lateral flow of tubes of electric induction producing an accumulation of tubes in the case of a dielectric current.

According to the *magnetic* definition the term "current" is used as a name for the strength of the equivalent magnetic

shell; or more generally the deduction from Ampère's Theorem may be used, which states that—

$$\text{magnetomotive force} = 4\pi \times \text{current} \times n.$$

Thus the magnetic meaning of "current strength" is the work done in taking unit pole round a curve linked once with the circuit, divided by 4π.

Or in place of this definition we may use the formula applied to a tangent galvanometer, namely—

$$f = \frac{(\text{current}) \times lm}{r^2}, \text{ or } (\text{current}) \times m = \frac{fr^2}{l}.$$

This last expression will be used in the present chapter, as it exhibits the relation of pole-strength to current most directly. It is mathematically equivalent to the other two definitions.

We shall use the symbols C and i to distinguish the electrical and magnetic meanings of current respectively.

412. Two Orders of Definition.

The above summary shows one set of quantities (electrical), the units of which depend on the unit of charge; and another set (magnetic), the units of which depend on the unit of pole-strength. We may, however, make all the quantities of the two groups depend on *either* e or m by means of a "connecting link". The link is supplied by the assumption that the quantity termed current in the electrical system is identical with the quantity of the same name defined by magnetic properties. In support of the assumption we have the experimental fact that C (as measured by a voltameter or condenser discharge) is proportional to i as measured by a tangent galvanometer or ballistic galvanometer[1]; or in symbols—

$$C \propto i.$$

Hence the assumption that $C = i$ can only involve the suppression of a constant factor (not necessarily a mere numeric).

[1] In the condenser and ballistic galvanometer the time integrals only of C and i are obtained, but the proportionality of the time integrals supports the assumption.

The two orders of definition are termed—

The electromagnetic order if we commence with m;
„ electrostatic „ „ „ e.

413. Leading Equations.

In the electromagnetic order of definition the principal equations are—

$$\text{(a) } f = \frac{mm'}{d^2};$$

$$\text{(b) } f = \frac{ilm}{r^2} \text{ and } i = C;$$

$$\text{(c) } C = \frac{Q}{t}.$$

In the electrostatic order of definition we have—

$$\text{(a) } f = \frac{ee'}{d^2};$$

$$\text{(b) } C = \frac{Q}{t} \text{ and } C = i;$$

$$\text{(c) } f = \frac{ilm}{r^2}.$$

The following examples will illustrate the application of the above equations:—

1. Show how the unit of capacity is arrived at in the electromagnetic order.

(i) Unit pole is defined from the equation $f = \frac{mm'}{d^2}$, as in Art. 16.

(ii) Make use of the unit pole so defined, and obtain unit current from the equation $f = \frac{ilm}{r^2}$ (Art. 269). By the assumption $i = C$.

(iii) Substitute the unit current in the equation $C = Q/t$, and so obtain the definition of unit charge or quantity (e or Q). (Art. 290.)

(iv) Unit electric field is then defined from $F = \frac{f}{e}$.
„ „ potential is defined from $V = F \times d$.
„ „ capacity is defined from $S = Q/V$.

2. Show how the unit of magnetic moment is arrived at in the electrostatic order.

(i) Unit charge is defined with reference to the equation $f = \frac{ee'}{d^2}$, Art. 117.

(ii) Make use of the unit charge so defined to obtain definition of unit current from the equation $C = Q/t$, Art. 239. By assumption $C = i$.

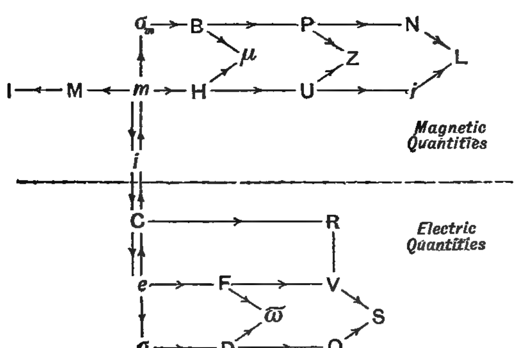
(iii) Substitute the unit current in the equation $f = \frac{ilm}{r^2}$, and so obtain the definition of unit magnetic pole, namely: "Unit pole is the quantity of magnetism which, when placed at the centre of a circle of unit radius in which a unit current is flowing, experiences a unit force for each unit length of the wire".

(iv) Unit magnetic moment is then defined from $M = m \times l$.

The diagram below will assist the student to distinguish the relations of the various units. The electromagnetic and electrostatic systems differ only in the order of defining charge, current, and magnetic pole.

electromagnetic order: $m \rightarrow i \rightarrow C \rightarrow e$,
electrostatic order: $e \rightarrow C \rightarrow i \rightarrow m$.

In the diagram this double order is shown by the double lines. The derivation of other quantities is shown by the single lines. Corresponding to each arrow there is an equation in the foregoing list—



In theoretical work it is usual to express both electromagnetic and electrostatic units in terms of the centimetre, gram, and second. This is merely a matter of convenience; any other consistent system of units, *e.g.* the foot, pound, second system might be used.

414. Relative Magnitude of the Units.

Measurements have been made of several quantities in terms of both electrostatic and electromagnetic units, and the relative magnitude of these units has thus been ascertained. For example, the capacity of a condenser in electrostatic units may be calculated, and its potential-difference when charged may be measured with an absolute electrometer. The charge in electrostatic units is then ascertained from the product S × V. If the condenser is now discharged through a ballistic galvanometer, the charge is determined in electromagnetic units.

It has been found that the electromagnetic unit of quantity is much larger than the electrostatic unit, the relation being—

1 C.G.S. electromagnetic unit of charge = 30,000,000,000 C.G.S. electrostatic units.

Since unit current transfers unit charge per second, the ratio of the units of current is expressed by the same number.

Unit pole and current are connected by the equation—

$$i \, . \, m = \frac{fd^2}{l}.$$

Putting f, d, l each equal to unity, we see that when i is large m is small, and vice versa. Thus the electromagnetic unit of m is $\frac{1}{3 \times 10^{10}}$ of the electrostatic unit. The relative magnitudes of the two units of each of the three principal quantities are therefore as below.

Quantity.	Electromagnetic.	Electrostatic.
Unit pole	1	30,000,000,000
„ current	30,000,000,000	1
„ charge	30,000,000,000	1

It may be shown from the definitions of permeability and permittivity that the equations for the forces between point poles and point charges for any medium are—

$$f = \frac{mm'}{\mu d^2} \text{ and } f = \frac{ee'}{\omega d^2}.$$

In the electromagnetic system, when the medium is air—

$$f = \frac{mm'}{d^2} \text{ and } f = \frac{ee'}{\omega_0 d^2},$$

where ω_0 is the permittivity of free ether (vacuum, or, practically, air) in electromagnetic units.

In the electrostatic system—

$$f = \frac{mm'}{\mu_0 d^2} \text{ and } f = \frac{ee'}{d^2},$$

where μ_0 is the permeability of free ether in electrostatic units.

Hence from the relative values of the two units we see that—

$$\mu_0 = \frac{1}{9 \times 10^{20}}, \quad \omega_0 = \frac{1}{9 \times 10^{20}}.$$

The permittivity of free ether in the electrostatic system and the permeability in the electromagnetic system are both necessarily unity, owing to the mode of defining unit charge and unit pole respectively.

It is shown in the electromagnetic theory of light that the velocity of propagation of a light wave in any medium should be given by the equation—

$$v = \sqrt{\frac{1}{\omega\mu}}.$$

Hence if electromagnetic units are adopted, $\mu_0 = 1$ for free ether, and v should be $1/\sqrt{\omega_0}$, or $\omega_0 = 1/v^2$. Now the best determinations of the velocity of light give $v = 3 \times 10^{10}$, which gives $\omega_0 = 1/(9 \times 10^{20})$—a result in perfect agreement with that obtained above. If electrostatic units are adopted, $\omega_0 = 1$ for free ether and $\mu_0 = 1/v^2 = 1/(9 \times 10^{20})$.

In media for which μ has practically the same value as for a vacuum, the index of refraction $n = \sqrt{\frac{\omega_1}{\omega_0}} = \sqrt{K}$—a result which for many substances has been confirmed experimentally.

415. Dimensions of Physical Quantities.

Length, force, and time may be regarded as primary sense-perceptions on which we base our reasoning respecting the mechanical properties of material bodies. But it is customary to substitute the idea of mass (derived from "force" and "acceleration") in place of force. Units of length, mass, and time are therefore adopted as the primary units in terms of which units of other quantities are defined. Thus a body has unit velocity when it traverses unit length in unit time; unit acceleration when one unit velocity is added per unit time. Unit force is such that it generates in unit mass a unit acceleration.

A formula indicating how a quantity involves other quantities in terms of which it is defined without reference to numerical relations is called a *dimensional formula.*

The dimensions of velocity are proportional directly to the distance traversed, and inversely proportional to the time taken. This is expressed by the formula—

$$[v] = \frac{[L]}{[T]}.$$

The brackets may be omitted when it is understood that dimensions only are referred to.

Acceleration is directly proportional to the additional velocity acquired, and inversely proportional to the time taken to acquire it. Hence the dimensions of acceleration are velocity ÷ time, or—

$$\frac{L}{T} \div T = \frac{L}{T^2}.$$

This is also written with a negative index thus, LT^{-2}. Acceleration is said to be of *one* dimension in length, and *negative two* in time.

Similarly we have—

$$\begin{aligned} &\text{Force} &&= \text{mass} \times \text{acceleration} &&= \mathrm{M \,.\, LT^{-2}}. \\ &\text{Work or energy} &&= \text{force} \times \text{length} &&= \mathrm{ML^2T^{-2}}. \\ &\text{Power or activity} &&= \text{work per unit time} &&= \mathrm{ML^2T^{-3}}. \end{aligned}$$

Area $= \mathrm{L^2}$. Volume $= \mathrm{L^3}$. Density $=$ mass per unit volume $= \mathrm{ML^{-3}}$.

Purely numerical multipliers and ratios are of no dimensions. Hence pure numerics, as π, do not enter into dimensional equations. Thus the equation—

$$\text{Area of a circle} = \pi r^2$$

is dimensionally

$$\mathrm{L^2} = \mathrm{L^2}.$$

The expression for the time of swing of a pendulum—

$$t = 2\pi\sqrt{\frac{l}{g}}$$

is dimensionally

$$\mathrm{T} = \sqrt{\frac{\text{length}}{\text{force per unit mass}}}$$

$$= \sqrt{\frac{\mathrm{L}}{\mathrm{MLT^{-2}} \div \mathrm{M}}} = \mathrm{T}.$$

Note.—Most equations relating to physical phenomena reduce to identities when expressed dimensionally. But observe—

(1) If an equation expressing any physical fact contains a quantity not defined independently of that fact, the equation amounts to a definition of the quantity in question. In this case both sides of the equation necessarily reduce to the same dimensions.

(2) If all the quantities involved are defined independently of the fact expressed, then the two sides of the equation may or may not reduce to the same dimensions. If all the spacial and dynamical processes which make the quantities interdependent have been taken into account, the dimensional equation becomes an identity.

416. Dimensions of Electrical Quantities.

In dealing with electrical quantities we consider the dimensions of their measures only—electromagnetic or electrostatic.

EXAMPLE 1.—To find the dimensions of electric charge on the electromagnetic system.

Referring to the diagram, we see that the order of definition is—

$$m \longrightarrow i \longrightarrow \mathrm{C} \longrightarrow e.$$

m. Formula for the measure of m is—

$$f = \frac{mm'}{d^2}. \quad \therefore \text{ with equal poles } m^2 = fd^2.$$

$$\text{Dimensional equation is } [m]^2 = \mathrm{MLT^{-2}L^2}$$
$$= \mathrm{ML^3T^{-2}}$$
$$[m] = \mathrm{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}}.$$

i. Formula is $f = \dfrac{ilm}{d^2}.$

$$\therefore i = \frac{fd^2}{lm}.$$

$$\text{Dimensional equation } [i] = \frac{\mathrm{MLT^{-2}\,.\,L^2}}{\mathrm{L\,.\,M\ L^{3}T^{-1}}}$$
$$= \mathrm{M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}}.$$

C. By assumption $[\mathrm{C}] = [i]$.

e. Formula is $\mathrm{C} = e \div t.$

$$\therefore e = \mathrm{C} \times t.$$

$$\text{Dimensional equation } [e] = \mathrm{M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1} \times T}$$
$$= \mathrm{M^{\frac{1}{2}}L^{\frac{1}{2}}}.$$

Thus the dimensions of electric charge are $\mathrm{M^{\frac{1}{2}}L^{\frac{1}{2}}}$ in the electromagnetic system.

EXAMPLE 2.—To find the dimensions of inductance in the electrostatic system.

Referring to the diagram, we see that the order of definition is—

$$e\text{—C—}i\text{—}m \begin{array}{c} \diagup \mathrm{B—P—N} \diagdown \\ \diagdown \mathrm{H—U—}i \diagup \end{array} \mathrm{L}.$$

The equations expressing measures and dimensions are as follows:—

e. $f = \dfrac{ee'}{d^2}. \quad \therefore [e^2] = [fd^2].$

$$[e] = \mathrm{M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1} \times L^2 = M^{\frac{1}{2}}L^{\frac{3}{2}}\,.\,T^{-1}}.$$

C. $\mathrm{C} = \dfrac{e}{t}.$

$$\therefore [\mathrm{C}] = \mathrm{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1} \div T = M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}}.$$

i. Assumed identical with C.

m. $f = \dfrac{ilm}{d^2}. \quad m = \dfrac{fd^2}{il}. \quad [m] = \dfrac{\mathrm{MLT^{-2} \times L^2}}{\mathrm{M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}L}} = \mathrm{M^{\frac{1}{2}}L}\,.$

H. $\mathrm{H} = \dfrac{f}{m}. \qquad [\mathrm{H}] = \dfrac{\mathrm{MLT^{-2}}}{\mathrm{M^{\frac{1}{2}}L^{\frac{1}{2}}}} = \mathrm{M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2}}.$

U. $U = H \times d$. $[U] = M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$.

i. Same dimensions as U. By Ampère's theorem $4\pi n \,.\, i = U$ and $4\pi n$ is a numeric. Last two values were unnecessary, but form a confirmation of the previous value [C].

B. $B = 4\pi \frac{m}{a}$. $[B] = M^{\frac{1}{2}}L^{\frac{1}{2}} \div L^2 = M^{\frac{1}{2}}L^{-\frac{3}{2}}$.

P. $P = B \times a$. $[P] = M^{\frac{1}{2}}L^{\frac{1}{2}}$.

N. $N = P \times n$, and n, the number of turns in a coil, is a mere numeric. Hence $[N] = [P] = M^{\frac{1}{2}}L^{\frac{1}{2}}$.

L. $L = \frac{N}{i}$. $\therefore [L] = M^{\frac{1}{2}}L^{\frac{1}{2}} \div M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2} = L^{-1}T^2$.

The process of derivation may be shortened by noticing that the following sets of quantities are of the same dimensions:—

m, P, and N; U and i; e or Q.

Dimensions of the other electrical and magnetic quantities may be obtained in the manner illustrated in the above examples.

Comparing the dimensions of a given quantity in the two systems we find that these differ. According to the nature of the quantity dealt with the ratio has always the dimensions of a velocity, the square of a velocity, or the reciprocal of one of these. This is exemplified in the following.

	Electrostatic Measure.	Electromagnetic Measure.	Ratio.
Charge...................	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}$	LT^{-1}
Pole.......................	$M^{\frac{1}{2}}L^{\frac{1}{2}}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	$L^{-1}T$
Current..................	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	LT^{-1}
Capacity................	L	$L^{-1}T^2$	L^2T^{-2}
Permittivity............	—	$L^{-2}T^2$	L^2T^{-2}
Permeability...........	$L^{-2}T^2$	—	$L^{-2}T^2$

The velocity is that of an electromagnetic wave travelling through free ether, or the velocity of light.

PRACTICAL UNITS

417. The units adopted in electrical practice are **all electromagnetic.** (The electrostatic system is only used for theo-

retical purposes as a rule.) For a few quantities the C.G.S. electromagnetic units are adopted. The remaining quantities are expressed in terms of special units which are multiples of the C.G.S. units.

(1) Practical units belonging to the *C.G.S. system.*

Quantity.	Name of Unit.
Magnetic pole	—
" force	Gauss
" induction	Gauss or "line"
" flux	Maxwell

In the case of resistance and E.M.F. the C.G.S. units are exceedingly small compared with the quantities ordinarily measured. To avoid the use of very large numbers units are chosen—the ohm and the volt—whose magnitudes are respectively 10^9 and 10^8 times those of the C.G.S. units. The ohm, volt, and other units derived from them may be said to belong to a system in which the unit of length is 1 earth-quadrant ($= 10^9$ cm.), the unit of mass $\frac{1}{10^{11}}$ gram, and the unit of time 1 second. We may term this for brevity the "quadrant" system.

(2) Practical units belonging to the *Quadrant system.*

Quantity.	Name of Unit.	Relation to C.G.S. Unit.
Resistance	Ohm	$= 10^9$ C.G.S.
Electromotive force (and potential-difference)	Volt	$= 10^8$ "
Current	Ampere	$= \frac{1}{10}$ "
Quantity (charge)	Coulomb	$= \frac{1}{10}$ "
Capacity	Farad	$= \frac{1}{10^9}$ "
Linkage	—	$= 10^8$ "
Inductance	Henry	$= 10^9$ "
Work (or Energy)	Joule	$= 10^7$ "
Activity (or Power)	Watt	$= 10^7$ "

If an equation only involves quantities which are included in the quadrant system, the equation will hold when the quantities are expressed in practical units or C.G.S. units. Thus the equation $W = \frac{1}{2}SV^2$ will hold (without the introduction of any other factor) when W is in joules, S in farads, V in volts.

But if the equation involves quantities from both the C.G.S. and quadrant systems of electromagnetic units, a power of 10 is necessary as a factor. Thus for tangent galvanometer,

$C = H \cdot \frac{r}{2\pi n} \cdot \tan \delta$, when all quantities are in C.G.S. units,

$C = 10 \frac{r}{2\pi n} H \tan \delta$, when C is in amperes, and H, r, in C.G.S. units.

All the units of the quadrant system are of a convenient magnitude with the exception of the unit of capacity. The farad is a much larger capacity than any met with in practice, and a secondary unit, the microfarad, is therefore adopted in actual measurements.

Secondary units of the other quantities distinguished by the prefixes *milli* (= one thousandth), *kilo* (= one thousand), *micro* (= one millionth), *mega* (= one million) are also adopted when convenient.

(3) *Mixed Units.*—Some quantities are expressed in terms of units derived partly from the quadrant system and partly from the C.G.S. system. Thus electric force may be expressed in kilovolts per centimetre.

Energy is frequently expressed in watt-hours, or in Board of Trade units (= 1000 watt-hours).

Activity is frequently expressed in terms of horse-power.

418. **Remarks on Practical Units.**

Electrical measurement may be said to have begun with Ohm's enunciation of the law of conduction, but the units of resistance and E.M.F. used were quite arbitrary, some portion of the circuit or a length of copper wire being chosen as a standard of reference. One unit of this kind (introduced

about 1860), much used on the Continent, was Siemens's unit—represented by a column of mercury 1 sq. mm. in cross-section and 1 metre long at 0° C. It was afterwards discovered that resistance could be measured independently of all other electrical quantities, and could be expressed in terms of a velocity; the C.G.S. unit being therefore equivalent to a velocity of 1 cm. per second. Actual measurement showed that the Siemens unit was very nearly 10^9 C.G.S. units, the value being $\cdot 98 \times 10^9$. It was therefore considered advisable to adopt a new unit which should be exactly 10^9 C.G.S. units. This was accordingly done, the new unit being termed the Ohm. A determination of the ohm in terms of a mercury column was first made by the British Association in 1864, and since that time many other determinations have been made. As a result of these the legal ohm is defined thus:

"The ohm, which has the value 10^9 in terms of the centimetre and second of time, is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14·4521 grammes in mass, of a constant cross-sectional area, and of length 106·3 centimetres."

Again, the E.M.F. of a Daniell cell was found to be slightly greater than 10^8 C.G.S. units, and it was therefore decided to adopt a potential-difference of exactly 10^8 C.G.S. units as the standard of P.D. and E.M.F. This was termed the volt.

From these practical units of R and V the units of C and the other quantities in the quadrant system can be derived. For example, since the ohm $= 10^9$ C.G.S. and the volt $= 10^8$ C.G.S., the unit of current (the ampere) $= \frac{1}{10}$ C.G.S. unit.

Determination of Standards.—Although the units of resistance and E.M.F. are those on which the *definition* of the other units of the quadrant system depends, this order is not suitable for the accurate determination of material standards, since a method of direct determination of E.M.F. in terms of dynamical units of the requisite accuracy has not yet been devised. At present our material standards depend on determinations

of **resistance** and **current.** The following are two important examples:—

Lorenz's Determination of Resistance.—Using the rotating disc experiment described in Art. 345, and balancing the E.M.F. generated against the P.D. at the ends of the resistance to be measured (on the potentiometer principle), the resistance was determined in terms of velocity.

Lord Rayleigh's Determination of Current.—Using a current balance to determine the attraction between two coils, the value of the current was determined in terms of the intensity of gravity and the dimensions of the apparatus, but independently of all other electrical and magnetic quantities. To preserve the result in terms of a material standard the value of the current was utilized to determine the electrochemical equivalent of silver. The silver voltameter was thus established as a secondary standard.

Current and resistance standards having been obtained, the other quantities can be measured. Thus the E.M.F. of a Clark cell is determined with a standard resistance and silver voltameter, using a potentiometric method.

QUESTION

What are the practical units of quantity, current, resistance E.M.F., capacity, energy, and power? Give a table showing the connections between these units and their values in C.G.S. absolute electromagnetic units. (1905.)

CHAPTER XXXIII

TERRESTRIAL MAGNETISM—ATMOSPHERIC ELECTRICITY

419. We have seen that the earth is surrounded by a magnetic field which can be imagined to come from a large magnet situated at the earth's centre and making a small angle with the earth's axis of rotation. The earth's magnetic

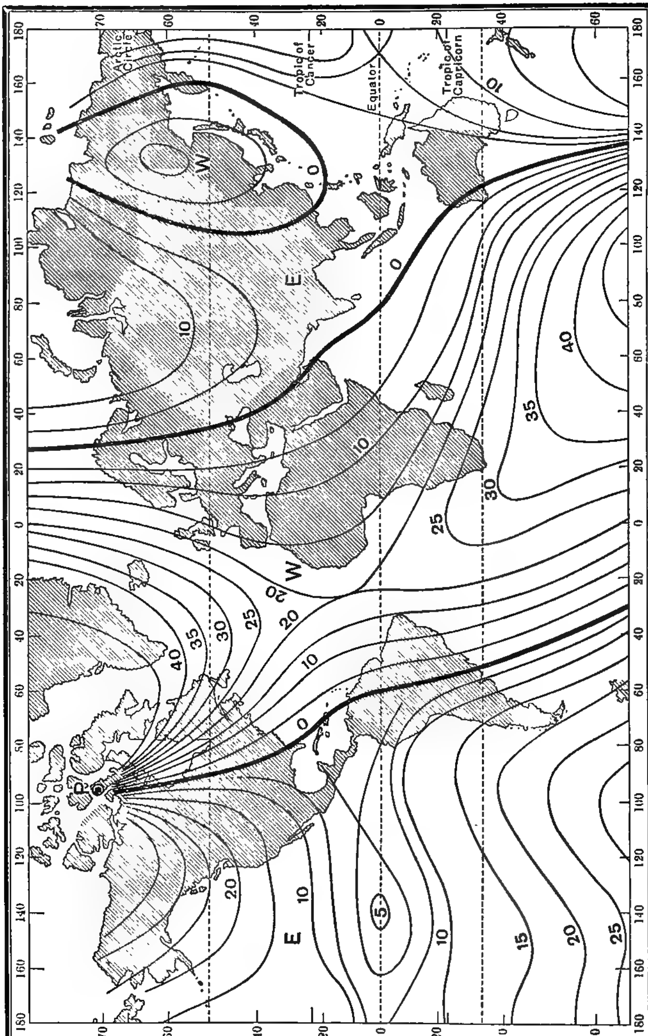


Fig. 248.—Isogonic Lines

field at any place is completely specified by the three magnetic elements—

(1) The declination (θ);
(2) The dip, or inclination (δ);
(3) The horizontal magnetic intensity (H).

The vertical and total intensities can be calculated from H and δ. The methods of determining the magnetic elements, for laboratory purposes, have already been given.

420. Magnetic Surveys.

A complete knowledge of the earth's magnetic field can only be obtained by extensive surveys. In survey work the same general principles of measurement as are used in laboratory determinations are applied. For details of the instruments used at sea or in field work the student is referred to special treatises. We must here confine ourselves to a consideration of the principal results obtained. The variations in the magnetic elements over the earth's surface are conveniently represented by charts.

Declination.—Charts of declination may be constructed by drawing lines on the map to represent at each point the direction of the magnetic meridian. The differences of declination are, however, better shown by marking on the map lines which join places of equal declination. These are termed **isogonic lines.** They must not be confused with magnetic meridian lines. The general form of the isogonic lines is shown in fig. 248. The particular lines which join places of no declination are termed *agonic lines.* These are shown by the bolder lines on the map. One of the agonic lines is a closed curve, and is termed from its location the *Siberian oval.* It will be seen that the agonic lines divide the map into two regions of easterly and two of westerly declination.

Dip.—The variations in magnetic dip are represented similarly by drawing lines through all places having the same dip. These loci, termed **isoclinic lines,** follow the general directions of the parallels of latitude (fig. 249). The line joining

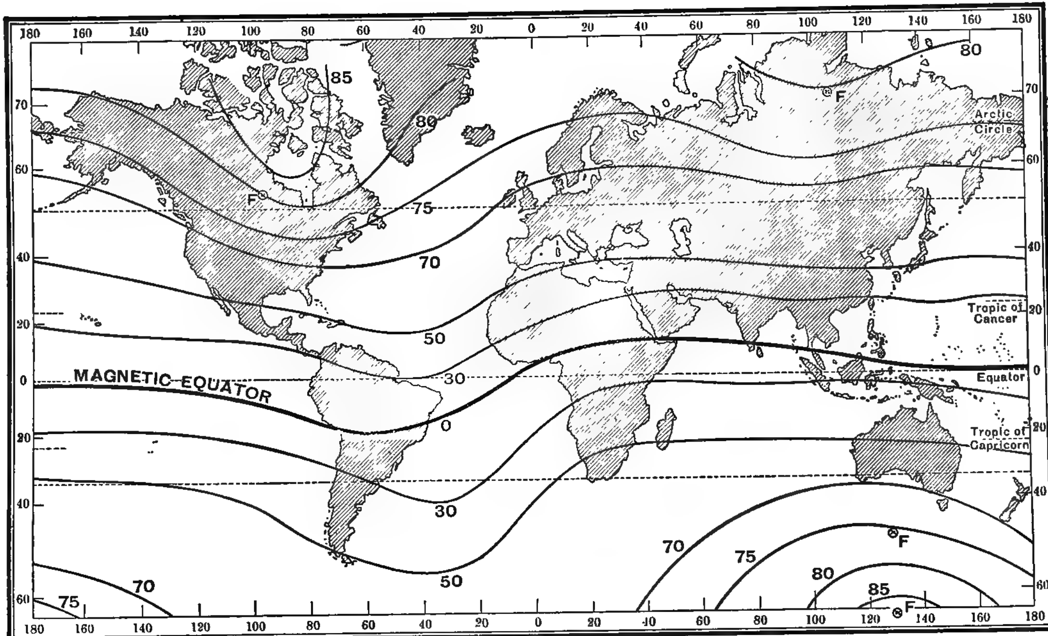


Fig. 249.—Isoclinic Lines

places where the dip is 0° is termed the *aclinic line* or *magnetic equator*. One portion of this line lies north of the geographical equator, and the other portion south of it. The two points on the earth's surface where the dip is 90° are termed the *magnetic poles.*

The northern magnetic pole is located in Boothia, about 70° 5′ N. latitude and 96° 40′ W. longitude. The southern magnetic pole has not yet been reached, but is estimated to be in the region 73° S. latitude and 143° E. longitude.

Horizontal Force.—The variations of horizontal force are represented in fig. 251. Lines joining places where the horizontal force is the same are termed **isodynamic lines.**

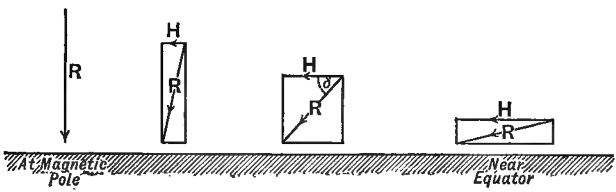


Fig. 250

These lines well show the complicated character of the distribution of the earth's magnetic force. It will be observed that the horizontal force decreases towards the magnetic poles. The reason for this will be evident from fig. 250. Near the poles the total force R is large, but owing to the large dip there is only a small horizontal component. This component vanishes at the magnetic poles, and is greatest near the magnetic equator; but it will be seen from a comparison of the maps that the magnetic equator itself is not a line of maximum horizontal force.

It must also be noticed that the total force does not reach a maximum value at the magnetic poles. It is strongest at four places, where its direction is not quite vertical. These places are termed the *magnetic foci* of the earth. Their positions are shown at F on the map (fig. 249). The northern foci are widely separated (American focus 52° N. latitude, 90° W.

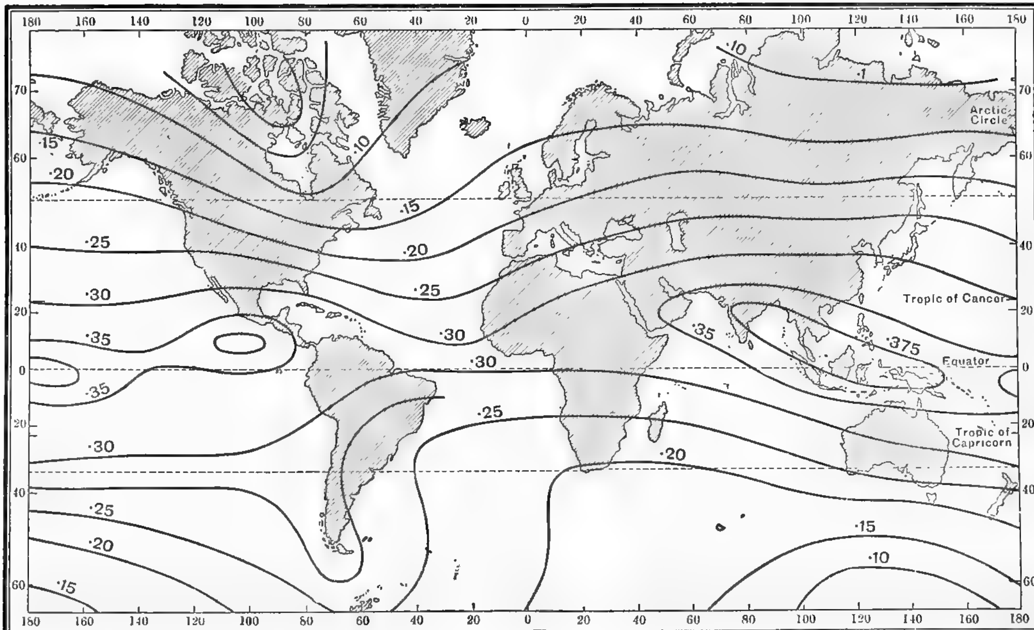


Fig. 251.—Isodynamic Lines

longitude; Siberian focus 70° N. latitude, 115° E. longitude). The southern foci are comparatively close together and are situated south of Australia.

In Navigation Charts the declination is indicated by small diagrams of the compass-needle. These are marked on the map at a sufficient number of points, but closer together near the coast lines.

Secular Variations.—Records of magnetic observations show that the elements undergo slow changes within certain limits. The following table gives the declination and dip in London for various epochs.

Date.	Declination.	Dip.
1580.........	11° E.	—
1600.........	—	72°
1657.........	0°.	—
1700.........	9° W.	—
1820.........	(Max.) 24° 30′ W.	70°
1900.........	16° 53′ W.	67°

These variations indicate that the magnetic poles of the earth revolve round the geographical poles, the period of revolution being nearly 500 years.

At the present time the declination in England is decreasing, the rate being about 7 minutes of arc yearly. The dip changes at a slower rate, decreasing at present about 1 minute of arc yearly.

421. Continuous Records.

Diurnal and Annual Variations.—Besides the very slow changes termed secular, there are very small and periodic variations. These are detected by the sensitive recording instruments used in magnetic observatories.

For observations of declination a set of small magnets is attached to a mirror suspended as in a deflection magnetometer. A beam of light is directed on the mirror, and is reflected so

that it is brought to a focus on a drum covered with sensitized (photographic) paper. The drum is kept in steady rotation by clockwork, so that a smooth line is recorded on the paper if the magnet remains stationary. A deflection of the magnet causes the spot of light to deflect parallel to the axis of the drum. Variations in deflection are thus recorded in the irregular curve traced out.

Changes in the horizontal force are determined in a similar manner, but the magnet has a bifilar suspension, and the suspension points are adjusted so that the magnet lies (normally) magnetic E. and W. By this means the effect of changes in the *direction* of the horizontal field is made negligible, and the magnet is only affected by a change in the *magnitude* of H.

The variations in vertical intensity are recorded by the use of a light steel magnet supported on agate knife-edges like the beam of an ordinary balance, and weighted so that normally it remains horizontal. A mirror is attached to the magnet, and a photographic record is obtained.

The daily range or *diurnal* variation in declination varies with the season of the year; it amounts to a few minutes of arc only. The diurnal variation in dip is about one minute of arc. The horizontal force varies also, the changes being of the order ·0002 gauss. The changes in dip and total force cannot be conveniently observed directly; they are calculated from the changes in V and H.

The mean daily value of the elements undergoes a slow variation having a period of one year. This is called the *annual* variation.

Magnetic Storms.—A sudden and irregular oscillation of the suspended magnets is sometimes observed. These "magnetic storms" are most frequent during displays of the Aurora Borealis. It has also been shown that there is some connection between solar activity (indicated by sun-spots) and terrestrial magnetic disturbances.

ATMOSPHERIC ELECTRICITY

422. Lightning.

The identity of the effects produced by a lightning flash with those due to an electric spark discharge puts the electrical origin of lightning almost beyond doubt. The disruptive, heating, magnetic, and physiological effects are all represented. The electric discharge which produces the lightning may take place between neighbouring clouds or between a cloud and the earth. In the latter case the charge on the cloud induces a charge of opposite kind on the surface of the earth. Franklin showed by means of his famous kite experiment that electricity could be drawn from a storm-cloud.

Photographs of lightning flashes have been obtained. These do not show the traditional zigzag shape of the discharge path. The line of discharge breaks up into twigs and branches. Photographs taken on the cinematograph principle show that a series of discharges occurs in rapid succession in a single "flash". It is possible therefore that the discharge may be oscillatory. The duration of the discharge is usually an exceedingly small fraction of a second.

Thunder owes its duration to several causes. The sound from the earth end of a flash reaches the observer earlier than that from the cloud end, and the branching character of the discharge will cause the sound to vary in intensity during the interval. Echoes among the clouds impart a rumbling character to the sound. The cause of "sheet" or "summer" lightning, which is unaccompanied by thunder, has not yet been determined.

It has been found by experience that prominent buildings may be protected to a large extent from the disruptive discharge by the use of lightning conductors. These are usually copper rods running the full height of the building, and ending in spikes at the top. The bottom end of the rod must be kept in good connection with moist earth. The conductor must be kept clear of all metallic pipes, rods, etc., used on the build-

ing. These should be connected separately to earth. To be as efficient as possible the conductor should be made several square centimetres in cross-sectional area, and should take the form of a flat band to offer as large a surface as practicable. On account of the suddenness of the discharge the conduction is confined to the "skin" of the conductor.

The sharp spikes at the top of the conductor discharge electricity into the atmosphere of opposite kind to that of the inducing cloud. The earth thus loses its induced charge, and the tendency to a disruptive discharge is diminished.

423. Normal Electrification of the Atmosphere.

The air is not free from electrification even under ordinary conditions. In fine weather the potential at a point in the air is, as a rule, higher than that of the earth. The best way of demonstrating this condition of the atmosphere, and subjecting it to measurement, is by the *water-dropping collector*, invented by Lord Kelvin.

A copper vessel is provided with a long horizontal jet. The vessel is filled with water, and this is allowed to escape in drops from the extremity of the jet. The whole arrangement is insulated and connected to the *needle* of an electrometer. The quadrants are maintained at a constant difference of potential by means of a battery. Hence the deflection of the electrometer indicates the potential of the jet. If the electric field induces any charge on the extremity of the jet, this is carried off by the water drops, so that in the course of a few minutes the *tip* of the jet is uncharged. It is then at the same potential as the surrounding air, and this is indicated by the deflection of the electrometer. (The mean potential of the quadrants must be made zero by earthing the centre of the battery. See formula, Art. 194.) For field observations a portable electrometer may be substituted for the quadrant instrument, and a flame may be used to replace the water jet. Lord Kelvin used blotting-paper impregnated with lead nitrate and rolled into sticks. This forms a slow-burning match, the fumes from which carry off any induced charge on the tip.

Results.—Experiments made in this way show that under ordinary circumstances and in fine weather the potential in

the air is positive with respect to the earth. But in wet weather the potential may be negative. The potential gradient is very variable, but may be placed normally at 75 to 150 volts per metre. The electric field is directed downwards, so that the earth's surface is negatively charged, the corresponding positive charge being in the atmosphere. Equipotential surfaces are horizontal planes over open districts, but over hills, buildings, etc., the potential gradient is steeper and the equipotential surfaces are nearer together. This is to be expected, since the surface charge of the earth will naturally collect more densely on these projecting portions.

Earth Currents.—Telegraphic lines which are connected to earth at points a considerable distance apart are sometimes traversed by currents of considerable strength, owing to a difference in potential at the earth-plates. They are most frequent in lines running north and south. The cause is possibly atmospheric.

The causes of atmospheric electricity and terrestrial magnetism have not yet been determined.

424. **The Aurora Borealis.**

As seen in the northern hemisphere the aurora consists of a pale luminous arch, sometimes of a delicate crimson tint, situated in the direction of the northern magnetic pole. It is frequently intersected by radial streamers, and often presents a flickering appearance. The aurora is seldom seen except in high latitudes. A similar phenomenon, the *aurora australis*, occurs in the antarctic regions.

It is probable that the aurora consists of an electric discharge in the upper regions of the atmosphere surrounding the pole. It presents a bright-line spectrum similar to that produced by ordinary vacuum tubes, but lines are present which do not correspond to those of any known element.

QUESTIONS

1. What is an isoclinal line? Give a general account of the arrangement of the isoclinal lines in a magnetic map of the earth's surface. (1905.)

2. Describe the water-dropping collector, and explain its action and the purpose to which it may be applied. (1902.)

3. What is an *isogonal line*? Describe the general form of the isogonals over the surface of the earth. How are the observations made which are used in determining the isogonals? (1906.)

APPENDIX

1. Proportions and Equations.

Most experiments by which physical laws are proved establish only *proportional* relations between the quantities measured. To convert these proportions into equations we must introduce "constant numerical coefficients or factors".

Thus if $a \propto b$,

then $a = (\text{a constant}) \times b$.

A simple example will illustrate this point. When we ascend a flight of steps the height of ascent is *proportional* to the number of steps. In symbols—

$$h \propto n.$$

We cannot, however, say that the height is *equal* to the number of steps. But if the height of each step is known, say 9 inches, then we may write—

$$h = 9 \times n.$$

Thus the introduction of a numerical multiplier converts the proportion into an equation. If h is in feet, then—

$$h = \tfrac{3}{4}n.$$

When a proportion is converted into an equation **the value of the constant depends on the units** in which the various quantities are expressed.

2. Useful Approximations.

By the binomial theorem (for this see works on Algebra) we have—

$$\frac{1}{a+x} = \frac{1}{a}\left(1 - \frac{x}{a} + \frac{x^2}{a^2} - \ldots\ldots\ldots\right).$$

If x is very small compared with a, x^2/a^2 and higher powers may be neglected. Hence—

$$\frac{1}{a+x} = \frac{1}{a}\left(1 - \frac{x}{a}\right) \text{ approx.} \quad \ldots\ldots\ldots\ldots(1)$$

Similarly we may show that when $\frac{x}{a}$ is very small—

$$\frac{1}{(a+x)^2} = \frac{1}{a^2}\left(1 - \frac{2x}{a}\right), \quad \ldots\ldots\ldots\ldots(2)$$

$$\frac{1}{\sqrt{(a+x)}} = \frac{1}{\sqrt{a}}\left(1 - \frac{x}{2a}\right). \quad \ldots\ldots\ldots\ldots(3)$$

3. **Mean Value of Force along a Short Path.**

The magnetic (or electric) force due to a unit pole (or unit charge) at a distance d is—

$$f_1 = \frac{1}{d^2}.$$

At a slightly greater distance $(d + x)$ the force is—

$$f_2 = \frac{1}{(d+x)^2}.$$

The mean or average value f is half the sum of these.

$$\therefore f = \frac{f_1 + f_2}{2}$$

$$= \frac{1}{2}\left\{\frac{1}{d^2} + \frac{1}{(d+x)^2}\right\} = \frac{1}{2}\,\frac{d^2 + 2dx + x^2 + d^2}{d^2(d+x)^2}.$$

Since x is very small (in comparison with d) its square may be neglected. Hence—

$$f = \frac{1}{2}\,\frac{2d^2 + 2dx}{d^2(d+x)^2} = \frac{d(d+x)}{d^2(d+x)^2} = \frac{1}{d(d+x)} \quad \ldots\ldots(4)$$

Or, the mean value of the force is the reciprocal of the *product* of the two nearly equal distances.

4. Solid Angles.

The circular measure of a *plane* angle is obtained by drawing a circle with the vertex of the angle as centre, and taking the ratio—

$$\frac{\text{Arc subtended}}{\text{Radius}}.$$

A *solid* angle is the tapering space included by the faces of a pyramid, or by the curved surface of a cone. Solid angles are expressed in spherical measure. Let a number of spheres be described with the vertex of the cone (or pyramid) as

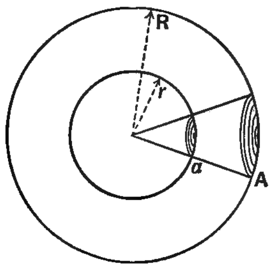


Fig. 252

centre. A portion of the surface of each sphere is included within the cone. The ratio—

$$\frac{\text{Area included}}{\text{Square of the radius}},$$

is the same for each sphere (fig. 252). This constant ratio is taken as the measure of the solid angle (ω).

Thus, $$\omega = \frac{a}{r^2} = \frac{A}{R^2} = \text{etc.} \quad\ldots\ldots\ldots\ldots\ldots(5)$$

If the cone opens out until its surface becomes flat the surface included is a hemisphere.

$$\therefore \left.\begin{array}{l}\text{the solid angle subtended}\\ \text{at the centre of a sphere}\\ \text{by the hemisphere}\end{array}\right\} = \frac{\frac{1}{2}(4\pi r^2)}{r^2} = 2\pi \quad\ldots\ldots(6)$$

If the portion of the sphere included is greater than a hemisphere (as when the cone turns inside out), the solid angle subtended becomes greater than 2π; and in the limiting case we have—

Solid angle corresponding to the whole surface of a sphere

$$= \frac{(4\pi r^2)}{r^2} = 4\pi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Small Solid Angles.

If a solid angle is very narrow its value may be expressed very approximately in terms of the area of an oblique section. If an oblique section, of area a, is made at a distance r from the vertex,

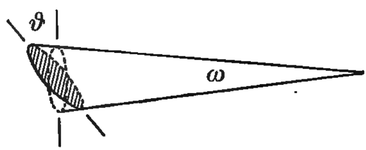


Fig. 253

the area of the normal section at the same mean distance is $a \cos \theta$, where θ is the angle between the planes of the two sections (fig. 253).

$$\therefore \omega = \frac{a \cos \theta}{r^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

5. Law of Inverse Squares.

The following deductions from the law are important:—

Gauss's Theorem.—Let a pole (strength m) be situated at P (fig. 254). Draw round P a *closed* surface of any shape. Take any *very small* area AB on the surface, and let ω_1 be the solid angle which this area subtends at P. Let r be the distance of Q (the mid-point of AB) from P. Let CQD be the portion of a sphere, drawn with P as centre and PQ as radius, included within the solid angle.

Then, if F is the resultant magnetic force at Q due to m—

$$F = \frac{m}{r^2}.$$

Let F_1 be the component of this at right angles to the surface AB.

$$F_1 = F \cos \theta$$
$$= \frac{m}{r^2} \cos \theta,$$

where θ is the angle between the places AB and CD.

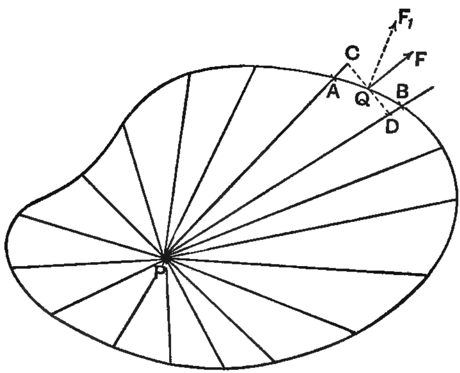


Fig. 254

Now since AB is a *small* area, F_1 has sensibly the same value at all points of this area. Multiplying F_1 by the area over which it is constant, we obtain $F_1 a_1$.

But $$F_1 a_1 = \frac{m a_1}{r^2} \cos \theta$$
$$= m\omega_1 \text{ [by (8)]} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

We may fill up the whole space round P with small solid angles ω_2, ω_3, etc., and these will be subtended by small areas a_2, a_3, etc. If F_2, F_3, etc., are the *components at right angles to the surface* (in directions taken *outwards* from the surface) of the magnetic force at the points considered, then—

$$F_2 a_2 = m\omega_2.$$
$$F_3 a_3 = m\omega_3, \text{ etc.}$$

Hence by addition—

$$F_1a_1 + F_2a_2 + F_3a_3 + \ldots\ldots + F_na_n$$
$$= m(\omega_1 + \omega_2 + \omega_3 + \ldots\ldots + \omega_n).$$

But the sum of the solid angles for a closed surface is the solid angle representing a complete sphere, namely 4π.

$$\therefore\ F_1a_1 + F_2a_2 + \ldots\ldots + F_na_n = m_1 \times 4\pi.$$

If there are other poles m_2, m_3, etc., inside the surface similar expressions hold for these. Hence if F_1, F_2, etc., represent the total components at right angles to the surface—

$$F_1a_1 + F_2a_2 + \ldots\ldots + F_na_n = 4\pi \times (\text{total quantity of magnetism included within the surface}).$$

The left-hand member is called the "total normal outward flux". Hence—

(Total normal flux) = 4π × (total quantity of magnetism included)..............................(10)

This relation is termed "Gauss's Theorem". It must be observed that—

(*a*) The surface may be of any shape, but must be completely closed.

(*b*) Outward flux is considered; an actual inward flux is reckoned as a negative outward one.

(*c*) The quantity of magnetism must be reckoned algebraically, S magnetism being taken as negative.

(*d*) The outward flux due to magnetic poles *outside* the surface considered is zero. The inward flux across one part of the surface is balanced by an outward flux across another part.

(*e*) If there is no magnetism within the surface the outward flux is zero.

The theorem applies also to the flux of force due to a system of electric charges, e being substituted for m in the above expression.

We have assumed that the medium is air.

Deductions from Gauss's Theorem.

(1) **The flux of force is constant at all parts of a tube of force.**

Let the surface chosen be a portion of a narrow tube of force, bounded by two plane sections a_1, a_2. Let F_1, F_2 be the magnetic (or electric) forces over these plane sections. There is no component force at right angles to the curved surface of the tube. The total outward flux is due to the ends, and, since no charge is included, this flux, by Gauss's Theorem, is zero.

$$\therefore \; F_1a_1 - F_2a_2 = 0,$$

or

$$F_1a_1 = F_2a_2, \qquad (11)$$

which proves the proposition.

(2) **The force close to a plane area carrying a uniform layer of magnetism** (or electric charge) **and due to this layer is $2\pi\sigma$.**

Let the surface chosen be a flat cylinder having its plane ends parallel to the surface and on opposite sides of it. By symmetry it is evident that there is no flux across the curved surface. The fluxes across the ends are both outwards (or both inwards). Thus—

$$\begin{aligned} Fa + Fa &= 4\pi m \\ &= 4\pi a\sigma \\ \therefore \; F &= 2\pi\sigma \qquad (12) \end{aligned}$$

It must be noticed that this is not necessarily the actual force; it is merely the component due to the charge considered.

6. Current Variation.

When a current is increasing or decreasing in strength there is a temporary self-induced E.M.F. due to the change in the number of tubes of induction linked with the cir-

cuit. If C is the current at any instant, the linkage is given by—

$$N = LC \qquad (L = \text{inductance.})$$

$$\text{The induced E.M.F.} = -\frac{dN}{dt}$$

$$= -\frac{d(LC)}{dt} = -L\frac{dC}{dt},$$

if L is constant.

Thus, if the impressed (or battery) E.M.F. is E, the *effective* E.M.F. is—

$$E - L\frac{dC}{dt}.$$

Hence by Ohm's Law—

$$CR = E - L\frac{dC}{dt},$$

or
$$E = L\frac{dC}{dt} + CR \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

This is the characteristic equation for a circuit of negligible capacity.

7. Transient Discharge.

The quantity of electricity discharged inductively round a circuit may be calculated thus:

Let E′ be the induced E.M.F. at any instant, N the number of linkages at the same instant. Then—

$$E' = \frac{dN}{dt} \quad \text{(numerically).}$$

If C′ is the instantaneous value of the *induced* current—

$$C' = \frac{E'}{R} = \frac{1}{R}\frac{dN}{dt};$$

$$\therefore C'dt = \frac{dN}{R},$$

or
$$dq = \frac{1}{R}dN.$$

Hence by integration—

$$Q = \frac{N}{R} \quad \text{(14)}$$

Division of Discharge in Parallel Branches.

Let V be the potential-difference between the junctions of the two branches at any instant. Then, if at the same instant the currents in the branches are C_1, C_2 respectively; and if R_1, R_2 are the respective resistances, and L_1, L_2 the inductances of the branches—

$$\left.\begin{aligned} V &= C_1R_1 + L_1\frac{dC_1}{dt} \\ V &= C_2R_2 + L_2\frac{dC_2}{dt} \end{aligned}\right\} \text{ by (13).}$$

$$\therefore\ C_1dtR_1 + L_1dC_1 = C_2dtR_2 + L_2dC_2,$$

or $$R_1\int C_1dt + L_1\int dC_1 = R_2\int C_2dt + L_2\int dC_2.$$

Now $\int dC_1$ and $\int dC_2$ vanish since the current returns to its initial value after the inductive discharge.

$$\therefore\ R_1\int C_1dt = R_2\int C_2dt,$$

$$R_1\int dq_1 = R_2\int dq_2,$$

$$R_1Q_1 = R_2Q_2,$$

$$\frac{Q_1}{Q_2} = \frac{R_2}{R_1} \quad \text{(15)}$$

Thus the *quantities* discharged are inversely proportional to the resistances of the branches.

8. Data for Induction Curves.

The following are typical values of B, from which the normal curves of induction for three important varieties of iron may be drawn[1]:—

[1] From *Modern Electric Practice.*

H.	Cast Iron.	Wrought Iron.	"Dynamo" Cast Steel.	H.	Cast Iron.	Wrought Iron.	"Dynamo" Cast Steel.
5	1900	9,000	8,150	45	6450	16,300	16,900
10	3000	12,200	12,100	50	6700	16,500	17,140
15	3900	13,600	14,000	60	7150	16,800	17,450
20	4550	14,450	15,000	70	7530	17,000	17,750
25	5100	15,050	15,700	80	7900	17,220	18,000
30	5500	15,500	16,200	90	8250	17,400	18,200
35	5870	15,800	16,500	100	8570	17,580	18,400
40	6180	16,100	16,750	125	9200	17,930	18,900

ANSWERS TO EXERCISES

CHAPTER III (p. 59)

1. Unaltered. **3.** 17 : 24. **4.** 360°.

CHAPTER IV (p. 79)

4. 64 : 125. **6.** 30°. **8.** ·0048, ·0024, ·00134 gauss, $\phi = 26° \, 34'$.

CHAPTER V (p. 96)

2. 3,771,000 dyne.

CHAPTER VII (p. 133)

3. Without leakage 1681, with leakage 1930.

CHAPTER IX (p. 167)

3. $\frac{1}{2}$ mg. **7.** 121·24. **8.** ·1 dyne per unit charge.

CHAPTER XI (p. 201)

1. 500 ergs, against electric forces. **5.** 2·553 ergs per unit charge.

CHAPTER XII (p. 224)

2. 5 units. **4.** (*a*) First density : second density = 1 : 2; (*b*) 4 : 7.
5. 1 : 5 in each case. **7.** 1 : $2\frac{2}{3}$.

CHAPTER XIII (p. 248)

1. 84 : 325.
2. K = 3.
3. From first to second.
4. In air condenser.
5. Before 33, after, 5.
6. ·272.
7. 11, 5, 2.
8. 4 : 9. **9.** 6·77.
11. 636·5 centimetres
12. 200.
13. (*a*) Condenser : shell = 1 : 2.
(*b*) r less than $\frac{1}{2}$ R.
14. Charges are 60, −60, 300/11, 360/11, −360/11, 120/11, 240/11, −240/11, 0. Potentials are 105/11, 50/11, 20/11, 0.
15. First jar : P.D. = E, charge = EF. Second jar : if battery insulated, charge and P.D. negligible; if middle of battery earthed, P.D. = E/2, charge = EF/2.
16. A : B = 2 : 1. **17.** 4 : 1 : 1.

CHAPTER XIV (p. 262)

2. 35 units. **3.** Energy = F, P.D. = 2F/Q.

CHAPTER XVI (p. 305)

7. (*a*) V reduced in ratio 4 : 3; (*b*) 12·5 cm.

CHAPTER XIX (p. 352)

2. In series. **3.** $1\frac{1}{3}$ amp., V = $\frac{2}{3}$, $V_E = 3\frac{1}{6}$.

CHAPTER XX (p. 372)

1. ·034 C.G.S., or 34 amp. **3.** (*a*) In smaller; (*b*) 30°. **6.** ·14 gauss. **7.** 2·45 ohms, 117·6 ohms. **8.** 600/3, 1400/3, 4000/3.

CHAPTER XXI (p. 387)

2. 1/5 gauss.

CHAPTER XXII (p. 401)

1. 2×10^9 ergs. **2.** 2 : 1. **3.** 10/11, 1/11 of battery current; $H_1 : H_2$ = 10 : 1. **4.** 1·66 amp.

CHAPTER XXIV (p. 444)

4. 11·75 c.c. **6.** Increased in ratio 1 : $\sqrt{2}$.

CHAPTER XXV (p. 455)

1. ·0032 volt or 3·2 millivolts.

CHAPTER XXVI (p. 477)

6. ·00005938 volt or 59·38 microvolts.

CHAPTER XXVII (p. 491)

1. New P.D. = 3 volts. **3.** 89 volts. **4.** $\frac{7}{253}$. **5.** $\frac{1}{3}$ ampere flows in path ADBCA; no current in AB nor in CD when pos. poles are to A and B.

CHAPTER XXVIII (p. 509)

1. 288 amp. **2.** (*a*) 950 watts; (*b*) 100 amp.; (*c*) 95 % : 50 %. **4.** 89 %; 90·1 %.

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2. $w = \pi r^2 HC = 31{\cdot}42$ ergs. **3.** $F = \frac{1}{5}\pi \times$(amp. per unit width).

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